ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 10, April 2014

The Effect of Black Body Radiation and Electron Inertia on the Jeans Instability of Rotating and Magnetized Gaseous Plasma of Interstellar Medium

R.K.Pensia, V.Prajapat, V.Kumar, G.S.Kachhawa, D.L.Suthar
Department of Physics, Govt. Girls College, Neemuch
Department of Physics, Pacific University, Udaipur
Department of Physics, Mewar University, Chitorgarh

Abstract:-The effect of black-body radiation and electron inertia on the Jeans instability of rotating and magnetized gaseous plasma of interstellar medium has been studied with the help of relevant MHD equations using normal mode analysis. Rotation is taken parallel and perpendicular to the magnetic field for both, the longitudinal and transverse modes of propagation. The jeans criterion of instability is modified to give the stabilizing effect of radiation pressure. The stabilizing effect of magnetic field is observed only of transverse mode of propagation where as finite election inertia destabilizing effect. The rotation stabilizes only along the magnetic field for transverse mode. The stabilizing effect of rotation is comparatively more effect.

Key words: **Jeans wave number, Interstellar medium (ISM) Magneto hydro dynamics (MHD) and Finite electron inertia.**

I. INTRODUCTION

The gravitational instability is one of the fundamental concepts of modern astrophysical hot plasma gas cloud and it is connected with the fragmentation of interstellar matter in regard to star formation. James Jeans [1] first studied this instability problem and suggested that an infinite homogeneous self-gravitating fluid is unstable for all wave number which is less than critical Jeans wave number. In terms of the wave length of a fluctuation, Jeans criterion says that λ should be greater than a critical value $\lambda_j = \sqrt{\frac{\pi C^2}{G\rho}}$ which is named the Jeans length. In this formula G is the gravitational constant, ρ is the unperturbed matter density and $C = \sqrt{\frac{K_B}{m}}$ is the sound speed for adiabatic perturbation. K_B is the Boltzmann's constant, T is the physical temperature and m is the mass of the particle. In this connection Jeans[2] discussed the conditions under which a fluid becomes gravitational unstable under the action of its own gravity. Nowadays, for any relevant length scales in the Universe [Stars, galaxies, asteroid, planetary rings, clusters, etc]; such as instability has been recognized as the key mechanism to explain the gravitational formation of structure and their evolution in the linear regime. The Jeans problem has been extensively investigated under varying assumptions. A comprehensive account of these investigations has been given by Chandra sekhar[3] in his monograph on hydrodynamic and hydro magnetic stability. The problem of an isothermal gas sphere subjected to external pressure has been studied by Ebert[4] and he found that disturbances of length scale approximately equal to the Jeans length based on the central density were unstable to gravitational collapse. Since $\lambda_j \propto \rho^{-1/2}$ and Jeans mass $M \propto \rho\lambda_j^3 \propto \rho^{-1/2}$; this considerably reduced the minimum unstable mass and demonstrated that O star could form in the centre of an interstellar cloud. Hunter[5] studied the growth of perturbations in a gravitationally contracting isothermal gas cloud and he found that perturbations with initial scale of the order of or less than the Jeans length grew less rapidly relative to the back ground density than did perturbation of substantially larger dimension. In this connection, many investigators have discussed the Jeans instability of homogeneous plasma considering the effects of various parameters [6-13] In addition to these magnetic fields can provide pressure support and inhibit the contraction and fragmentation of interstellar clouds. The magnetic field interacts directly only with the ions, electrons and charged grains in the gas collision of the ions with the predominately neutral gas in clouds are responsible for the indirect coupling of the magnetic field to the bulk of the gas. Langer [14] demonstrated that the degree to which the magnetic pressure is important and depends upon the field strength and the fractional abundance. In the interstellar medium (ISM), a large amount of energy is injected by the stars, which leads to the formation of shock waves; but when these shock waves weaken, they become large amplitude hydro magnetic Alfven waves. In this connection many investigators have discussed the contribution of the magnetic field in the ISM [15-18]. In the interior of hot and large plasma clouds regions, where the temperature is rather high while the density is low, it is important to take into account the radiative processes. In the recent ISM observations, it has been established as a fact that the radiative processes plays an important role in the star formation and molecular cloud condensation process in connection with thermal instability. The ISM structure

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 10, April 2014

shows that the radiative processes are the major cause for the condensation of large astrophysical compact objects. Taking the radiative processes in consideration, Vranjes and Cadez[19] and Vranjes[20] have investigated the influence of radiative processes on gravitational instability. Inutsuka et al.[21] have studied the propagation of shock waves into a warm neutral medium taking into account radiative heating and cooling, thermal conduction and viscosity terms. Thus the aim of the present paper is to study the effect of radiation pressure on the Jeans self-gravitational instability of a homogeneous hot plasma gas with finite election inertia and rotation.

II. EQUATIONS OF THE PROBLEM

The MHD equations of infinite, homogeneous, viscous, uniformly magnetized rotating plasma a cloud incorporating the radiative processes and finite electron inertia may be given as follows as:

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p + \vec{\nabla}\phi + \frac{1}{4\pi\rho}\left(\vec{\nabla}\times\vec{H}\right)\times\vec{H} + 2\left(\vec{v}\times\vec{\Omega}\right) \quad (1)$$

$$\frac{D\rho}{Dt} = -\rho\vec{\nabla}.\vec{v} \quad (2)$$

$$\frac{\partial\vec{H}}{\partial t} = \vec{\nabla}\times\left(\vec{v}\times\vec{H}\right) + \frac{C^2}{4\pi\omega_{pe}^2}\nabla^2 h\partial t \quad (3)$$

$$\nabla^2\phi = -4\pi G\rho \quad (4)$$

$$\frac{dT}{T} = (\Gamma-1)\frac{d\rho}{\rho} \quad (5)$$

Operator $\frac{D}{Dt}$ is substantial derivative given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}.\vec{\nabla} \quad (6)$$

Where $\rho, \vec{v}, p, \phi, \vec{H}(0,0,H), G, T, \vartheta$ and $\vec{\Omega}(\Omega_x, 0, \Omega_v)$ are denote the density, velocity, pressure gravitational potential magnetic field universal gravitational constant, temperature and kinetic viscosity and rotation respectively ω_{pe} is the electron plasma frequency.

For an enclosure containing matter and radiation, Chandrasekhar(16) has defined Γ as The gas pressure

$$\Gamma = 1 + \frac{\Gamma_1 - b}{4 - 3b} \quad (7)$$

Pressure $\Gamma_1 = \frac{b + (4-3b)^2(\gamma-1)}{b + 12(\gamma-1)(1-b)}$ (8)

$$b = \frac{p_g}{p_g + p_r} \quad (9)$$

The gas pressure $p_g = \rho RT$, the radiation pressure $p_r = \frac{1}{3}\alpha_R T^4$

the total pressure $p = p_g + p_s$

III. LINEARZIED PERTURBATION EQUATIONS

In the unperturbed state the fluid is assumed to be at rest. The field variables in perturbed state may be taken as $\rho + \delta\rho, \vec{v}, p + \delta p, \phi + \delta\phi, \vec{H} + \vec{h}, T + \delta T$. After linearization equation (1) to (5) become as

$$\frac{\partial\vec{v}}{\partial t} + RT\left(\frac{\vec{\nabla}\delta\rho}{\rho}\right) + R(1 + 4R_p)\vec{\nabla}\delta T - \vec{\nabla}\delta\phi - \frac{\left(\vec{\nabla}\times\vec{h}\right)\times\vec{H}}{4\pi\rho} - 2\left(\vec{v}\times\vec{\Omega}\right) - \vartheta\nabla^2\vec{v} = 0 \quad (10)$$

$$\frac{\partial\delta\rho}{\delta t} + \rho\left(\vec{\nabla}.\vec{v}\right) = 0 \quad (11)$$

$$\frac{\partial\vec{h}}{\partial t} + \vec{\nabla}\times\left(\vec{v}\times\vec{H}\right) + \frac{1}{4\pi\omega^2 p_e}\frac{\partial}{\partial t}\nabla^2 h = 0 \quad (12)$$

$$\nabla^2\delta\phi + 4\pi\phi G\delta\rho = 0 \quad (13)$$

$$\frac{\partial\delta T}{\partial t} - (\Gamma-1)\frac{T}{\rho}\frac{\partial\delta\rho}{\partial t} = 0 \quad (14)$$

Γ is assumed to be constant, R denote gas constant and $R_p = \frac{P_{ro}}{P_{go}}$

IV. DISPERSION RELATION

Let us assume the perturbation of all the quantities very as

$$\exp.\left[i\left(K_x x + K_z z + \omega t\right)\right] \quad (15)$$

Where ω is the growth rate of the perturbations and K_x and K_z are the wave number of perturbation along the x-axis and z-axis respectively such that

$$K_x^2 + K_z^2 = K^2 \quad (16)$$

Using equation (12) to (15) in Equation (10) and writing algebraic amplitude Equation of (10) and (11), we get

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 10, April 2014

$$\left(\omega^2 - \frac{V^2K^2}{\alpha}\right)\upsilon_x + 2i\omega\Omega_z\upsilon_y + \left(\Omega_j^2 + C^2K^2A\right)\frac{K_x}{K^2}\omega s = 0$$

(17)

$$-2i\omega\Omega_z\upsilon_x + \left(\omega^2 - \frac{V^2K_z^2}{\alpha}\right)\upsilon_y + 2i\omega\Omega_x\upsilon_z = 0$$

(18)

$$-2i\omega\Omega_x\upsilon_y + \omega\upsilon_z + \left(\Omega_j^2 + C^2K^2A\right)\frac{K_z}{K^2}\omega s = 0$$

(19)

$$K_x\upsilon_x + K_z\upsilon_z + \omega s = 0$$

(20)

Where $s = \frac{\partial\rho}{\rho}$ the condensation of the medium is

$V^2 = \frac{H^2}{4\pi\rho}, v$ is the Alfven velocity

$c^2 = RT, c$ is velocity of sound

$$\Omega_j^2 = C^2K^2 - 4\pi G\rho$$

(21)

$$\alpha = \left(1 + \frac{C^2K^2}{\omega_{pe}^2}\right)$$

(22)

$$A = \frac{(\Gamma - 1)}{1 + 4R_p}$$

(23)

Now equations (17), (18), (19) and (20) written in the matrix form

$$\begin{vmatrix} \omega^2 - \frac{V^2K^2}{\alpha} & 2i\omega\Omega_z & 0 & \left(\Omega_j^2 + i^2K^2A\right)\frac{K_x\omega}{K^2} \\ -2i\omega\Omega_z\upsilon_x & \omega^2 - \frac{V^2K_z^2}{\alpha} & 2i\omega\Omega_x & 0 \\ 0 & -2i\Omega_x & \omega & \left(\Omega_j^2 + C^2K^2A\right)\frac{K_z}{K^2}\omega \\ K_x & 0 & K_z & \omega \end{vmatrix} \begin{vmatrix} \upsilon_x \\ \upsilon_y \\ \upsilon_z \\ s \end{vmatrix} = 0$$

(24)

The determinant of the matrix (24) gives the dispersion relation as taking $i\omega = \sigma,$ is

$$\sigma^6 + \sigma^4\left[4\Omega^2 + \frac{V^2}{\alpha}\left(K^2 + K_z^2\right) + \left(\Omega_j^2 + C^2K^2A\right)\right]$$
$$+\omega^2\left[4\left(\frac{\Omega_j^2 + C^2K^2A}{K^2}\right).\left(\Omega_xK_x + \Omega_zK_z\right)^2 + \frac{V^2K_z^2}{\alpha}\left\{\frac{V^2K^2}{\alpha} + 2\left(\Omega_j^2 + C^2K^2A\right)\right\}\right]$$
$$+\frac{V^2K_z^4}{\alpha^2}\left[\Omega_j^2 + C^2K^2A\right] = 0$$

(25)

V. DISCUSSION

Equation (24) represents the general dispersion relation of an infinite homogeneous, viscous, magnetized rotating plasma cloud incorporating the radiative process. Equation (24) does not allow a positive real or a complex root whose real part is positive and so the system is stable. It follows that when $\Omega_j^2 + C^2K^2A < 0$ then one of the root of equation (24) is positive, that means instability occurs with condition.

$$K^2 = \frac{K_j^2}{\frac{1}{\gamma}\left[1 + (\Gamma - 1)(1 + 4R_p)\right]}$$

(26)

When K_j is Jeans wave number

$K_j^2 = \frac{4\pi G\rho}{C_s^2}$ and $C_s^2 = rRT$

Thus the condition of instability given by Jeans is modified by radiation. The radiation pressure stabilizes the system. The magnetic field and viscosity do not affect the condition of instability in this generalized case. For negligible radiation pressure $R_p \longrightarrow 0$ and $\Gamma = \Gamma_1 = r$ the condition of instability reduces to the original Jeans expression. For gas pressure $p_g << p_r$,the radiation pressure, $\Gamma = \Gamma_1 = \frac{4}{3}$.

The condition of instability becomes

$$K^2 = \frac{K_j^2}{\frac{4}{3}\left(1 + R_p\right)}$$

(27)

A. *Longitudinal Wave Propagation*

For longitudinal wave propagation $K_Z = K$ and $K_x = 0$ the Equation (25) reduces to

$$\sigma^6 + \sigma^4\left[4\Omega^2 + \frac{2V^2K^2}{\alpha} + \left(\Omega_j^2 + C^2K^2A\right)\right] + \sigma^2\left[\begin{array}{l} 4\Omega_z^2\left(\Omega_j^2 + C^2K^2A\right) \\ + \frac{V^2K^2}{\alpha}\left\{\frac{V^2K^2}{\alpha} + 2\left(\Omega_j^2 + C^2K^2A\right)\right\} \end{array}\right]$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 10, April 2014

$$+\frac{V^4K^4}{\alpha^2}\left[\Omega_j^2+C^2K^2A\right]=0 \qquad (28)$$

It is clear from equation (28) the condition of instability remains the same as given by the equation (26) for both the cases of rotation taken in parallel and perpendicular directions to magnetic field.

We write the dispersion relation (28) is non dimensional form in terms of self gravitation as

$$\sigma_*^6+\sigma_*^4\left[4\Omega_*^2+\frac{2V_*^2K_*^2}{\alpha}+K_*^2-1+AK_*^2\right]$$
$$+\sigma_*^2\left[4\Omega_z^2*\left(K_*^2-1+K_*^2A\right)+\frac{V_*^2K_*^2}{\alpha}\left\{\frac{V_*^2K_*^2}{\alpha}+2\left(K_*^2-1+K_*^2A\right)\right\}\right]$$
$$+\frac{V_*^2K_*^2}{\alpha^2}\left[K_*^2-1+K_*^2A\right]=0 \qquad (29)$$

Where the various non-dimensional parameters are defined as

$$K_*=\frac{KC}{(4\pi G\rho)^{1/2}},\qquad V_*=\frac{V}{C},\qquad \Omega_*=\frac{\Omega}{(4\pi G\rho)^{1/2}} \qquad (30)$$

The variation of non-dimensional growth rate verses non-dimensional wave number are shown in fig. (1-2)

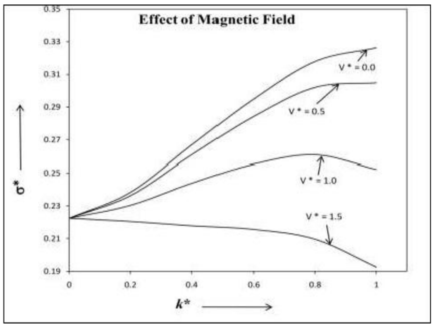

Fig. 1

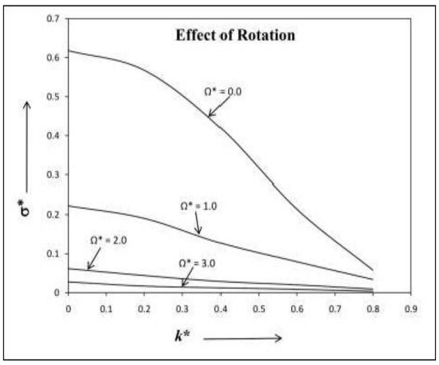

Fig. 2

In fig.(1) we have depicted the non-dimensional growth rate versed non-dimensional wave number for various arbitrary value of magnetic field $V*$ = 0.0, 0.5, 1.0, 1.5 and the value of other parameter are fixed.

From fig.(1), we notice that the growth rate of the instability for non-magnetized medium ($V*$ = 0) is higher in comparison with magnetized medium ($V*$ > 0). It is also noted that the value growth rate is decreased with increasing magnetization of medium. Hence, we conclude that the increasing magnetic field tends to stabilize the system. In fig.(2) we have depicted the non-dimensional growth rate versed non-dimensional wave number for variation in the normalized rotational effect $\Omega*$ = 0.0, 1.0, 2.0, 3.0 and the value of other parameter are fixed .

From fig.(2), we conclude that the growth rate of the instability decreased with increasing values of rotation parameter, for non-rotating medium ($\Omega*$ = 0) the growth rate of the instability is maximum while for higher values of rotation it tends to minimize. Thus, we conclude that rotation parameter reduces the growth rate of the instability and maintain the stability of system.

B. Transverse Wave Propagation

For transverse wave propagation

$K_Z=0, K_x=K$ Equation (22) reduces to

$$\sigma^4+\sigma^2\left[4\Omega^2+\frac{V^2K^2}{\alpha}+\Omega_j^2+C^2R^2A\right]+4\Omega_x^2\left(\Omega_j^2+C^2K^2A\right)=0 \qquad (31)$$

Equation (27) does not allow a positive real or a complex root whose real part is positive and so the system is stable. From equation (27), it is obvious that for rotation taken in perpendicular direction to magnetic field the condition of instability is same as given by equation (28).

When rotation is taken parallel to magnetic field $\Omega_x=0$, Equation (31) reduces to

$$\sigma^2+4\Omega^2+\frac{V^2R}{\alpha}+\Omega_j^2+C^2K^2A=0 \qquad (32)$$

The condition of instability is given by

$$K^2=\frac{4\pi G\rho-4\Omega^2}{C_s^2\left[1+(\Gamma-1)(1+4R_p)\right]+\frac{V^2}{\alpha}} \qquad (33)$$

This is the modified Jeans condition of instability due to magnetic field, finite electron inertia, and rotation and radiation pressure. It is clear from Equation (32) that the rotation and magnetic field give the stabilizing effect along with radiation where as finite election inertia destabilizing effect.

VI. CONCLUSION

To Summarize, we have dealt that the stabilizing effect of radiation pressure exists in general to modify the Jeans expression while the rotation and magnetic field stabilize the system only when wave propagation in transverse direction to magnetic field with rotation taken along the direction of magnetic field. The finite electron inertia destabilizes the system only when wave propagates in transverse direction to magnetic field with rotation taken along the direction of magnetic field. it is also observed that the critical wave length increases with the increases of radiation pressure.

REFERENCES

[1] J. H. Jeans, Philoshph. Trans., 1 (1902) 199.

[2] J. H. Jeans, Astronomy and Cosmogny (Cambridge University Press, Cambridge (1929)

[3] S. Chandrasekhar, Hydrodynamic and hydro magnetic stability, (Clarendon Press, Oxford (1961)

[4] R. Ebert, Z. Astrophys.,42 ,263

[5] J. H. Hunter, M. N. R. A. S., 133 (1965) 239

[6] P. K. Bhatia, Phys. Fluids, 10 (1967) 1652

[7] R. K. Chhajlani and P. Purohit, Beitr. Plasma Phys., 24 (1984) 195

[8] R. K. chhajlani and D. S .Vaghela, Astrophys. Space Sec., 134(1987) 301

[9] F. J. herrengger, Plasma Phys., 8 (1972) 393

[10] R. C. Sharma and K. P. Thakur, Astrophys. Space sci., 81(1992) 95

[11] S. Ali. P. K .Shukla,Phys, Scr.,73(2006)359

[12] R. P. Prajapati, G. D. Sone and R. K .Chhajlani, Phys. Plasma., 15(2008)062108

[13] M. Salimullah, M. Jamil, HA shah and G. Murtaza, Phys. Plasmas., 16(2009), 014502

[14] D. W. Langer, Astrophysical Journal., 225 (1978)95

[15] P. Hennebelle and M. Perault, Astrophys., 359 (2000)1124

[16] DFalceta-Goncalves, MC DeJule and V Jatenco- Pereira, Astrophys J., 597 (2003)970

[17] C. F. Mckee and E.G. Zweibel, Astrophys. J., 440(1995)686

[18] F. DeColle and A .Raga, Astrophys space Sci., 293 (2004)173

[19] J. Vranjes and V. Codez, Astrophys. Space Sci., 164 (1990)329

[20] J. Vranjes, Astrophys space Sci., 173 (1990) 293

[21] S. Inutsuka, H. Koyama, and T Inoue, AIP conf. Proc., 784 (2005)318

AUHTOR'S PROFILE

Dr. R.K. Pensia, Professor of Physics, Govt. Girls College, Neemuch (M.P.) India

Vikas Prajapat, Research Scholar, Pacific University, Udaipur (Raj.) India

Vishal Kumar, Research Scholar, Pacific University, Udaipur (Raj.) India

G.S. Kachhawa, Research Scholar, Pacific University, Udaipur (Raj.) India

D.L. Suthar, Research Scholar, Mewar University, Chittorgarh (Raj.) India