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Increased Robust Stability of Nonlinear Systems Based on Hyperbolic Umbilic Catastrophe Theory

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Abstract*—this paper presents an approach to assuring robustness of a dynamic system. Its primary focus is on robust stability, not robust performance, in the presence of parameter variations and model uncertainties. The proposed control scheme makes use of a catastrophe function with three control factors and two behavior axes. Based on our analysis, the paper presents the conditions for converging to equilibrium points and explains how to implement a controller to achieve robust stability. As a practical case study, design of a control system for a robotic manipulator and the corresponding numerical simulations are presented.

Index Terms—**catastrophe function, equilibrium, robust stability, robust control.**

I. INTRODUCTION

In a number of applications where a system is characterized by nonlinear and time-varying behavior the issue of stability and performance becomes prominent. It is further complicated by the fact that when a mathematical model is used as part of the control strategy, it is typically assumed to closely match the real process. However, the process-model mismatch often results in sub-standard performance or even stability issues. To alleviate this, adaptive or robust control approaches can be employed. In this paper, we concentrate on improving stability characteristics of a dynamic system with uncertain parameters, which can also be subjected to time variations. An example of such a system is a mechanical manipulator that is typically a highly nonlinear system prone to changes in dynamic performance with different payloads and different operating conditions. As such, it requires a special control strategy that assures stability over a range of parameter variations. Some traditional robust control solutions are reported in [1], [2] and [3]. Our approach described in the following sections is different and it is based on catastrophe functions [4].

II. ROBUST STABILITY DISCUSSION

Robust stability is discussed in many previously published papers. However, many of them address either linear systems or nonlinear systems with specific constraints. Successful results were reported when catastrophe theory [5], [6] was employed to achieve robust stability of control systems with uncertain parameters. For example, in [7] it was proven that cusp catastrophe can be used to assure stability of a rock slope and the range of parameter variations was extended from 0.9 (for traditional controllers) to 1.1. However, this approach is based on only one of the known catastrophe functions and the range of stability was not very large. Similarly, in [8] the same function was used to analyze crowd jam in public buildings and it was demonstrated that catastrophe theory is a preferred method for obtaining critical equilibrium points that do not depend on the initial conditions. An example of a nonlinear control system considered in [9] shows that, without time-delay, all solutions asymptotically approach zero. When the method of Lyapunov functions is applied, it is possible to demonstrate that asymptotic stability of zero solutions can be achieved in time-delay systems as well. The results reported in [10] and [11] are of particular interest, where increased robustness based on catastrophe theory lead to structurally stable systems. In particular, [10] presents both analysis and synthesis steps of the process. However, linear systems are still emphasized and most of the non-linear systems with uncertainties are considered only as special cases. In addition, the examples discussed in these papers demonstrate stability when some system parameters (or their ratios) are negative and the systems become unstable when the values are positive. This limits the range of robustness. Therefore, one of our goals is to build structurally stable control systems that remain stable in spite of wider variation of both parameters of the plant and the controller.

II. PROPOSED CONTROL SYSTEM

Consider the following nonlinear state equation

$$\dot{x} = f(x(t), u(t)) \tag{1}$$

Our further discussion will concentrate on a model given by equation (1) and linearized around a certain operating point. Let our linearization result in the controllable canonical form

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{2}$$

Variation of parameters in equation (2) can be expected within certain limits due to both nonlinear nature of the process and its time-varying characteristics; therefore, a controller that stabilizes the system over a broad range of variations of parameters $a_1(t)$ and $a_2(t)$ is highly desired.

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Our approach to designing a control system with robust stability is based on catastrophe theory, which is a branch of bifurcation theory. Out of seven standard forms of functions [12] we propose the hyperbolic umbilic catastrophe [13], which has three control factors and two behavior axes. Combined with a second-order plant, our proposed controlled system applied to a linearized version given by (2) assumes the following form:

$$\frac{dx_1}{dt}=x_2$$
$$\frac{dx_2}{dt}=-a_1x_1-a_2x_2+x_2+ \quad (3)$$
$$+[-x_1^3-x_2^3-k_1x_1x_2+k_2x_2+k_3x_1]$$

The hyperbolic umbilic catastrophe, shown in the brackets in the above equation, has control factors k_1, k_2, and k_3 and behavior axes x_1 and x_2. We chose this form of control function, as opposed to linear feedback control, because it provides more degrees of freedom, more tuning options, and greater potential to improve stability of the system, as will be shown below. Other forms of nonlinear controls can also be considered; however, hyperbolic umbilic catastrophe combines the above advantages with relative simplicity, even when compared to traditional linear controllers that are not very suitable for assuring robust stability.
Equilibrium points of the above system can be found by solving the following algebraic equations [14]:

$$-x_{1S}^3-x_{2S}^3-k_1x_{1S}x_{2S}+(k_2-a_2)x_{2S}+(k_3-a_1)x_{1S}=0,$$
$$x_{2S}=0 \quad (4)$$

A trivial solution to the above equations yields

$$x_{1S,1}=0,\ x_{2S,1}=0 \quad (5)$$

And two other solutions can be obtained from

$$-x_{1S}^3+(k_3-a_1)x_{1S}=0, \quad (6)$$
$$x_{2S}=0 \quad (7)$$

The resultant equilibrium points are found as follows:

$$x_{1S}=+\sqrt{k_3-a_1},\ x_{2S}=0 \quad (8)$$
$$x_{1S}=-\sqrt{k_3-a_1},\ x_{2S}=0 \quad (9)$$

We can use the original nonlinear function *f(x(t))* given by (1), put it in the linearized form, combine with our proposed control system, as shown by (3) and then perform Taylor series expansion:

$$\frac{dx_1}{dt}=x_2$$
$$\frac{dx_2}{dt}=-3(x_{1S})^2x_1-k_1x_{2S}x_1+(k_3-a_1)x_1- \quad (10)$$
$$-3x_{1S}x_1^2-x_1^3-3(x_{2S})^2x_2-k_1x_{1S}x_2+$$
$$+(k_2-a_2)x_2-3x_{2S}x_2^2-x_2^3$$

The same equation in the matrix-vector form is given as follows:

$$\frac{dx}{dt}=Gx+h(x(t)) \quad (11)$$

When performed around the equilibrium point given by (5), the expansion (10) reduces to the following form:

$$\frac{dx_1}{dt}=x_2$$
$$\frac{dx_2}{dt}=(k_3-a_1)x_1+(k_2-a_2)x_2-x_1^3-x_2^3 \quad (12)$$

where matrix G of the linearized portion and a nonlinear term *h(x)* are

$$G=\begin{bmatrix}0 & 1\\ k_3-a_1 & k_2-a_2\end{bmatrix} \quad (13)$$

and

$$h(x(t))=\begin{bmatrix}0\\ -x_1^3-x_2^3\end{bmatrix}, \quad (14)$$

respectively. Performing Taylor series expansion around the equilibrium point given by (8) results in

$$\frac{dx_1}{dt}=x_2$$
$$\frac{dx_2}{dt}=-2(k_3-a_1)x_1+\left(k_2-a_2-k_1\sqrt{k_3-a_1}\right)x_2- \quad (15)$$
$$-3\sqrt{k_3-a_1}x_1^2-x_1^3-x_2^3$$

The corresponding linear and nonlinear parts of the system are

$$G=\begin{bmatrix}0 & 1\\ -2(k_3-a_1) & k_2-a_2-k_1\sqrt{k_3-a_1}\end{bmatrix} \quad (16)$$

$$h(x(t))=-3\sqrt{k_3-a_1}x_1^2-x_1^3-x_2^3 \quad (17)$$

Finally, when using the equilibrium point given by (9), the resultant state equation can be written as follows

$$\frac{dx_1}{dt}=x_2$$
$$\frac{dx_2}{dt}=-2(k_3-a_1)x_1+\left(k_2-a_2+k_1\sqrt{k_3-a_1}\right)x_2+ \quad (18)$$
$$+3\sqrt{k_3-a_1}x_1^2-x_1^3-x_2^3$$

and the individual components are

$$G=\begin{bmatrix}0 & 1\\ -2(k_3-a_1) & k_2-a_2+k_1\sqrt{k_3-a_1}\end{bmatrix} \quad (19)$$

and

$$h(x(t))=3\sqrt{k_3-a_1}x_1^2-x_1^3-x_2^3 \quad (20)$$

It can be observed that the equilibrium points given by equations (8) and (9) converge to (5) when $k_3 =$ and diverge from it when $k_3 \neq$. It is also expected that, since the original system is nonlinear and time-varying,

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coefficients $a_0(t)$ and $a_1(t)$ in its linearized model are going to change within a certain range. This can be accounted for in our proposed controller by proper selection of its parameters. Then the problem of designing a system with robust stability can be formulated as a problem of finding a solution that would not be susceptible to those variations in model parameters.

III. ROBUST STABILITY ANALYSIS

The kind of robustness that we are trying to achieve can be formulated as the ability of our system to remain stable in spite of perturbations of its parameters a_i and convergence to one of the equilibrium points, as long as some of the conditions discussed in this section are met. We can analyze the equilibrium conditions derived in (5), (8) and (9) to determine if the linearized system is stable at those points. This can be achieved by finding the Eigen values of the matrix G. For example, it can be demonstrated that stability of the system given by (12) at point (5) is guaranteed if

$$Re\left\{\frac{k_2-a_2}{2} \pm \sqrt{\frac{(k_2-a_2)^2}{4} + (k_3-a_1)}\right\} < 0 \tag{21}$$

At the same time, stability conditions for equilibrium points (8) and (9) derived from (16) and (19), respectively, are:

$$\mathrm{Re}\left\{-\frac{k_1\sqrt{k_3-a_1}-(k_2-a_2)}{2} \pm \pm\sqrt{\frac{\left(k_1\sqrt{k_3-a_1}-(k_2-a_2)\right)^2}{4} - 2(k_3-a_1)}\right\} < 0 \tag{22}$$

and

$$Re\left\{\frac{k_1\sqrt{k_3-a_1}+(k_2-a_2)}{2} \pm \pm\sqrt{\frac{\left(k_1\sqrt{k_3-a_1}+(k_2-a_2)\right)^2}{4} - 2(k_3-a_1)}\right\} < 0 \tag{23}$$

Finding reasonable values of control factors $k_1 - k_3$ that satisfy the above conditions for a practical range of time-varying parameters a_1 and a_2 is relatively simple. However, since the original system is nonlinear, additional stability analysis should be performed. We can utilize Lyapunov's first method to determine system behavior around the equilibrium points [15]. As per this method, the equilibrium point x_e=0 is stable if for any Γ>0 there exists γ>0, such that when $|x(0)| < \gamma$, then $|x(t)| < \Gamma$ for all $t \geq 0$. In addition, if for $|x(0)| < \gamma x(t) \to 0$ as $t \to \infty$, then x_e is asymptotically stable [15]. Even if the equilibrium point is not zero, there is no loss of generality, since a shifted system of coordinates can be assumed. Consider the following description of a nonlinear system

$$\frac{dx(t)}{dt} = G_N\, x(t), \tag{24}$$

Where

$$G_N = \left.\frac{\partial f(x(t))}{\partial x}\right|_{x_e}, \tag{25}$$

which corresponds to one of the nonlinear models given by (12), (15) or (18). A solution to any of those systems of nonlinear differential equations can be expressed in the general form

$$x(t) = e^{G(t-t_0)}x_0 + \int_{t_0}^{t} e^{G(t-\tau)}h(x(\tau))d\tau \tag{26}$$

The following is also true as can be seen from (26)

$$|(x(t))| \le \left|e^{G(t-t_0)}\right||x_0| + \int_{t_0}^{t}\left|e^{G(t-\tau)}\right||h(x(\tau))|d\tau. \tag{27}$$

Then, in order to achieve asymptotic stability of natural motion of a nonlinear system defined by matrix G, it is necessary to have real positive numbers k and δ, such that $\left|e^{Gt}\right| \le ke^{-\delta t}$ for any t. Hence,

$$|(x(t))| \le ke^{-\delta(t-t_0)}|x_0| + k\int_{t_0}^{t}\left|e^{-\delta(t-\tau)}\right||h(x(\tau))|d\tau. \tag{28}$$

Depending on the expression for *h(x(t))*, (28) can be in one of the following forms:

a) For $h(x(t)) = -x_1^3 - x_2^3$

$$|x_1(t)| \le ke^{-\delta(t-t_0)}|x_{10}| + +k\int_{t_0}^{t}\left|e^{-\delta(t-t_0)}\right||-x_1^3(\tau)|d\tau \tag{29}$$

b) For $h(x(t)) = -3\sqrt{k_3-a_1}x_1^2 - x_1^3 - x_2^3$

$$|x_1(t)| \le ke^{-\delta(t-t_0)}|x_{10}| + +k\int_{t_0}^{t}\left|e^{-\delta(t-t_0)}\right|\left|-3\sqrt{k_3-a_1}x_1^2(\tau) - x_1^3(\tau)\right|d\tau \tag{30}$$

c) For $h(x(t)) = 3\sqrt{k_3-a_1}x_1^2 - x_1^3 - x_2^3$

$$|x_1(t)| \le ke^{-\delta(t-t_0)}|x_{10}| + +k\int_{t_0}^{t}\left|e^{-\delta(t-t_0)}\right|\left|3\sqrt{k_3-a_1}x_1^2(\tau) - x_1^3(\tau)\right|d\tau \tag{31}$$

It can also be demonstrated that for a given ε>0 there exists σ>0, such that

$$\left|-x_1^3(t)\right| \le \frac{\varepsilon}{k}|x_1(t)|; \quad \left|-x_1^2\right| \le \frac{\varepsilon}{k}|x_1(t)| \tag{32}$$

It should be noted that the above condition does not hold globally, but only when $|x_1(t)| \le \sigma$; however, when dealing with practical systems, including robotic manipulators, this condition is often inherent. The following expressions can be obtained for our three equilibrium points:

a) $$e^{\delta(t-t_0)}|x_1(t)| \le k|x_{10}| + \varepsilon\int_{t_0}^{t} e^{\delta(t-t_0)}|x_1(\tau)|d\tau \tag{33}$$

b) $$e^{\delta(t-t_0)}|x_1(t)| \le \le k|x_{10}| + \varepsilon(3\sqrt{k_3-a_1}+1)\int_{t_0}^{t} e^{\delta(t-t_0)}|x_1(\tau)|d\tau \tag{34}$$

c) $$e^{\delta(t-t_0)}|x_1(t)| \le k|x_{10}| + \varepsilon\left(-3\sqrt{k_3-a_1}+1\right) * *\int_{t_0}^{t} e^{\delta(t-t_0)}|x_1(\tau)|d\tau \tag{35}$$

Application of Bellman-Gronwall inequality [16], [17] to (33)-(35) results in

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a) $e^{\delta(t-t_0)}|x_1(t)| \leq ke^{\varepsilon(t-t_0)}|x_{10}|$ (36)
b) $e^{\delta(t-t_0)}|x_1(t)| \leq ke^{\varepsilon(3\sqrt{k_3-a_1}+1)(t-t_0)}|x_{10}|$ (37)
c) $e^{\delta(t-t_0)}|x_1(t)| \leq ke^{\varepsilon(-3\sqrt{k_3-a_1}+1)(t-t_0)}|x_{10}|$ (38)

The above can be further modified as follows:

a) $|x_1(t)| \leq ke^{-(\delta-\varepsilon)(t-t_0)}|x_{10}|$ (39)
b) $|x_1(t)| \leq ke^{-\{\delta-\varepsilon[3\sqrt{k_3-a_1}+1](t-t_0)\}}|x_{10}|$ (40)
c) $|x_1(t)| \leq ke^{-\{\delta-\varepsilon[-3\sqrt{k_3-a_1}+1](t-t_0)\}}|x_{10}|$ (41)

If ε is selected such that

a) $\varepsilon < \sigma$ (42)
b) $\varepsilon[3\sqrt{k_3-a_1}+1] < \sigma$ (43)
c) $\varepsilon[-3\sqrt{k_3-a_1}+1] < \sigma$ (44)

Then $|x_1(t)| \leq k|x_{10}|$ if $|x_1(t)| \leq \sigma$. Therefore, if $|x_{10}| \leq \sigma/k$, then

a) $|x_1(t)| \leq \sigma e^{-(\delta-\varepsilon)(t-t_0)}|x_{10}|$
for any $t \geq t_0$ (45)
b) $|x_1(t)| \leq \sigma e^{-\{\delta-\varepsilon[3\sqrt{k_3-a_1}+1](t-t_0)\}}|x_{10}|$
for any $t \geq t_0$ (46)
c) $|x_1(t)| \leq \sigma e^{-\{\delta-\varepsilon[-3\sqrt{k_3-a_1}+1](t-t_0)\}}|x_{10}|$
for any $t \geq t_0$ (47)

The results presented by (45)-(47) prove asymptotic stability of the original nonlinear system that utilizes a controller designed in the form of hyperbolic umbilic catastrophe. Our analysis also indicates that the equilibrium point given by (5) is asymptotically stable if $(k_2 - a_2) <$ and $(k_3 - a_1) <$, and it becomes unstable when $(k_2 - a_2) >$ and $(k_3 - a_1) >$. However, the equilibrium points given by (8) and (9) are feasible when $(k_3 - a_1) >$, and they can also be asymptotically stable when conditions given by equations (22) and (23) are satisfied. The set of conditions derived in this section can be used to select proper values of controller parameters. It appears that control factors k_2 and k_3 are crucial for robust stability assurance, and control factor k_1 can be used to further improve dynamic characteristics of the system.

IV. CASE STUDY

As a practical case, we can select a mechanical system whose operation can be described by a set of nonlinear relationships. For example, dynamics of a robotic manipulator with n joints can be presented by a second-order differential equation given in the matrix-vector form as follows [18]:

$$M(\theta)\ddot{\theta} + N(\theta,\dot{\theta}) + G(\theta) + H(\dot{\theta}) = U \quad (48)$$

where $\theta(t)$ is the vector of joint positions, $U(t)$ is the vector of applied joint torques, M is the inertia matrix, $N(\theta,\dot{\theta})$ is the Coriolis and centrifugal torque vector, $G(\theta)$ is the gravity loading vector, and $H(\dot{\theta})$ is the frictional torque vector. Each element in M, N, G and H is a nonlinear function that describes manipulator dynamics where all joints are interconnected by "coupling torques" [19]. Therefore, the model can be represented by a set of n second-order nonlinear scalar differential equations in the following form:

$$m_{ii}(\theta)\ddot{\theta}_i + \sum_{\substack{j=1 \\ i \neq j}}^{n} m_{ij}(\theta)\ddot{\theta}_j + n_i(\theta,\dot{\theta}) + g_i(\theta) + (\dot{\theta}) = u_i \quad (49)$$

By introducing a state vector

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \quad (50)$$

equation (49) can be represented as a nonlinear state equation (with subscript i omitted for simplicity) in the form given by (1) Since the state variables in vector x represent position and velocity of manipulator joints, our linearization results in the controllable canonical form given by equation (2), hence all previous analysis still applies.

V. SIMULATION RESULTS

Robust stability conditions derived in the previous section can be demonstrated by numerical simulations. The following figure shows a Simulink block diagram of the system with a feedback controller based on a hyperbolic umbilic catastrophe function.

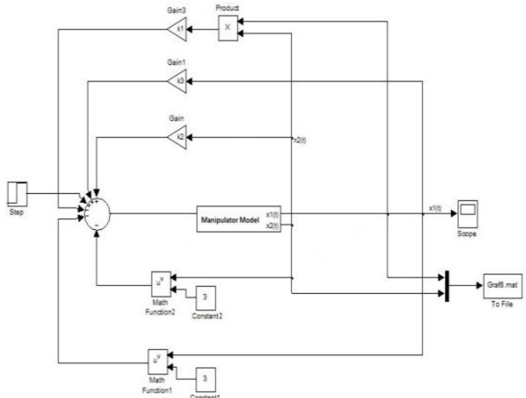

Fig 1. Simulation diagram of a control system with robust stability characteristics

Since we are only concerned with robust stability and do not consider robust performance, the input to the above system should be set to zero and only natural motion to an equilibrium point from arbitrary initial conditions is considered [20]. The model of the manipulator and the controller are set in such a way that, in the linearized form, the following parameters of the system are used in four different numerical simulations:

a) k_1=2,(k_2- a_2)=-3, (k_3- a_1)=-1,
b) k_1=2,(k_2- a_2)=3, (k_3- a_1)=1, (unstable),
c) k_1=-2,(k_2- a_2)=3, (k_3- a_1)=5,

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d) k_1=2,(k_2- a_2)=-3, (k_3- a_1)=5,
e) k_1=2,(k_2- a_2)=3, (k_3- a_1)=5.

The first set of parameters results in an asymptotically stable system whose convergence process is shown in Fig. 2. It can be clearly seen that the equilibrium point in this case is given by equation (5).

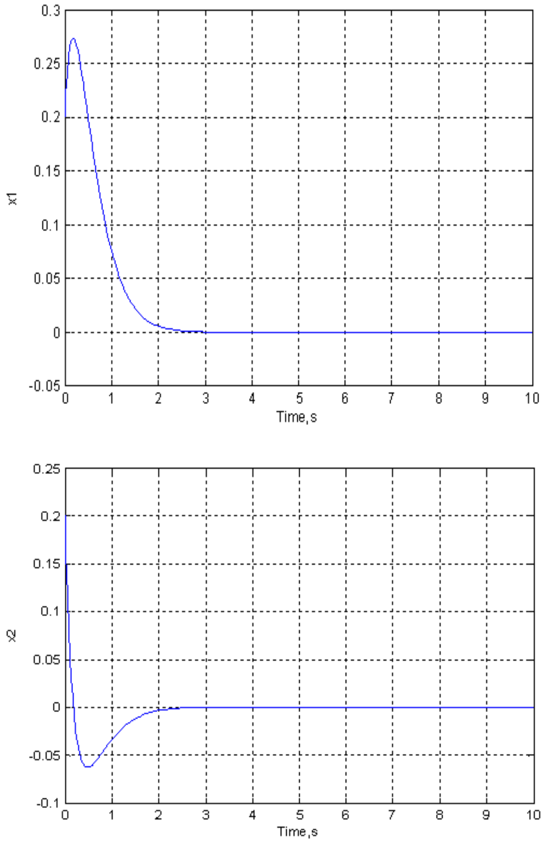
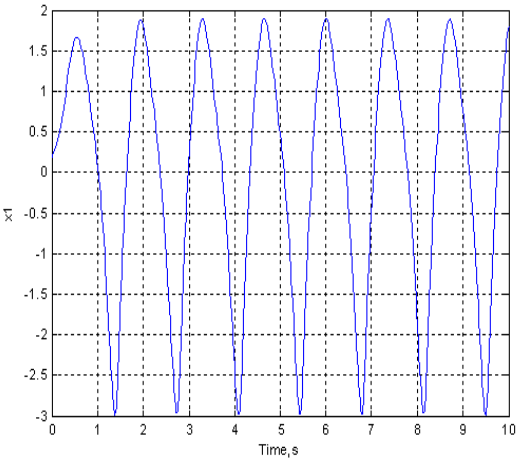


Fig 2. Numerical simulation results: experiment 1

If both (k_2- a_2) and (k_3- a_1) are made positive, as in case *b)*, the system loses its stability as shown in Fig. 3.



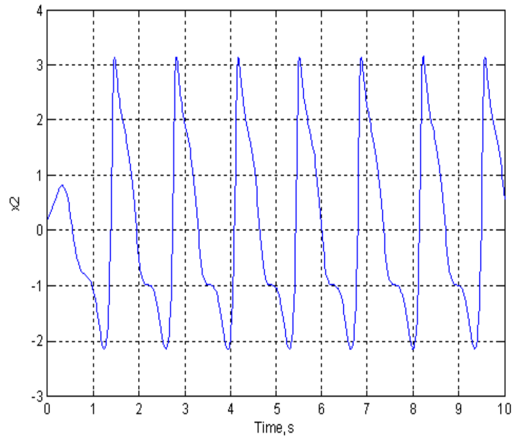


Fig 3. Numerical simulation results: experiment 2

The waveforms presented in Fig. 4 – Fig. 6 show stable performance for a different set of parameters and converge to equilibrium points given by (8) or (9).

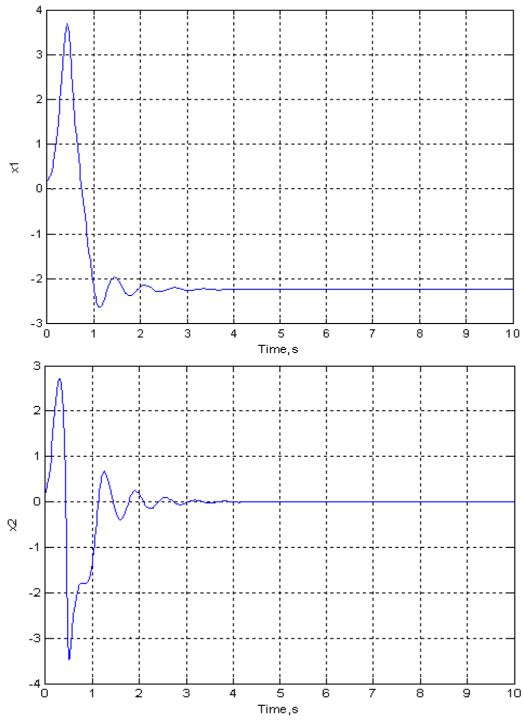
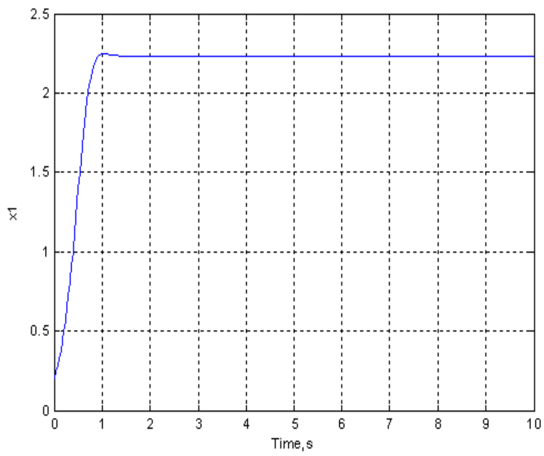


Fig 4. Numerical simulation results: experiment 3



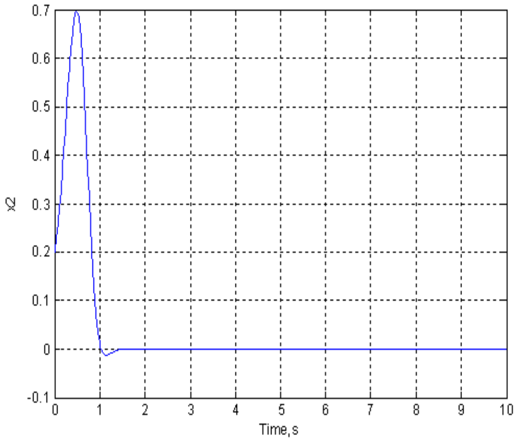


Fig 5. Numerical simulation results: experiment 4

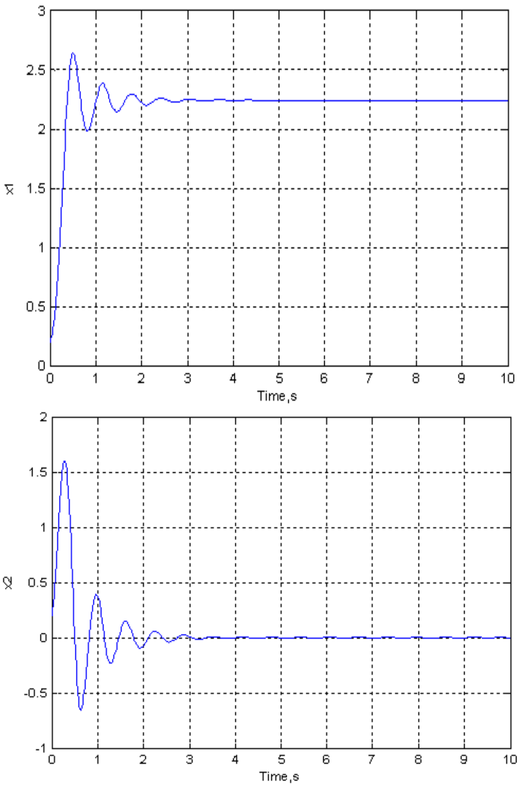


Fig 6. Numerical simulation results: experiment 5

The waveforms in Fig. 4 and Fig. 6 are of particular interest, since they demonstrate stability when conditions given by (22) and (23) are satisfied. The results presented above illustrate that it is possible for the system to converge to an equilibrium point even if dynamics of the controlled process suddenly changes. This is assured by having proper values of control factors (primarily k_2 and k_3), as was discussed in the previous sections. In addition, the remaining control factor k_1 can be used to improve dynamic performance of the convergence process, but this property is not considered within the scope of this paper.

VI. CONCLUSION

This paper discusses an approach to increasing robust stability of a dynamic system with uncertain model parameters. Our topology is based on a controller in the form of a hyperbolic umbilic catastrophe function with three control factors, which can be applied to a nonlinear dynamic system described by a set of second-order nonlinear differential equations. Conditions for convergence to one of the equilibrium points are derived and the results of numerical simulations indicate that this approach is suitable for assuring robust stability of a system over a significant range of parameter variations. One of the main advantages of our approach is that it can guarantee stability for both positive and negative values of the system parameters that were previously demonstrated in traditional linear controllers to work only when having negative values, such as in negative feedback cases. This feature of our system increases its range of robust stability. Furthermore, traditional analysis of robustness relies on finding a single parameter that assures stability of the system, but it often requires intense numerical calculations. Our proposed approach based on catastrophe theory relies on finding several parameters, but using simple calculations. Finally, our approach offers several equilibrium points unlike many traditional techniques that consider only one.

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