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Simplex Method: An Alternative Approach

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Abstract- In this paper, an alternative approach to the Simplex method of solution of linear programming is suggested. The method sometimes involves less iteration than in the Simplex method or at the most equal number. This powerful technique is illustrated through the problems.

Key words: **Basic feasible solution, optimum solution, conventional simplex method.**

I. INTRODUCTION

The linear programming has its own importance in obtaining the solution of a problem.

To maximize Z = Cx

Subject to Ax=b

$x \geq 0$

Where

x = n x 1 column vector

A = m x n coefficient matrix

b = m x 1 column vector

C = 1x n row vector

and the columns of A are denoted by $y_1, y_2, \ldots\ldots y_n$ There are four methods for solution of linear programming problem. These methods can be classified as:

(i) The graphical method
(ii) The systematic trial and error method
(iii) The vector method
(iv) Simplex method

The simplex method is the most general and powerful.

II. THE SIMPLEX ALGORITHM

For the solution of any L.P.P., by simplex algorithm, the existence of an initial basic feasible solution is always assumed. The steps for the computation of an optimum solution are as follows:

Step 1. Check whether the objective function of the given L.P.P. is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximizing it by using the result
Minimum z = - Maximum (-z)

Step 2. Check whether all bi (i=1,2…..m) are non-negative. If any one of bi is negative then multiply the corresponding in equation of the constraints by -1, so as to get all bi (i=1,2…..m) non-negative.

Step 3. Convert all the in equations of the constraints into equations by introducing slack and/or surplus variables in the constraints. Put the costs of these variables equal to zero.

Step 4. Obtain an initial basic feasible solution to the problem in the form $x_B=B^{-1}b$ and put it in the first column of the simplex table.

Step 5. Compute the net evaluations z_j-c_j (j=1,2…..n) by using the relation $z_j - c_j= c_By_j - c_j$ where $y_j = B^{-1}a_j$. Examine the sign z_j-c_j

(i) If all $(z_j-c_j) \geq 0$ then the initial basic feasible solution x_B is an optimum basic feasible solution

(ii) If at least one $(z_j-c_j) < 0$, proceed on to the next step.

Step 6. If there are more than one negative (z_j-c_j), then choose the most negative of them. Let it be (z_r-c_r) for some j=r.

(i) If all $y_{ir} \leq 0$, (i=1,2…..m), then there is an unbounded solution to the given problem.

(ii) If at least one $y_{ir} > 0$, (i=1,2…..m), then the corresponding vector y_r enters the basis y_B

Step7. Compute the ratios $\left\{\frac{x_{Bi}}{y_{ir}}, y_{ir} > 0, i = 1,2, \ldots\ldots m\right\}$ and choose the minimum of them. Let the minimum of these ratios be $\frac{x_{Bk}}{y_{kr}}$. Then the vector y_k will leave the basis y_B. The common element y_{kr}, which is in the kth row and the rth column is known as the leading element (or pivotal element) of the table.

Step 8. Convert the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zeroes by making use of the relations:

$$\hat{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir}, \quad i = 1,2\ldots\ldots\ldots m+1,\ i \neq k$$

$$\hat{y}_{kj} = \frac{y_{kj}}{y_{kr}}, \quad j = 1,2,3\ldots\ldots n.$$

Step 9. Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or there is an unbounded solution.

III. ALTERNATIVE METHOD

In alternative method of solution to LPP first four steps are same.

Step 5. Compute the net evaluations z_j-c_j (j=1,2…..n) by using the relation $z_j - c_j= c_By_j - c_j$ where $y_j = B^{-1}a_j$.

Also compute $\frac{zj - cj}{\Sigma yj}, \forall\, yij$

(i) If all $(z_j-c_j) \geq 0$ then the initial basic feasible solution x_B is an optimum basic feasible solution

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(ii) If at least one $(z_j\text{-}c_j) < 0$, proceed on to the next step.

Step 6. If there are more than one negative $(z_j\text{-}c_j)$, then choose the entering vector corresponding to which $\frac{zj-cj}{\Sigma yj}$, is most negative. Let it be $\frac{zr-cr}{\Sigma yj}$ for some j=r. and rest of the procedure is same as that of Simplex method.

It is shown that if we choose the entering vector y_j such that $\frac{zj-cj}{\Sigma yj}$ is most negative, then the iterations required are fewer in some problems. This has been illustrated by giving the solution of problems. We have also shown that either the iterations required are same or less but iterations required are never more than those of the simplex method.

PROBLEM 1:

Maximize $z = 12x_1+20x_2+18x_3+40x_4$

Subject to: $4x_1+9x_2+7x_3+10x_4 \leq 6000$;

$x_1+x_2+3x_3+40x_4 \leq 4000$;

$x_1, x_2, x_3, x_4 \geq 0$

SOLUTION

Maximize $z = 12x_1+20x_2+18x_3+40x_4+0(x_5+x_6)$

S.t. $4x_1+9x_2+7x_3+10x_4+ x_5= 6000$;

$x_1+x_2+3x_3+40x_4+ x_6= 4000$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

First Iteration

			12	20	18	40	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	θ
0	y_5	6000	**4**	9	7	10	1	0	**1500**
0	y_6	4000	1	1	3	40	0	1	4000
		$z_j\text{-}c_j$	-12	-20	-18	-40	0	0	
		$\sum y_j$	5	10	10	50	1	1	
		$\frac{zj-cj}{\Sigma yj}$	-12/5	-2	-9/5	-4/5	0	0	

y_1 is entering into the basis and y_5 leaves the basis.

Second Iteration

			12	20	18	40	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	θ
12	y_1	1500	**1**	9/4	7/4	5/2	1/4	0	600
0	y_6	2500	1	-5/4	5/4	**75/2**	-1/4	1	**200/3**
		$z_j\text{-}c_j$	0	7	3	-10	3	0	

y_4 is entering into the basis and y_6 leaves the basis.

Third Iteration

			12	20	18	40	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
12	y_1	4500/3	**1**	7/3	3/2	0	4/15	-1/15
40	y_4	200/3	0	-1/30	1/3	**1**	1/150	2/75
					0			
		$z_j\text{-}c_j$	0	20/3	4/3	0	14/5	4/15

Since $z_j\text{-}c_j \geq 0$, an optimum solution has been reached.
Solution is y1 = 4500/3, y4= 200/3

PROBLEM 2:

Maximize $z = 2x_1+x_2+3x_3$

Subject to $x_1+x_2+2x_3 \leq 5$;

$2x_1+3x_2+4x_3= 12$;

$x_1, x_2, x_3 \geq 0$

SOLUTION:

Maximize $z = 2x_1+x_2+3x_3+0x_4\text{-}Mx_5$

Subject to $x_1+x_2+2x_3 + x_4= 5$;

$2x_1+3x_2+4x_3+x_5= 12$;

$x_1, x_2, x_3, x_4, x_5 \geq 0$

First Iteration

			2	1	3	0	-M	
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	
0	y_4	5	1	1	2	1	0	
-M	y_5	4	2	3	4	0	1	
		$z_j\text{-}c_j$	-2M-2	-3M-1	-4M-3	0	0	

c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_4	5	1	1	2	1	0
1	y_5	4	2	3	4	0	1
		$z_j\text{-}c_j$	-2M-2	-3M-1	-4M-3	0	0

Third Iteration

			2	1	3	0	-M
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
2	y_1	3	1	0	2	3	-1
1	y_2	2	0	1	0	-2	1
		$z_j\text{-}c_j$	0	0	1	4	M-1

Since $z_j\text{-}c_j \geq 0$, an optimum solution has been reached.
Optimum solution is $x_1= 3$, $x_2=2$.

PROBLEM 3:

Maximize $z = 5x_1+2x_2$

Subject to $4x_1+2x_2 \leq 16$;

$3x_1+x_2 \leq 9$;

$3x_1\text{-}x_2 \leq 9$;

$x_1, x_2 \geq 0$

SOLUTION:

Maximize $z = 5x_1+2x_2+0(x_3+x_4+x_5)$

Subject to $4x_1+2x_2+x_3 =16$;

$3x_1\text{-}x_2+ x_4 = 9$;

$3x_1\text{-}x_2+x_5 = 9$;

$x_1, x_2 \geq 0$

First Iteration

			5	**2**	**0**	**0**	**0**
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
0	y_3	**16**	**4**	**2**	**1**	**0**	**0**

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0	y_4	**9**	**3**	**1**	**0**	**1**	**0**
0	y_5	**9**	**3**	**-1**	**0**	**0**	**1**
		z_j-c_j	**-5**	**-2**	**0**	**0**	**0**
		$\frac{zj - cj}{\sum yj}$	**-1/2**	**-1**			

y_2 is entering into the basis and y_3 leaves the basis.

Second Iteration

			5	2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
2	y_2	8	2	1	1/2	0	0
0	y_4	1	1	0	-1/2	1	0
0	y_5	17	5	0	1/2	0	1
		z_j-c_j	**-1**	**0**	**1**	**0**	**0**

Third Iteration

			5	2	0	0	0
c_B	y_B	X_B	y_1	y_2	y_3	y_4	y_5
2	y_2	**6**	**0**	**1**	**3/2**	**-2**	**0**
5	y_1	**1**	**1**	**0**	**-1/2**	**1**	**0**
0	y_3	**12**	**0**	**0**	**1/2**	**-5**	**1**
		z_j-c_j	**0**	**0**	**1**	**0**	**0**

Since z_j-c_j ≥ 0, an optimum solution has been reached.
Optimum solution is x_1= 1, x_2=6 and x_3=12.

IV. CONCLUSION

It is observed that if we solve the above problems by the alternative method, the iterations required for optimum solution are less as compared to the simplex method. Also in third problem if we use simplex method we come across with a tie for outgoing vector and it requires six iterations to solve the problem whereas by alternative method the problem is solved at third iteration and tie doesn't arise.

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