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Thermal Stresses of a Solid Cylinder with Internal Heat Source

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***Abstract-** In this paper, an attempt has been made to study thermoelastic response of a solid circular cylinder, in which source are generated according to the linear function of the temperature, with boundary conditions of the radiation type, by applying integral transform techniques. The results are obtained in terms of Bessel's functions in the form of infinite series. Numerical calculations are carried out for a particular case of a cylinder made of Aluminum metal and the results are depicted graphically.*

Keywords and Phrases: **Thermoelastic response, solid cylinder, temperature distribution, thermal stress, integral transform.**

I. INTRODUCTION

Nowacki [5] has determined steady-state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face circular edge. **Wankhede** [9] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However there aren't many investigations on transient state. **Roy Choudhuri** [8] has succeeded in determined the quasi static thermal stresses in a circular place subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. in a recent work, same problems have been solved by **Noda et al**. [6] and **Deshmukh et al**. [1]. In all aforementioned investigations an axisymmetrically heated plate has been considered. Recently **Nasser** [3,4] proposed the concept of heat sources in generalized thermo elasticity and applied to a thick plate problem. They have not however considered any thermo elastic problem with boundary conditions of radiation type in which source are generated according to the linear function of the temperature, which satisfies the time dependent heat conduction equation. From the previous literature regarding circular solid cylinder as considered, it was observed by the author that no analytical procedure has been established considering internal heat source generation within the body.

This paper is concerned with the transient thermo elastic problem in a circular solid cylinder in which sources are generated according to the linear function of temperature occupying the space

$$D=\{(x,y,z)\in R^3\ :\ 0\le (x^2+y^2)^{1/2}\le a,\quad -h\le z\le h\}$$

where $r=(x^2+y^2)^{1/2}$ with radiation type boundary conditions.

II. STATEMENT OF THE PROBLEM

Consider a solid circular cylinder in which sources are generated according to the linear function of temperature. The material is isotropic, homogeneous and all properties are assumed to be constant. Heat conduction with internal heat source and the prescribed boundary conditions of the radiation type is considered. The equation for heat conduction in cylindrical coordinates is

$$k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial z^2}\right]+\Theta(r,z,t,\theta)=\frac{\partial\theta}{\partial t} \qquad (1)$$

Where $\Theta(r,z,t,\theta)$ is the source function and $k=\lambda/\rho C$ being the thermal conductivity of the material, ρ is the density and C is the calorific capacity, assumed to be constant.

For convenience, we consider the under given functions as the superposition of the simpler function [2]

$$\Theta(r,z,t,\theta)=\Phi(r,z,t)+\Psi(t)\,\theta(r,z,t) \qquad (2)$$

and

$$T(r,z,t)=\theta(r,z,t)\,\exp\left[-\int_0^t\Psi(\zeta)d\zeta\right] \qquad (3)$$

$$\chi(r,z,t)=\Phi(r,z,t)\,\exp\left[-\int_0^t\Psi(\zeta)d\zeta\right]$$

or for the sake of brevity, we consider

$$\chi(r,z,t)=\frac{\delta(r-r_0)\delta(z-z_0)}{2\pi r_0}\exp(-\omega t),$$

$$0\le r_0\le a,\quad -h\le z_0\le h,\quad \omega>0\ .$$

Substituting equations (2) and (3) in the heat conduction equation (1), one obtains

$$k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\chi(r,z,t)=\frac{\partial T}{\partial t} \qquad (4)$$

Where k is the thermal diffusivity of the material of the cylinder (which is assumed to be constant), subject to the initial and boundary conditions

$$T(r,z,t)=f(r,z)\quad\text{at}\quad t=0\quad\text{for all}\quad 0\le r\le a,\ -h\le z\le h \qquad (5)$$

$$T(r,z,t)=0\ \text{at}\ r=a\ \text{for all}\ -h\le z\le h,\ t>0 \qquad (6)$$

$$\left[T+k_1\frac{\partial T}{\partial z}\right]_{z=h}=\exp(-\omega t)\delta(r-r_0)\quad\text{for all}\ 0\le r\le a,\ t>0. \qquad (7)$$

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$$\left[T + k_2 \frac{\partial T}{\partial z}\right]_{z=-h} = \exp(-\omega t)\delta(r - r_0) \quad \text{for all} \quad 0 \le r \le a,\ t > 0. \tag{8}$$

where $\delta(r - r_0)$ is the Dirac Delta function having $0 \le r_0 \le a$, $\omega > 0$ is a constant; $\exp(-\omega t)\delta(r - r_0)$ is the additional sectional heat available on its surface at $z = -h, h$; k_1 and k_2 are radiation constants on the upper and lower surface of the cylinder respectively. The Navier's equations without the body forces for axisymmetric two-dimensional thermo elastic problem can be expressed as [6]

$$\nabla^2 u_r - \frac{u_r}{r^2} + \frac{1}{1-2\nu}\frac{\partial e}{\partial r} - \frac{2(1+\nu)}{1-2\nu} a_t \frac{\partial \theta}{\partial r} = 0$$
$$\nabla^2 u_z - \frac{1}{1-2\nu}\frac{\partial e}{\partial z} - \frac{2(1+\nu)}{1-2\nu} a_t \frac{\partial \theta}{\partial z} = 0 \tag{9}$$

Where u_r and u_z are the displacement components in the radial and axial directions, respectively and the dilatation e as

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}$$

The displacement function in the cylindrical coordinate system are represented by the Goodier's thermo elastic displacement potential $\phi(r, z, t)$ and Michel's function M as [6]

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z} \tag{10}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2} \tag{11}$$

in which Goodier's thermo elastic potential must satisfy the equation

$$\nabla^2 \phi = \left(\frac{1+\nu}{1-\nu}\right) a_t \theta \tag{12}$$

and the Michell's function M must satisfy the equation

$$\nabla^2(\nabla^2 M) = 0 \tag{13}$$

where $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$

The component of the stresses are represented by the use of potential ϕ and Michel's function M as

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right\} \tag{14}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right\} \tag{15}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left((2-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\} \tag{16}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left((1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\} \tag{17}$$

Where G and ν are the shear modulus and Poisson's ratio respectively.
The boundary conditions on the traction free surface of a solid cylinder are

$$\sigma_{rr} = \sigma_{rz} = 0 \text{ at } r = a \tag{18}$$

The equations (1) to (18) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying finite Hankel transform and finite Marchi-Fasulo transform to the equation (4) we get

$$\frac{d\bar{T}^*}{dt} + kp^2\bar{T}^* = ke^{-\omega t}\left(\frac{P_m(h)}{k_1} - \frac{P_m(-h)}{k_2}\right) + \bar{\chi}^* \tag{19}$$

Where $\bar{T}$ is the Hankel transform of T and μ_n is the Hankel transform parameter, $\bar{\bar{T}}^*$ is the finite Marchi-Fasulo transform of $\bar{T}$ and λ_m is the Marchi-Fasulo transform parameter.

$$P_m(z) = Q_m \cos(a_m z) - W_m \sin(a_m z)$$
$$Q_m = a_m(k_1 + k_2)\cos(a_m h)$$
$$W_m = 2\cos(a_m h) + (k_2 - k_1)a_m \sin(a_m h)$$

The eigen values a_m are the positive roots of the characteristic equation

$$[k_1 a\cos(ah) + \sin(ah)][\cos(ah) + k_2 a\sin(ah)]$$
$$= [k_2 a\cos(ah) - \sin(ah)][\cos(ah) - k_1 a\sin(ah)]$$

The solution of differential equation (19) is given by

$$\bar{T}^*(\mu_n, \lambda_m, t) = e^{-kp^2 t}[B(m)e^{-(\omega - kp^2)t} + 2kp^2\int \bar{\chi}^* e^{kp^2 t} dt + \bar{f}^*]$$

where $B(m) = \frac{-k}{kp^2 + \omega}\left(\frac{P_m(h)}{k_1} - \frac{P_m(-h)}{k_2}\right)$

Now, applying inversion of finite Marchi-Fasulo transform and Hankel transform to the equation (20) to get

$$T = \frac{2}{a^2}\sum_{m,n=1}^{\infty}\frac{J_0(r\xi_n)}{[J_0'(a\xi_n)]^2}\frac{P_m(z)}{\lambda_m}e^{-kp^2 t}$$
$$\times[B(m)e^{-(\omega - kp^2)t} + 2kp^2\int \bar{\chi}^* e^{kp^2 t} dt + \bar{f}^*] \tag{21}$$

The solution for the displacement function are represented by the Goodier's thermo elastic displacement potential ϕ governed by equation (12) is given by

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$$\phi=\left(\frac{1+\nu}{1-\nu}\right)\frac{r^2 a_t}{2a^2}\sum_{m,n=1}^{\infty}\frac{J_0(r\xi_n)}{[J_0'(a\xi_n)]^2}\frac{P_m(z)}{\lambda_m}e^{-kp^2 t}$$
$$\times[B(m)e^{-(\omega-kp^2)t}+e^{2kp^2 t}\int\bar{\chi}^* e^{-kp^2 t}dt+\bar{f}^*] \tag{22}$$

The solution for Michell's function M are assumed so as to satisfy the governed condition of equation (13) as

$$M=\left(\frac{1+\nu}{1-\nu}\right)\frac{r^2 a_t}{2a^2}\sum_{m,n=1}^{\infty}\frac{J_0(r\xi_n)}{[J_0'(a\xi_n)]^2}e^{-kp^2 t}$$
$$\times[A_n J_0(\beta_n r)+C_n(\beta_n r)J_1(\beta_n r)]\cosh(\beta_n z)$$
$$\times[B(m)e^{-(\omega-kp^2)t}+e^{2kp^2 t}\int\bar{\chi}^* e^{-kp^2 t}dt+\bar{f}^*] \tag{23}$$

Using equations (22) and (23) in (10) and (11), the displacement components are obtained as

$$u_r=-\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2a^2}\sum_{m,n=1}^{\infty}\frac{\beta_n\sinh(\beta_n z)}{[J_0'(a\xi_n)]^2}e^{-kp^2 t}$$
$$\times[B(m)e^{-(\omega-kp^2)t}+e^{2kp^2 t}\int\bar{\chi}^* e^{-kp^2 t}dt+\bar{f}^*]$$
$$\times\Big\{r^2 J_0(r\xi_n)[A_n J_0'(\beta_n r)+C_n(\beta_n r)J_1'(\beta_n r)]+C_n\beta_n J_1(\beta_n r)]$$
$$+\left[A_n J_0(\beta_n r)+C_n(\beta_n r)J_1(\beta_n r)]-\frac{P_m(z)}{\lambda_m\beta_n\sinh(\beta_n z)}\right]$$
$$\times[r^2 J_0'(r\xi_n)+2rJ_0(r\xi_n)]\Big\} \tag{24}$$

$$u_z=\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2a^2}\sum_{m,n=1}^{\infty}\frac{\cosh(\beta_n z)}{[J_0'(a\xi_n)]^2}e^{-kp^2 t}$$
$$\times[B(m)e^{-(\omega-kp^2)t}+e^{2kp^2 t}\int\bar{\chi}^* e^{-kp^2 t}dt+\bar{f}^*]$$
$$\times\Big\{\frac{r^2 J_0(r\xi_n)P_m'(z)}{\lambda_m\cosh(\beta_n z)}+2r^2(1-\nu)J_0(r\xi_n)$$
$$\times[A_n J_0''(\beta_n r)+C_n(\beta_n r)J_1''(\beta_n r)+2C_n\beta_n J_1'(\beta_n r)]$$
$$+[4r^2(1-\nu)J_0'(r\xi_n)+10r(1-\nu)J_0(r\xi_n)]$$
$$\times[A_n J_0'(\beta_n r)+C_n(\beta_n r)J_1'(\beta_n r)+C_n\beta_n J_1(\beta_n r)]$$
$$+[2r^2(1-\nu)J_0''(r\xi_n)+10r(1-\nu)J_0'(r\xi_n)$$
$$+(8-8\nu+\beta_n^2 r^2-2\nu\beta_n^2 r^2)J_0(r\xi_n)]$$
$$\times[A_n J_0(\beta_n r)+C_n(\beta_n r)J_1(\beta_n r)]\Big\} \tag{25}$$

The stress components can be calculated by substituting the values of thermoelastic displacement potential ϕ from equation (22) and Michell's function M from equation (23) in equation (14) to (17), one obtain the stress functions as

$$\sigma_{rr}=-2G\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2a^2}\sum_{m,n=1}^{\infty}\frac{\sinh(\beta_n z)}{[J_0'(a\xi_n)]^2}e^{-kp^2 t}$$
$$\times[B(m)e^{-(\omega-kp^2)t}+e^{2kp^2 t}\int\bar{\chi}^* e^{-kp^2 t}dt+\bar{f}^*]$$
$$\times\Big\{\frac{1}{\lambda_m\sinh(\beta_n z)}[rP_m(z)J_0'(r\xi_n)+(2P_m(z)+r^2 P_m''(z))J_0(r\xi_n)]$$
$$-\beta_n r^2(\nu-1)J_0(r\xi_n)\left[\begin{matrix}A_n J_0''(\beta_n r)+C_n(\beta_n r)J_1''(\beta_n r)\\+2C_n\beta_n J_1'(\beta_n r)\end{matrix}\right]$$
$$-[2(\nu-1)\beta_n r^2 J_0'(r\xi_n)+r\beta_n(5\nu-4)J_0(r\xi_n)]$$
$$\times[A_n J_0'(\beta_n r)+C_n(\beta_n r)J_1'(\beta_n r)]+C_n\beta_n J_1(\beta_n r)]$$
$$-[(\nu-1)\beta_n r^2 J_0''(r\xi_n)+r\beta_n(5\nu-4)J_0'(r\xi_n)$$
$$+\beta_n(\nu\beta_n^2 r^2+4\nu-2)J_0(r\xi_n)]$$
$$\times[A_n J_0(\beta_n r)+C_n(\beta_n r)J_1(\beta_n r)]\Big\} \tag{26}$$

$$\sigma_{\theta\theta}=-2G\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2a^2}\sum_{m,n=1}^{\infty}\frac{\sinh(\beta_n z)}{[J_0'(a\xi_n)]^2}e^{-kp^2 t}$$
$$\times[B(m)e^{-(\omega-kp^2)t}+e^{2kp^2 t}\int\bar{\chi}^* e^{-kp^2 t}dt+\bar{f}^*]$$
$$\times\Big\{\frac{P_m(z)}{\lambda_m\sinh(\beta_n z)}[r^2 J_0''(r\xi_n)+4rJ_0'(r\xi_n)+\left(\frac{r^2 P_m''(z)+2P_m(z)}{P_m(z)}\right)J_0(r\xi_n)]$$
$$-\nu\beta_n r^2 J_0(r\xi_n)[A_n J_0''(\beta_n r)+C_n(\beta_n r)J_1''(\beta_n r)]+2C_n\beta_n J_1'(\beta_n r)]$$
$$-r\beta_n[2\nu r J_0'(r\xi_n)+(5\nu-1)J_0(r\xi_n)]$$
$$\times[A_n J_0'(\beta_n r)+C_n(\beta_n r)J_1'(\beta_n r)+C_n\beta_n J_1(\beta_n r)]$$
$$-\beta_n[\nu r^2 J_0''(r\xi_n)+r(5\nu-1)J_0'(r\xi_n)+(\nu\beta_n^2 r^2+4\nu-2)J_0(r\xi_n)]$$
$$\times[A_n J_0(\beta_n r)+C_n(\beta_n r)J_1(\beta_n r)]\Big\} \tag{27}$$

$$\sigma_{zz}=-2G\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2a^2}\sum_{m,n=1}^{\infty}\frac{\beta_n\sinh(\beta_n z)}{[J_0'(a\xi_n)]^2}$$
$$\times e^{-kp^2 t}[B(m)e^{-(\omega-kp^2)t}+e^{2kp^2 t}\int\bar{\chi}^* e^{-kp^2 t}dt+\bar{f}^*]$$
$$\times\Big\{\frac{P_m(z)}{\lambda_m\beta_n\sinh(\beta_n z)}[r^2 J_0''(r\xi_n)+5rJ_0'(r\xi_n)+4J_0(r\xi_n)]$$
$$-(2-\nu)r^2 J_0(r\xi_n)[A_n J_0''(\beta_n r)+C_n(\beta_n r)J_1''(\beta_n r)]+2C_n\beta_n J_1'(\beta_n r)]$$
$$-(2-\nu)r[2rJ_0'(r\xi_n)+5J_0(r\xi_n)]$$
$$\times[A_n J_0'(\beta_n r)+C_n(\beta_n r)J_1'(\beta_n r)+C_n\beta_n J_1(\beta_n r)]$$
$$-[r^2(2-\nu)J_0''(r\xi_n)+5r(2-\nu)J_0'(r\xi_n)$$
$$+(8-4\nu+\beta_n^2 r^2-\nu\beta_n^2 r^2)J_0(r\xi_n)]$$
$$\times[A_n J_0(\beta_n r)+C_n(\beta_n r)J_1(\beta_n r)]\Big\} \tag{28}$$

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$$\sigma_{rz} = 2G\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2a^2}\sum_{m,n=1}^{\infty}\frac{\cosh(\beta_n z)}{[J_0'(a\xi_n)]^2}\; e^{-kp^2 t}$$
$$\times[B(m)e^{-(\omega-kp^2)t} + e^{2kp^2t}\int \bar{\chi}^* e^{-kp^2t}dt + \bar{f}^*]$$
$$\times\left\{\frac{P_m'(z)}{\lambda_m \cosh(\beta_n z)}[r^2 J_0'(r\xi_n) + 2rJ_0(r\xi_n)]\right.$$
$$+(1-\nu)r^2 J_0(r\xi_n)[A_n J_1''(\beta_n r) + C_n(\beta_n r)J_1'''(\beta_n r)] + 3C_n\beta_n J_1''(\beta_n r)]$$
$$+(1-\nu)r[3rJ_0'(r\xi_n) + 7J_0(r\xi_n)]$$
$$\times\left[A_n J_0''(\beta_n r) + C_n(\beta_n r)J_1''(\beta_n r)] + 2C_n\beta_n J_1'(\beta_n r)\right]$$
$$+[(1-\nu)3r^2 J_0''(r\xi_n) + 14r(1-\nu)J_0'(r\xi_n) + (9-9\nu-\nu\beta_n^2 r^2)J_0(r\xi_n)]$$
$$\times[A_n J_0'(\beta_n r) + C_n(\beta_n r)J_1'(\beta_n r) + C_n\beta_n J_1(\beta_n r)]$$
$$+[(1-\nu)r^2 J_0'''(r\xi_n) + 7r(1-\nu)J_0''(r\xi_n)$$
$$+(9-9\nu-\nu\beta_n^2 r^2)J_0'(r\xi_n) - 2\nu r\beta_n^2 J_0(r\xi_n)]$$
$$\left.\times[A_n J_0(\beta_n r) + C_n(\beta_n r)J_1(\beta_n r)]\right\} \qquad (29)$$

IV. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(r,z) = z^2(a^2 - r^2)$ (30)

We first apply Hankel transform to equation (30) and then apply Marchi-Fasulo transform to get

$$\bar{f}^* = \frac{4aQ_n}{\beta_n^2 a_n^3}\left[\frac{2}{\beta_n}J_1(\beta_n a) - aJ_0(\beta_n a)\right]$$
$$\times[(a_n^2 h^2 - 2)\sin(a_n h) + 2a_n h\cos(a_n h)] \qquad (31)$$

Using equation (31) into equation (21) one obtains the expression for temperature distribution as

$$T = \frac{2}{a^2}\sum_{m,n=1}^{\infty}\frac{J_0(r\xi_n)}{[J_0'(a\xi_n)]^2}\frac{P_m(z)}{\lambda_m}e^{-kp^2t}$$
$$\times[B(m)e^{-(\omega-kp^2)t} + e^{2kp^2t}\int\bar{\chi}^* e^{-kp^2t}dt + \frac{4aQ_n}{\beta_n^2 a_n^3}$$
$$\times\left[\frac{2}{\beta_n}J_1(\beta_n a) - aJ_0(\beta_n a)\right]$$
$$\times[(a_n^2 h^2 - 2)\sin(a_n h) + 2a_n h\cos(a_n h)] \qquad (32)$$

V. CONCLUSION

In this problem, the temperature distributions, displacement and stress functions at the edge $z = h$ of a circular cylinder in which sources are generated according to the linear function of the temperature have been obtained where the cylinder is subjected to known heat source function $\exp(-\omega t)\delta(r - r_0)$. As a particular case mathematical model is constructed for $f(r,z) = z^2(a^2 - r^2)$ and performed numerical calculations. We develop the analysis for the temperature field by introducing the transformation techniques. Assigning suitable values to the parameters and functions in the equations of temperature, displacements and stresses respectively, expressions of special interest can be derived for any particular case of special interest.

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