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Thermal Deflection of a Finite Length Hollow Cylinder due to Heat Generation

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Abstract-We apply integral transformation techniques to study thermoelastic response of a finite hollow cylinder, in general in which sources are generated according to the linear function of the temperature, with boundary conditions of the radiation type. The results are obtained as series of Bessel functions. Numerical calculations are carried out for a particular case of a cylinder made of Aluminium metal and the results are depicted in figures.

Keywords: **Transient Response, Cylinder, Temperature Distribution, Thermal Deflection, Integral Transform.**

I. INTRODUCTION

Nowacki [4] has determined steady-state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge respectively. Roy Choudhary [5] discussed the normal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the upper face. This satisfies the time-dependent heat conduction equation. Recently Deshmukh studied stresses in a hollow circular disc due to internal heat generation within it. In this Paper, the work of Deshmukh et al [1] have studied the thermal deflection of the disc defined as $a \le r \le b$; $0 \le z \le h$. This problem deals with the determination of thermal deflection due to internal heat generation within it.

The cylinder is considered having arbitrary initial temperature and subjected to radiation type boundary conditions which are fixed at $(r = a)$ and $(r = b)$. The non homogeneous type boundary conditions are maintained on plane surfaces of the cylinder. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in series form in terms of Bessel's functions. The results for thermal deflection have been computed numerically and are illustrated graphically.

The results presented here will be useful in engineering problems particularly in aerospace engineering for stations of a missile body not influenced by nose tapering. The missile skill material is assumed to have physical properties independent of temperature, so that the temperature $T(r, z, t)$ is a function of radius, thickness and time only.

II. STATEMENT OF THE PROBLEM

Consider the cylinder of length h occupying the space D defined by $a \le r \le b$; $0 \le z \le h$. The cylinder is considered having arbitrary initial temperature and subjected to radiation type boundary conditions which are fixed at $(r = a)$ and $(r = b)$. The non homogeneous type boundary conditions are maintained at plane surfaces of the disc. For time $t > 0$, heat is generated within the cylinder at the rate $g(r, z, t)$. Under these conditions the thermal deflections in the cylinder due to heat generation are required to be determined. The differential equation satisfying the deflection function $\omega(r, t)$ is given by

$$\nabla^4 \omega = \frac{\nabla^2 M_T}{D(1-\nu)} \tag{1}$$

where M_T is the thermal moment of the cylinder defined as

$$M_T = a_t E \int_0^h T(r, z, t)\, z\, dz \tag{2}$$

D is the flexural rigidity of the cylinder denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{3}$$

where a_t, E and ν are the coefficients of the linear thermal expansion, Young's modulus and Poisson's ratio of the cylinder material respectively and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{4}$$

Since the curved surfaces of the cylinder is fixed and clamped,

$$\omega = \frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = a, b \tag{5}$$

The temperature of the plate $T(r, z, t)$ at time t satisfies the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r, z, t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{6}$$

With boundary conditions

$$T + k_1 \frac{\partial T}{\partial r} = G_1(z, t) \quad \text{at} \quad r = a, \ t > 0 \tag{7}$$

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$$T + k_2 \frac{\partial T}{\partial r} = G_2(z,t) \quad \text{at} \quad r = b, \ t > 0 \qquad (8)$$

$$T = f_1(r,t) \ \text{at} \ z = 0, \ t > 0 \qquad (9)$$

$$T = f_2(r,t) \ \text{at} \ z = \xi, \ t > 0 \ \text{(known)} \qquad (10)$$

$$T = F(r,t) \ \text{at} \ z = h, \ t > 0 \ \text{(unknown)} \qquad (11)$$

and initial condition is

$$T(r,z,t) = T_0 \ \text{in} \ a \le r \le b; \ 0 \le z \le h \ \text{for} \ t = 0 \qquad (12)$$

where k_1 and k_2 are radiation constants on curved surfaces and plane surfaces of the disc respectively and α is thermal diffusivity of the material of the disc.

Equations (1) – (12) constitute mathematical formulation of the problem.

III. SOLUTION OF THE PROBLEM

Applying Marchi-Zgrablich transform to the equation (6) one obtains

$$\frac{d^2\overline{T}}{dr^2} + \frac{\overline{g}}{k} - \mu_m^2 \overline{T} + \psi = \frac{1}{\alpha}\frac{d\overline{T}}{dt} \qquad (13)$$

Where $\overline{T}$ is the Marchi-Zgrablich transform of T and m is the Marchi-Zgrablich transform parameter,

$$\psi(z,t) = \frac{b}{k_2} S_0(\alpha,\beta,\mu_m b)\, G_2(z,t) - \frac{a}{k_1} S_0(\alpha,\beta,\mu_m a) G_1(z,t) \qquad (14)$$

Applying Laplace transform to equation (13) one obtains

$$\frac{d^2\overline{T}^*}{dr^2} - p^2 \overline{T}^* = \frac{-\overline{g}^*}{k} - \psi^* \qquad (15)$$

where $p^2 = \mu_m^2 + \dfrac{s}{\alpha}$.

Solution of the differential equation (15) is given by

$$\overline{T}^* = Ae^{pz} + Be^{-pz} + \text{P.I.} \qquad (16)$$

where A and B are arbitrary constants.

Using equation (9) and equation (10) in equation (16) one obtains

$$A + B + \psi(0) = \bar{f}_1^* \qquad (17)$$

$$Ae^{p\xi} + Be^{-p\xi} + \psi(\xi) = \bar{f}_2^* \qquad (18)$$

where $\psi(0) = P.I.|_{z=0}$ and $\psi(\xi) = P.I.|_{z=\xi}$

Solving (17) and (18) one obtains

$$A = \frac{\bar{f}_2^* - e^{-p\xi}\bar{f}_1^* + e^{-p\xi}\psi(0) - \psi(\xi)}{2\sinh p\xi} \qquad (19)$$

$$B = \frac{\bar{f}_1^* e^{p\xi} - \bar{f}_2^* + \psi(\xi) - e^{p\xi}\psi(0)}{2\sinh p\xi} \qquad (20)$$

Substituting the values of A and B in equation (16) one obtains

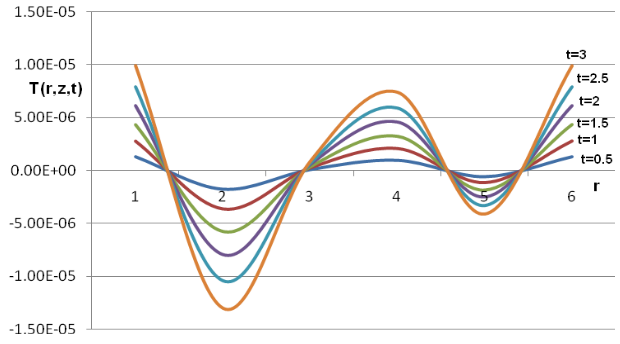
$$\overline{T}^* = [\bar{f}_2^* - \psi(\xi)]\frac{\sinh pz}{\sinh p\xi} - [\bar{f}_1^* - \psi(0)]\frac{\sinh p(z-\xi)}{\sinh p\xi} \qquad (21)$$

Applying inversion of Laplace transform and Marchi – Zgrablich transform to the equation (21) one obtains

$$T(r,z,t) = \frac{2\alpha\pi}{\xi^2}\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m} n(-1)^{n+1} e^{-\alpha t\left(\mu_m^2 + \frac{m^2\pi^2}{\xi^2}\right)}$$

$$\times \int_0^t \left\{ \sin\left[\frac{n\pi z}{\xi}\right](\bar{f}_2(t-u) - \phi) - \sin\left[\frac{n\pi(z-\xi)}{\xi}\right](\bar{f}_1(t-u) - \psi) \right\} du$$

(22)



Graph 1: T(r,z,t) versus r for different values of t

Graph 1. In this graph the temperature distribution T(r,z,t) tends to decrease along the radius between 1.5 to 3, 3 to 4.5 and 4.5 to 5.5, which shows a reduction in the rate of heat propagation in a sinusoidal form; while it tends to increase with heating time from t=0.5 to t=3.

$$F(r,t) = \frac{2\alpha\pi}{\xi^2}\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m} n(-1)^{n+1} e^{-\alpha t\left(\mu_m^2 + \frac{m^2\pi^2}{\xi^2}\right)}$$

$$\times \int_0^t \left\{ \sin\left[\frac{n\pi}{\xi}h\right](\bar{f}_2(t-u) - \phi) - \sin\left[\frac{n\pi}{\xi}(h-\xi)\right](\bar{f}_1(t-u) - \psi) \right\} du$$

(23)

IV. DETERMINATION OF THERMAL DEFLECTION $\omega(r,t)$

Using equation (22) in equation (2) one obtains

$$M_T = a_T E \frac{2\alpha\pi}{\xi^2}\int_0^h \sum_{m,n=1}^{\infty}\left\{ z\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m} n(-1)^{n+1} e^{-\alpha t\left(\mu_m^2 + \frac{m^2\pi^2}{\xi^2}\right)}\right.$$

$$\left. \times \int_0^t \left[\sin\left[\frac{n\pi}{\xi}z\right](\bar{f}_2(t-u) - \phi) - \sin\left[\frac{n\pi}{\xi}(z-\xi)\right](\bar{f}_1(t-u) - \psi) \right] du \right\} dz$$

We assume the solution of equation (1) satisfying condition (5) as

$$\omega(r,t) = \sum_{m=1}^{\infty} C_m(t)[S_0(k_1,k_2,\mu_m r) - S_0(k_1,k_2,\mu_m b)] \qquad (24)$$

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where μ_m are the positive roots of the transcendental equation

$$S_0(k_1, k_2, \mu_m a) - S_0(k_1, k_2, \mu_m b) = 0 \qquad (25)$$

It can be easily seen that

$$\omega = \frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = a, b$$

Hence solution (24) satisfies condition (5).

Now

$$\nabla^4 \omega = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \right)^2 \sum_{m=1}^{\infty} C_m [S_0(k_1, k_2, \mu_m r) - S_0(k_1, k_2, \mu_m b)] = 0$$

(26)

We use the well known result

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \right) S_0(k_1, k_2, \mu_m r) = -\mu_m^2 \, S_0(k_1, k_2, \mu_m r)] \qquad (27)$$

in equation (26) to obtain

$$\nabla^4 \omega = \sum_{m=1}^{\infty} C_m \mu_m^4 \, S_0(k_1, k_2, \mu_m r) \qquad (28)$$

And

$$\nabla^2 M_T = a_T E \frac{2\alpha\pi}{\xi^2} \int_0^h \sum_{m,n=1}^{\infty} \{ z \mu_m S_0(k_1, k_2, \mu_m r) \, n(-1)^{n+2} e^{-\alpha t \left(\mu_n^2 + \frac{n^2\pi^2}{\xi^2} \right)}$$

$$\times \int_0^t \left[\sin\left[\frac{n\pi}{\xi} z \right] (\bar{f}_2(t-u) - \phi) - \sin\left[\frac{n\pi}{\xi}(z-\xi) \right] (\bar{f}_1(t-u) - \psi) \right] du \Big\} dz$$

(29)

Using equation (28) and equation (29) in the equation (1) one obtains

$$\sum_{m=1}^{\infty} C_m \mu_m^4 S_0(k_1, k_2, \mu_m r) = a_T E \frac{2\alpha\pi}{\xi^2 D(1-\nu)}$$

$$\times \int_0^h \sum_{m,n=1}^{\infty} \{ z \mu_m S_0(k_1, k_2, \mu_m r) \, n(-1)^{n+2} e^{-\alpha t \left(\mu_n^2 + \frac{n^2\pi^2}{\xi^2} \right)}$$

$$\times \int_0^t \left[\sin\left[\frac{n\pi}{\xi} z \right] (\bar{f}_2(t-u) - \phi) - \sin\left[\frac{n\pi}{\xi}(z-\xi) \right] (\bar{f}_1(t-u) - \psi) \right] du \Big\} dz$$

(30)

On solving equation (30) one obtains

$$C_m(t) = a_T E \frac{2\alpha\pi}{\xi^2 D(1-\nu)} \times \int_0^h \sum_{n=1}^{\infty} \left\{ \frac{z}{\mu_m^3} \, n(-1)^{n+2} e^{-\alpha t \left(\mu_n^2 + \frac{n^2\pi^2}{\xi^2} \right)} \right.$$

$$\times \int_0^t \left[\sin\left[\frac{n\pi}{\xi} z \right] (\bar{f}_2(t-u) - \phi) - \sin\left[\frac{n\pi}{\xi}(z-\xi) \right] (\bar{f}_1(t-u) - \psi) \right] du \Big\} dz$$

(31)

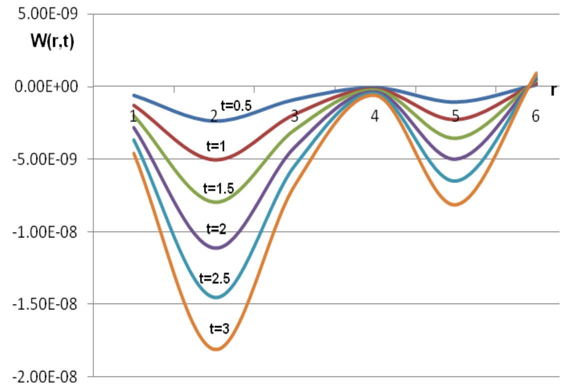
Using equation (31) in equation (24) one obtains

$$\omega(r,t) = \frac{2\alpha\pi \, a_t E}{\xi^2 D(1-\nu)} \int_0^h \sum_{m,n=1}^{\infty} \left\{ \frac{z}{\mu_m^3} \, n(-1)^{n+2} e^{-\alpha t \left(\mu_n^2 + \frac{n^2\pi^2}{\xi^2} \right)} \right.$$

$$\times \int_0^t \left[\sin\left[\frac{n\pi}{\xi} z \right] (\bar{f}_2(t-u) - \phi) - \sin\left[\frac{n\pi}{\xi}(z-\xi) \right] (\bar{f}_1(t-u) - \psi) \right] du \Big\} dz$$

$$\times [S_0(k_1, k_2, \mu_m r) - S_0(k_1, k_2, \mu_m b)]$$

(32)



Graph 2: W(r,t) versus r for different values of t

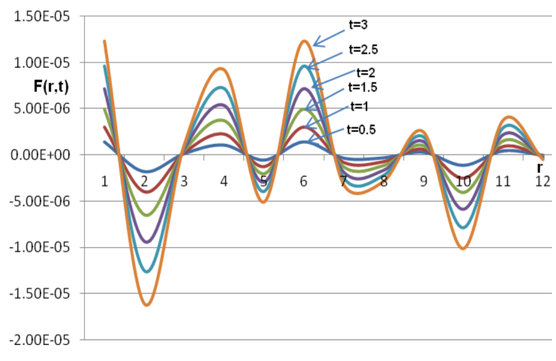
Graph 2. The thermal deflection W(r,t) decreases at different intervals of radius, and tends to decrease with heating time from t=0.5 to t=3. The graph shows a sinusoidal nature.

V. SPECIAL CASE AND NUMERICAL RESULTS

Setting $f_1(r,t) = \delta(r - r_0) \times (1 - e^{-t})$

$$f_2(r,t) = \delta(r - r_0) \times (1 - e^{-t}) \, e^{\xi} \qquad (33)$$

a =2, b =3 , h =1, k_1 = 0.25, k_2 =0.25, k =0. 86, r_0 = 0.75, t = 1 sec. $\xi = 0.5$, $\rho = \frac{2\alpha\pi}{\xi^2}$ in equation (23) one obtains the unknown temperature gradient as



Graph 3: F(r,t) versus r for different values of t

$$\frac{F(r,t)}{\rho} = \sum_{m,n=1}^{\infty} \frac{S_0(0.25, 0.25, \mu_m r)}{\mu_m} \, n(-1)^{n+1} \, e^{-\alpha \left(\mu_m^2 + (39.44) m^2 \right)}$$

$$\times \int_0^1 \{ \sin[(6.28)n] (\bar{f}_2(t-u) - \phi) - \sin[(3.14)n] (\bar{f}_1(t-u) - \psi) \} du$$

(34)

Graph 3. In this graph the unknown temperature gradient F(r,t) tends to decrease along the radius between 1.5 to 3, 3 to 4.5 and 4.5 to 5.5, which shows a reduction in the rate of heat propagation in a sinusoidal form; while it tends to increase with heating time from t=0.5 to t=3.

VI. CONCLUSION

In this paper, the work of Deshmukh et al. has been extended for two dimensional non-homogenous boundary value problem of heat conduction and the thermal deflection of the disc defined as $a \le r \le b$ and $0 \le z \le h$ have been studied. The present chapter deals with the determination of thermal deflection due to internal heat generation within the clamped annular disc. Using the formulas derived, calculations have been made using Mathcad, and graphs have been plotted for thermal conductivity and thermal deflection versus r and t for different values of time and radius. The thermal conductivity and thermal deflection of a finite length hollow cylinder made of aluminium have been determined in series form in terms of Bessel's functions and applying Marchi Zgrablich transform. The researchers have plotted the graphs taking the material properties of aluminium, and the numerical computation has been inferred accordingly.

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APPENDIX

The finite Marchi-Zgrablich integral transform is defined as

$$\bar{f}_p(n) = \int_a^b x f(x) \cdot S_p(k_1, k_2, \mu_n x)\, dx$$

and its inversion is given by

$$f(x) = \sum_{n=1}^{\infty} \frac{\bar{f}_p(n)\, S_p(k_1, k_2, \mu_n x)}{C_n}$$

Where

$$C_n = \frac{b^2}{2}\{S_p^2(k_1, k_2, \mu_n b) - S_{p-1}(k_1, k_2, \mu_n b) \cdot S_{p+1}(k_1, k_2, \mu_n b)\}$$

$$-\frac{a^2}{2}\{S_p^2(k_1, k_2, \mu_n a) - S_{p-1}(k_1, k_2, \mu_n a) \cdot S_{p+1}(k_1, k_2, \mu_n a)$$

$$S_p(k_1, k_2, \mu_n x) = J_p(\mu_n x)\{Y_p(k_1, \mu_n a) + Y_p(k_2, \mu_n b) - Y_p(\mu_n x)\{J_p(k_1, \mu_n a) + J_p(k_2, \mu_n b)\}$$

Where $J_p(\mu x)$ and $Y_p(\mu x)$ are Bessel's functions of first and second kind respectively of order p.

Operational Property:

$$\int_a^b x\left\{\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} - \frac{p^2}{x^2} f\right\} S_p(k_1, k_2, \mu_n x)dx = \frac{b}{k_2} S_p(k_1, k_2, \mu_n b)\left\{f + k_2 \frac{\partial f}{\partial x}\right\}_{x=b}$$

$$-\frac{a}{k_1} S_p(k_1, k_2, \mu_n a)\left\{f + k_1 \frac{\partial f}{\partial x}\right\}_{x=a} - \mu_n^2\, \bar{f}_p(n) \cdot$$

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Dr. N.W. Khobragade For being M.Sc in statistics and Maths he attained Ph.D. He has been teaching since 1986 for 27 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems and its application. Published more than180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and four students submitted their thesis in University for award of Ph.D Degree under their guidance.