

Color Image Compression Using Discrete Fractional Fourier Transform

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Abstract— *The rapid growth of digital imaging applications, including desktop publishing, multimedia, teleconferencing, and high-definition television has increased the need for effective and standardized image compression techniques. The purpose of image compression is to achieve a very low bit rate representation, while preserving a high visual quality of decompressed images. All the contemporary digital image compression systems use various mathematical transforms for compression. The compression performance is closely related to the performance by these mathematical transforms in terms of energy compaction and spatial frequency isolation by exploiting inter-pixel redundancies present in the image data. In this paper the fractional Fourier transform, which is a time-frequency distribution and generalization of Fourier transform, is used to compress the image with variation of its fractional order 'a'. It is found that by using the fractional Fourier transform, high visual quality decompressed image can be achieved for same amount of compression as that for Fourier transform. By adjusting 'a' to different values, the fractional Fourier transform can achieve low mean square error, better peak signal to noise ratio, a high compression ratio, while preserving good fidelity of decompressed image. The given image is subdivided and discrete fractional Fourier transform is applied for the subdivided image to form transformed coefficients and inverse discrete fractional Fourier transform is applied for reconstruction of original images. Discrete fractional Fourier transform is computed by eigen decomposition method. The significant improvement is observed using discrete fractional Fourier transform.*

Index Terms—discrete fractional Fourier transforms color image compression, mean square error, peak signal to noise ratio, decompression.

I. INTRODUCTION

Image compression is the technique to reduce the redundancies in image representation in order to decrease data storage requirements and transmit in efficient form ultimately reducing the bandwidth required for communication. This leads to develop efficient compression technique that will continue to be a design challenge for future communication systems for color and monochrome images. The representation of image is in terms of information and redundancy. Information is the portion of the image that must be preserved permanently in its original form in order to correctly interpret the meaning or purpose of the image. Redundancy is portion of the image that can be

removed when it is not needed or can be reinserted to interpret the image when needed. Image compression is a technique to reduce the redundancy in the image. In image decompression the reduced redundancy in image can be subsequently reinserted to recover the original image. There are two techniques of image compression namely lossless and lossy compression. Lossless compression is noiseless whereas lossy compression is noisy. The term noise refers to the error of reconstruction in the lossy compression because the reconstructed image is not identical to the original one. Lossless compression is preferred for artificial images like technical drawings, icons or comics and images of high value contents such as medical imagery or image scans made for archival purpose because it introduces compression artifacts especially when used at low bit rates. Lossy compression is suitable for natural images such as photos in applications where minor loss of fidelity is acceptable to achieve a substantial reduction in bit rate. The lossy compression is called visually lossless if it produces imperceptible differences. The methods generally used for lossless image compression are run-length encoding and entropy encoding whereas lossy compression uses various transforms for coding like DCT or the wavelet transform followed by quantization and entropy coding. Extremely good compression results with controllable degradation of image quality are achieved with transform based lossy image compression. The compression schemes may be static or dynamic. The mapping of actual information as a set of compressed code is fixed in static method whereas it changes over time in dynamic method. If the dynamic method adapts to changes in ensemble characteristics over time, it is called as adaptive. The adaptive dynamic method provides a compressed image file of such a size that can adaptively change for better compression efficiency. The main objective of image compression is to reduce the redundancy in an image while maintaining an acceptable visual quality for the decompressed image. The neighboring pixels contribute to redundancy of an image. In addition, the specialty of color image is the correlation between the color components. The multimedia data requires considerable storage capacity and transmission bandwidth. The existing image compression algorithms can not cope up with the present requirements for data storage capacity and data transmission bandwidth even

though there is a rapid progress in mass-storage density, processor speed and digital communication system performance. Signal and image processing community is extensively using fractional Fourier transform which belongs to the class of time-frequency representations. The fractional Fourier transform is a generalization of the ordinary Fourier transform, which is introduced to quantum mechanics by Namias [1], and applied to the signal processing community by Almeida [2]. The normal use of time-frequency representations involves use of a plane with two orthogonal axes that correspond to time and frequency. Consider a representation in which signal $x(t)$ represents time axis and its Fourier transform $X(f)$ as frequency axis, then the Fourier transform operator is represented as a change in signal representation corresponding to counterclockwise rotation of the axis by an angle $\pi/2$. As per the properties of Fourier transform two consecutive rotations of signal through an angle of $\pi/2$ results in inversion of time axis while signal remains unchanged for four such rotations. The fractional Fourier transform is a linear operator. In this transform the rotations of signal are not multiples of $\pi/2$. Thus, the signal is represented along the axis u forming an angle of α with the time axis. [3]. Fractional Fourier transform is a time-frequency distribution and an extension of the classical Fourier transform used in all applications where Fourier transform can be used. As fractional Fourier transform provides additional degree of freedom in terms of angle of rotation, one can expect extra gains in its applications. The parameter 'a' decides rotation angle and hence the interpretation of fractional Fourier transform. The fractional Fourier transform corresponds to classical Fourier transform when the parameter $a=1$ in rotation angle, $\alpha = a\pi/2$ and called identity operator when $a=0$. Fractional Fourier transform is a significant signal processing tool [4, 5]. This led to the development of discrete fractional Fourier transform [6]. The discrete fractional Fourier transform has properties like unitarity, index additivity, reduction to discrete Fourier transform when $\alpha = \pi/2$. Various applications of Fourier transform include communication domain, image processing and data compression. The generalization of discrete Fourier transform is the discrete fractional Fourier transform and applied to the compression of high resolution images. Decompressed is obtained based on the extra degree of freedom provided by discrete fractional Fourier transform and its fractional order. The discrete Fractional Fourier transform is applied to subdivided image to get transform coefficients, while inverse discrete fractional Fourier transform recovers the original images. Good image compression can be achieved by using discrete fractional Fourier transform in terms of performance parameters like peak signal-to-noise ratio, mean square error and compression ratio. An increase in compression ratio affects the image quality inversely [7]. Rajinder Kumar et al [8] used fractional Fourier transform for satellite image compression

for an optimum domain with more compression ratio, less mean square error and better peak signal-to-noise ratio. The filtering configurations based on fractional Fourier transform can be applied to image representation and compression. The coefficients of minimum mean square filtering configuration can approximately represent, store and transmit an image. The raw method provides an order of magnitude compression with moderate errors. The information content of image can be decomposed into fractional Fourier domains. Various domains provide different perspectives with reference to image information content [9].

II. DISCRETE FRACTIONAL FOURIER TRANSFORM (DFRFT)

The Fractional Fourier Transform (FRFT) belongs to the class of time–frequency representations that have been extensively used by the signal processing community. The normal time-frequency representation has two orthogonal axes corresponding to time and frequency. Generally the Fourier Transform operator is denoted as 'F' and the Fourier Transform operation is denoted as 'F^α[x (t)]', where $x(t)$ is the input signal which is to be transformed in to frequency domain using FFT and 'α' is the notation, which denotes how much time is to be taken for the FFT [5, 6, 8, 10].

Table 1 FFT Repeated Operation Results

| Operation | Result |
|-------------|---------|
| $F^0[x(t)]$ | $x(t)$ |
| $F^1[x(t)]$ | $X(w)$ |
| $F^2[x(t)]$ | $x(-t)$ |
| $F^3[x(t)]$ | $X(-w)$ |
| $F^4[x(t)]$ | $x(t)$ |

As shown in Table 1, it is clear that repeated FFT on signal $x(t)$ gives original signal itself after taking FFT four times $F^0[x(t)]$ means no FFT is taken so resultant signal is same as that of original signal. For $F^1[x(t)]$, the resultant signal is a transformed signal $X(w)$, which is in the frequency domain. Again taking FFT for two times, the resultant signal is the original signal, but it is the mirror image or time advanced signal [5, 10]. Similarly, for 3 times, the resultant signal is the mirror image of the 1 time FFT. At last for 4 times, the original signal is recovered which is same as that of recovered by taking the IFFT of transformed signal. In Fractional Fourier Transform, the power 'α' of the FFT operator 'F' varies in fractions, while on the other hand in FFT it is a complete integer. Figure 1 shows the working of fractional Fourier transform in time frequency plane. The ω indicates the frequency plane while t is the time plane. The α

indicates the power of the Fourier transform operator F [5, 10]. Whenever the FFT is calculated, the time domain plane axis t is shifted towards frequency plane ω . This indicates only one time FFT. Again, if the FFT is taken, then the resultant is the mirror image of the time domain signal and so on. This means that, original signal is obtained by rotating such plane by taking FFT 4 times.

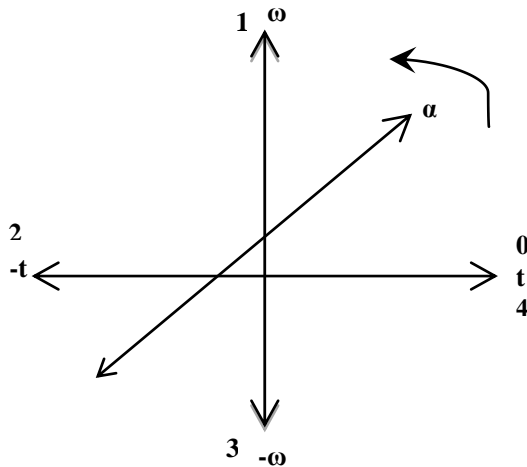


Fig 1 Concept of Fractional Fourier Transform

The concept of fractional transform begins from here. Generally the value of alpha i.e. power of Fourier transform operator 'F^{alpha}' changes from 0 to 1, 1 to 2, 2 to 3 and return to original from 3 to 4 by taking FFT each time. In fractional Fourier transform the value of this alpha i.e. power of Fourier transform operator 'F^{alpha}' changes from 0 to 1, for example 0, 0.1, 0.3, 0.5 and so on up to 1. If it goes beyond this range, the resultant is the mirror image of either time domain or frequency domain signal as shown in Figure 1. The fractional Fourier transform allows us to operate in between the frequency and the time domain. This transform gives the benefit of the middle rotation angle between frequency and time domain [6, 5, 8]. The computation of discrete fractional Fourier transform is based upon the eigen-decomposition of the kernel matrix of discrete Fourier transform. There are four distinct Eigen values [1, -j, -1, j] in the kernel matrix of discrete Fourier transform. The Eigen vectors construct a vector space same Eigen value because these Eigen vectors of discrete Fourier transform kernel are not uniquely determined. A matrix S to evaluate the eigen vectors of F with real values is defined [8] as shown in Figure 2, where $\omega = 2\pi N$. It satisfies the commutative property as,

$$SF = FS \tag{1}$$

$$S = \begin{bmatrix} 2 & 1 & 0 & 0 & \dots & 1 \\ 1 & 2\cos\omega & 1 & 0 & \dots & 0 \\ 0 & 1 & 2\cos 2\omega & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & \dots & 2\cos(N-1)\omega \end{bmatrix}$$

Fig 2 Matrix to evaluate the Eigen vectors of F with real values

The eigen vectors of S matrix are same as that of the eigen vectors of F having different corresponding eigen values. Due to symmetric property of S matrix, all eigen values of S matrix are real and the eigen vectors are orthonormal to each other [8]. The eigen decomposition of matrix S is written as,

$$S = \sum_{k=0}^{N-1} r_k v_k \tag{2}$$

where v_k is the eigenvector of the matrix S corresponding to the eigen value r_k . The eigen decomposition of discrete Fourier transform kernel matrix F is written as,

$$F = \sum_{k \in E1} v_k v_k^* + \sum_{k \in E2} (-j)v_k v_k^* + \sum_{k \in E3} (-1)v_k v_k^* + \sum_{k \in E4} (j)v_k v_k^* \tag{3}$$

where E1, E2, E3 and E4 is the set of indices for eigen vectors belongs to eigen values [1, -j, -1, j] respectively.

From (3) the eigen values of discrete Fourier transform kernel are determined [6]. By taking fractional powers of these eigen values, the transform kernel of discrete fractional Fourier transform can be easily defined as, [6, 8]

$$R^\alpha = F^{\frac{2\alpha}{\pi}} \tag{4}$$

$$\begin{aligned} &= \sum_{k=0}^{N-1} e^{-jN\alpha} v_k v_k^* && \text{For } N \text{ is Odd} \\ &= \sum_{k=0}^{N-2} e^{-jN\alpha} v_k v_k^* + e^{-jN\alpha} v_{N-1} v_{N-1}^* && \text{For } N \text{ is Even} \end{aligned}$$

where v_k is the eigen vector obtained from matrix S and R is rotational kernel matrix [8]. The discrete fractional Fourier transform of signal x(n) can be computed with,

$$X_\alpha(n) = R_\alpha x(n) = F^{\frac{2\alpha}{\pi}} x(n) = VD^{\frac{2\alpha}{\pi}} V^* x(n) \tag{5}$$

where, V is the even eigen vector and V* is odd eigen vector. The signal x(n) can also be recovered from its discrete fractional Fourier transform through an operation with parameter (-alpha), as,

$$x(n) = R_{-\alpha} X_\alpha(n) = F^{-\frac{2\alpha}{\pi}} X_\alpha(n) = VD^{-\frac{2\alpha}{\pi}} V^* X_\alpha(n) \tag{6}$$

Hence the inverse transform can be calculated by simply calculating the transformation kernel with (-alpha) parameter [8]. Table 2 shows the properties of discrete fractional Fourier transform. The discrete fractional Fourier transform can be one or two dimensional. In 2-dimensional discrete fractional Fourier transform there is one more rotation factor beta. The same mathematical formulae are used to calculate the 2-dimensional discrete fractional Fourier transform kernel. In 2 dimensional discrete fractional Fourier transform, there

are two kernels, one each for α and β ; but there should be one kernel for applying to the image. Hence in this situation, the tensor product of these two rotational kernel is calculated.

Table 2 Properties of DFRFT [6]

| Sr. No. | Property Name | Mathematical Representation |
|---------|-----------------|--|
| 1 | Unitary | $R_{\alpha}^* = R_{\alpha}^{-1} = R_{-\alpha}$ |
| 2 | Angle Aditivity | $R_{\alpha}R_{\beta} = R_{\alpha+\beta}$ |
| 3 | Time Inversion | $R_{\alpha}x(-n) = X_{\alpha}(-n)$ |
| 4 | Periodicity | $R_{\alpha+2\pi} = R_{\alpha}$ |
| 5 | Symmetric | $R_{\alpha}(a, b) = R_{\beta}(b, a)$ |

The tensor product of two kernels R_{α} and R_{β} is calculated as, [5, 8, 11].

$$R_{(\alpha,\beta)} = R_{\alpha} \otimes R_{\beta} \tag{7}$$

This 2-dimensional kernel is applied to quantity as forward and inverse transform. The forward 2-dimensional transform is calculated using,

$$X_{(\alpha,\beta)}(m, n) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} x(p, q)R_{(\alpha,\beta)}(p, q, m, n) \tag{8}$$

The inverse 2-dimensional transform is calculated using,

$$x_{(p,q)} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} X_{(\alpha,\beta)}(m, n)R_{(-\alpha,-\beta)}(p, q, m, n) \tag{9}$$

In this work 1-dimensional discrete fractional Fourier transform is used for transforming the image. The main reason behind choosing the 1-dimensional discrete fractional Fourier transform is that, it is faster than that of the 2-dimensional transform. While calculating the 2-dimensional transformation kernel, the tensor product is taken. This causes the variations in the size of the transformation kernel which is not preferred in image transformation. By using 1-dimensional discrete fractional Fourier transform the images can be easily transformed with less processing time. This 1-dimensional discrete fractional Fourier transform causes the increase in the speed of transformation. The properties of 1-dimensional and 2-dimensional discrete fractional Fourier transform are same. Considering all these advantages and disadvantages of the 1-dimensional and 2-dimensional discrete fractional Fourier transform, this work uses 1-dimensional discrete fractional Fourier transform for transforming the images. There are few important properties of α and the fractional transformation [5]. If $\alpha=1$, the 1-dimensional discrete fractional Fourier transform performs the conventional discrete Fourier transform. If $\alpha=0$, the 1-dimensional discrete fractional Fourier transform performs identity transform that is no transform. These two important characteristics of 1-dimensional discrete fractional Fourier transform are used for verifying the transformation kernel. If the kernel satisfies these two important properties then and then only one can say that, this transform is right otherwise it is calculated in wrong way. If this is correct then, this 1-dimensional

discrete fractional Fourier transform kernel is applied to each row of matrix and then to each column of the resultant matrix [10, 12].

III. COLOR IMAGE COMPRESSION USING DISCRETE FRACTIONAL FOURIER TRANSFORM

The well-known model for color images is RGB, as the recording of these images is in RGB format. In a digital color image the R, G, B color components contain 8 bits data each. These are the bits required for representing a pixel. A lot of data redundancy is observed in color images hence requiring large storage space. The transmission and storage cost of such color images is reduced with the help of image compression [13]. The block diagram of the implemented compression and decompression system for color images is as shown in Figure 3.

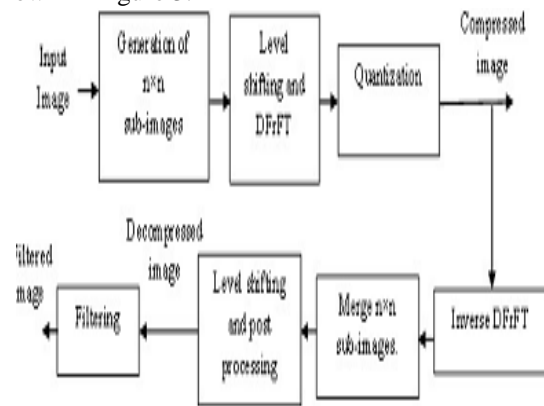


Fig 3 Block diagram of the implemented compression and decompression system

The flow of image compression starts from the block of input image to the compressed image. This is a method of image compression using transformation technique also called as Transform Compression. In this work, we have used one dimensional discrete fractional Fourier transform. The input used as uncompressed image for our experimentation purpose is bitmap image. For compression, the uncompressed image is divided into $n \times n$ non overlapping sub images like 4×4 , 8×8 or 16×16 . The fundamental reason behind subdividing the original image into sub images is that, it reduces the computational complexity hence requires less processing time. During the process of subdividing the input image, it is padded with zeros if the dimensions are not completely supported. The padded zeros do not affect the original image quality or the resultant image. After subdividing the uncompressed image, the sub images are processed one by one. The one dimensional discrete fractional Fourier transformation kernel is applied to each row and then to each column of the sub image matrix. This process is applied to all the blocks of the sub divided image. Since the input image is the colour image, this process is applied to each plane i.e. R, G and B plane. Before applying this transformation kernel, the level shifting is

performed on image, which is commonly performed in DCT image compression, same as in JPEG compression process. The transformation kernel is a matrix of the fractional transform coefficients concentrated at the corners. For transformation, the kernel must be centralized. Using some MATLAB functions, the corner coefficients are swapped from corner to the centre of the transformation kernel. This transform centralizing concept resembles the process of compacting or packing the contents. This process is like compressing the data without any mathematical manipulation involved in the centralizing process; only there is swapping of coefficients [6]. The image transformation time depends upon the size of image, image type and the processor used for image compression. The fractional compression of image is more time efficient on high speed processors as compared to low speed processors. After applying discrete fractional Fourier transform, the image is quantized. In image processing, digitizing the coordinate values is called 'Sampling' and that of amplitudes or pixel values is called 'Quantization' [14]. The goal of quantization is to keep redundant information out of the compressed representation. The quantization operation is irreversible and hence the information lost during the process can not be recovered back completely [10, 15]. In quantization the intensity of pixel values is shifted to nearest threshold value. Hence the total number of pixel intensities are greatly reduced. This results in increased compression level of that image. The quantization function or quantization coefficients are selected based on factors like sensitivity of human eye, low and high frequency coefficients, scale factor and normalization matrix. Then, this quantization function is applied to image for quantization. The standard quantization array used in our work is same as that of JPEG compression and shown in Table 3. This array can be user defined subject to the condition that the values should be in increasing manner from top left to bottom right indicating that the quantization is from lower to higher frequencies respectively.

Table 3 Standard Quantization Array 8x8

| | | | | | | | |
|----|----|----|----|-----|-----|-----|-----|
| 16 | 11 | 10 | 16 | 24 | 40 | 51 | 61 |
| 12 | 12 | 14 | 19 | 26 | 58 | 60 | 55 |
| 14 | 13 | 16 | 24 | 40 | 57 | 69 | 56 |
| 14 | 17 | 22 | 29 | 51 | 87 | 80 | 62 |
| 18 | 22 | 37 | 56 | 68 | 109 | 103 | 77 |
| 24 | 35 | 55 | 64 | 81 | 104 | 113 | 92 |
| 49 | 64 | 78 | 87 | 103 | 121 | 120 | 101 |
| 72 | 92 | 95 | 98 | 112 | 100 | 103 | 99 |

The performance of the compression algorithm using discrete fractional Fourier transform is evaluated based on the parameters like compression ratio, bits-per-pixel and compressed image size. Satisfactory values of these parameters indicates successful image compression,

otherwise compression process is repeated with different values of fractional order ranging from 0 to 1. The fractional image is different from original image as the compression process packs the original image into some pixels. Thus, the information content of neighbouring pixels are packed into a single pixel and such pixels are used during reconstruction process. This also reduces the neighbouring pixel amplitudes to zero. Thus there is no need to store a zero pixel because its contents are packed into a neighbouring pixel. Thus other zero valued pixels are rejected during calculating the image parameters as they do not have any information [8, 10]. Higher compression can be achieved by using discrete fractional Fourier transform. The input image compressed by using one dimensional discrete fractional Fourier transform is highly compressed version of the original image. After successful compression the image can be transmitted with the reduced bandwidth. At receiving station the image is decompressed. Image decompression process is exactly reverse process of that of the image compression process [14]. During the decompression process, the original image can be reconstructed. In decompression all the actions performed are opposite to that of during the forward compression as can be seen in Figure 3. The dequantization is performed on the compressed image by using standard quantization array which is exacty opposite to that of the quantization process. The image is reconstructed from the compressed image using the inverse discrete fractional Fourier transform. The inverse transformation kernel (6) can be easily computed by using '- α ' in (5). The method for applying this inverse transformation kernel to the compressed image is same as that of applying the forward transformation kernel to the uncompressed original image. After taking inverse transform of the compressed image, the contents of image are restored. The pixel intensities are restored back by level shifting to their original levels resulting into the reconstructed image with some degradation in image quality. These degradations can further be reduced using various image enhancement techniques available in image processing like filtering or histogram equalization etc. this is referred as image post processing. In this work, the reconstructed image quality is enhanced by filtering method. The filters used for the reconstructed image enhancement are average filter, median filter and Wiener filter. Performance of the reconstructed image and filtered image is measured in terms of various image quality measures like peak signal-to-noise ratio and mean square error alongwith compression ratio. The compression ratio calculated after image decompression is also known as Decompression Ratio. This factor should be ideally equal to one and practically nearly equal to one. The peak signal-to-noise ratio should be maximum, typically between 30 to 50 dB, and mean square error should be minimum. Since the mean square error indicates the error between reconstructed image and the original image it should be minimum. Generally, the image

whose peak signal-to-noise ratio is maximum, automatically it has very less mean square error and better image quality and hence one can say that image is properly recovered back. Various steps in the implemented algorithm for compression of color image are as follows:

1. Start
2. Read an Image.
3. Take input Parameters
4. Show Original Image.
5. Calculate Size and BPP of Original Image.
6. Subdivide Original image into sub-images.
7. Level Shifting.
8. Generate the 1D Discrete Fractional Fourier Transformation Kernel.
9. Apply Transformation kernel to each sub-image.
10. Apply Quantization on each sub-image.
11. Calculate Compression Ratio, BPP and Size of the compressed Image.
12. Calculate Processing Time.
13. Display Compressed Image.
14. Display calculated parameters i.e. Compression Ratio, BPP, Size of the compressed Image and Processing Time.

IV. RESULTS AND DISCUSSION

The simulation platform used for color image compression is the MATLAB in order to examine the validity of the color image compression using one dimensional discrete fractional Fourier transform. The implemented GUI code is applied to various uncompressed color images collected from website. The most important factor in the fractional image compression is (α) . Initially, with the change in this alpha from 0 to 1, the image is every time processed and various observations are noticed. The best results for two different images are reported here. Figure 4 shows results of the implemented algorithm for the sample image 'Chafa.bmp'. The memory required for storing the original image is 5.49 MB. The dimensions of this image are $1600 \times 1200 \times 3$. The bits per pixel are 8bpp. The image is compressed using one dimensional discrete fractional Fourier transform. Performance of the compression algorithm is analyzed on the basis of compression ratio, size of the compressed image and bits per pixel. The compression ratio comes out to be 635.84, with the size of the compressed image reduces to 379 kB and required 0.52 bits per pixel. This compressed image is then decompressed using inverse discrete fractional Fourier transform and filtered using average filter, median filter and Wiener filter. Performance of the decompression algorithm is analyzed on the basis of peak signal-to-noise ratio and mean square error. For the reconstructed image peak signal-to-noise ratio is observed as 62.44 dB and mean square error as 0.00194. Performance of the algorithm is further improved by applying the process of filtering. For the average filtered image peak signal-to-noise ratio is observed as 69.66 dB and mean square error as 0.0009432. For the

median filtered image peak signal-to-noise ratio is observed as 72.58 dB and mean square error as 0.00070386. For the Wiener filtered image peak signal-to-noise ratio is observed as 72.53 dB and mean square error as 0.00070771.

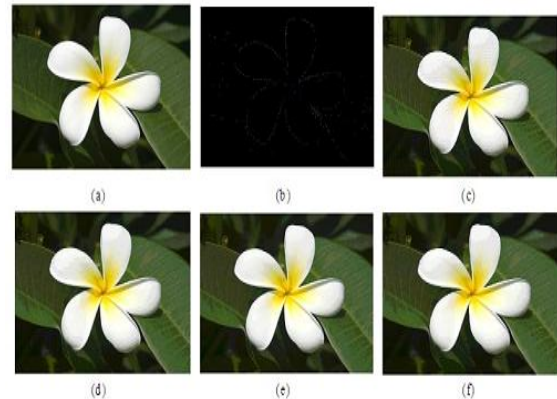


Fig 4 Results of the implemented algorithm for the sample image 'Chafa.bmp' (a) original image, (b) compressed image, (c) reconstructed image, (d) average filtered image, (e) Median filtered image, (f) Wiener filtered image.

Figure 5 shows results of the implemented algorithm for the sample image '512.bmp'. The memory required for storing the original image is 652 KB. The dimensions of this image are $512 \times 512 \times 3$. The bits per pixel are 8 bpp. The image is compressed using one dimensional discrete fractional Fourier transform. Performance of the compression algorithm is analyzed on the basis of compression ratio, size of the compressed image and bits per pixel. The compression ratio comes out to be 222.14, with the size of the compressed image reduces to 101 kB and required 7.1256 bits per pixel. This compressed image is then decompressed using inverse discrete fractional Fourier transform and filtered using average filter, median filter and Wiener filter. Performance of the decompression algorithm is analyzed on the basis of peak signal-to-noise ratio and mean square error. For the reconstructed image peak signal-to-noise ratio is observed as 62.59 dB and mean square error as 0.0019. For the average filtered image peak signal-to-noise ratio is observed as 63.54 dB and mean square error as 0.0017. For the median filtered image peak signal-to-noise ratio is observed as 69.08 dB and mean square error as 0.00076. For the Wiener filtered image peak signal-to-noise ratio is observed as 64.11 dB and mean square error as 0.0016. Thus it is observed that the performance of the reconstructed image is substantially improved after filtering. The peak signal-to-noise ratio value used to measure the difference between a decoded image and its original image. In general, larger the peak signal-to-noise ratio value, the better will be decoded image quality. Another parameter known as compression ratio used in this compression technique and it is defined as the ratio of size compressed image to the size of original image. Performance of the implemented compression algorithm is observed on

large database of color images of different sizes collected from website with variation in the fractional order.

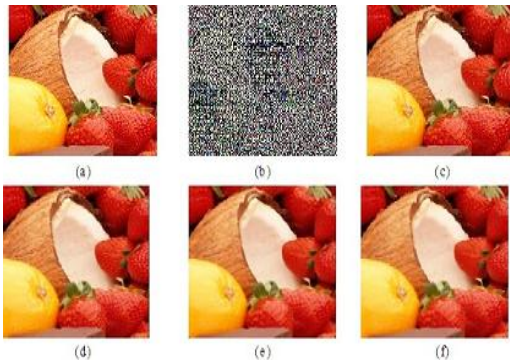


Figure 5 Results of the implemented algorithm for the sample image '512.bmp' (a) original image, (b) compressed image, (c) reconstructed image, (d) average filtered image, (e) median filtered image, (f) Wiener filtered image.

It is observed that fractional order as 0.86 is optimum domain which gives minimum mean square error corresponding to maximum peak signal-to-noise ratio and gives good visual quality of the image. The plots of mean square error and peak signal-to-noise ratio versus changing fractional order of the discrete fractional Fourier transform at fixed compression ratio for the color image 'Chafa.bmp' has been calculated and depicted in Figure 6 and 7. It is clearly observed from these figures that at optimum value of fractional order the mean square error is minimum and it increases further.

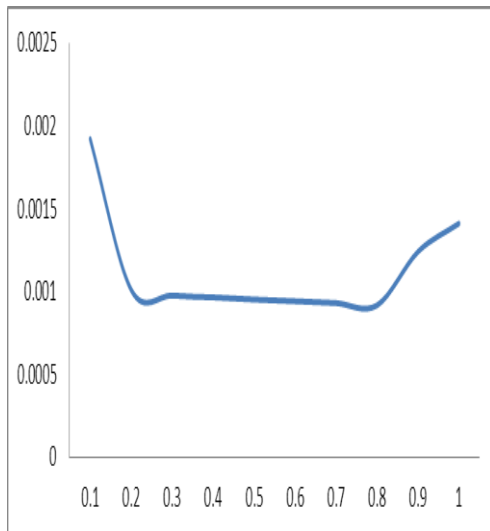


Fig 6 Mean square error versus fractional order for the sample image 'Chafa.bmp'

Similarly the peak signal-to-noise ratio is also maximum at the optimum value of fractional order and it reduces further. Generally, there is tradeoff between image quality and compression ratio. As compression ratio increases, the peak signal-to-noise ratio decreases and image quality degrades for constant fractional order, which is shown in Figure 8.

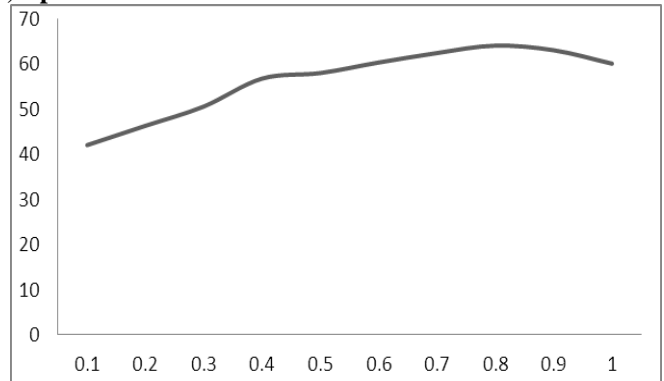


Fig 7 Peak signal-to-noise ratio versus fractional order for the sample image 'Chafa.bmp'

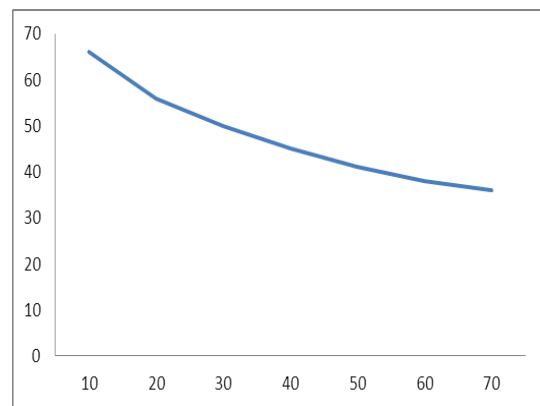


Fig 8 Peak signal-to-noise ratio versus compression ratio

V. CONCLUSIONS

The technique of color image compression using discrete fractional Fourier transform with the additional degree of freedom provided by the fractional orders achieve an optimum domain. At this optimum value of fractional order the compression ratio is more; mean square error is less with better peak signal-to-noise ratio. The good image quality is retained even after reconstruction of the image. Image enhancement techniques applied after reconstruction of the image further improves the results to a great extent. It also observed that with increase in compression ratio beyond certain limit the quality of image decreases.

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