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Thermal Deflection of a Thick Clamped Rectangular Plate

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Abstract- ***This paper is concerned with transient thermoelastic problem in which we need to determine the temperature distribution and thermal deflection of a thick clamped rectangular plate when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.***

Keywords- **Thick rectangular plate, transient problem, direct thermoelastic problem, finite Fourier sine transform, cosine transform and Marchi--Fasulo transform.**

I. INTRODUCTION

Deshmukh and Ingle [1] have derived thermal deflection of a thin circular plate due to partially distributed heat supply, Khobragade, and Wankhede [2] have investigated displacement function, temperature and stresses of a thin rectangular plate and Khobragade, Durge [3] have established displacement function, temperature and stresses of a thick rectangular plate. In the present paper, an attempt is made to determine the temperature distribution and thermal deflection at any point of the plate occupying the space *D*: $\{(x,y,z)\in R^3 : -a\le x\le a, 0\le y\le b, 0\le z\le h\}$ with the known boundary conditions. Finite Fourier sine transform and Laplace transform techniques are used to find the solution of the problem. Numerical estimate for the temperature distribution is obtained and depicted graphically.

II. STATEMENT OF THE PROBLEM

Consider a thick isotropic rectangular plate occupying the space *D*. The differential equation satisfied by the deflection ω *(x,y, t)*, is

$$D\nabla^4\omega(x,y,t) = \frac{-\nabla^2 M_T(x,y,t)}{1-\nu} \tag{1}$$

where ν is the Poisson's ratio of the plate material , M_T denote the thermal momentum of the plate and *D* denote the flexural rigidity,

Where $\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$

and the resultant thermal momentum M_T is defined as

$$M_T(x,y,t) = \alpha E\int_0^h zT(x,y,z,t)dz \tag{2}$$

Where α ,*E* are the linear coefficient of thermal expansion of the material, and Young's modulus respectively. Since the edge of the rectangular plate is fixed and clamped,

$$\omega = \frac{\partial^2\omega}{\partial x^2} = \frac{\partial^2\omega}{\partial y^2} = 0 \quad \text{at } x = a \text{ and } y=b \tag{3}$$

The temperature of the plate at time t satisfying the differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{4}$$

where k is the thermal diffusivity of the material of the plate,

subject to the initial and boundary conditions

$$T(x,y,z,0) = 0 \tag{5}$$

$$\left[T(x,y,z,t) + k_1\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = 0 \tag{6}$$

$$\left[T(x,y,z,t) + k_2\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = 0 \tag{7}$$

$$[T(x,y,z,t)]_{y=0} = 0 \tag{8}$$

$$[T(x,y,z,t)]_{y=b} = 0 \tag{9}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=0} = 0 \tag{10}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=h} = f(x,y,t) \tag{11}$$

Equations (1) to (11) constitute the mathematical formulation of the problem under consideration.

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III. SOLUTION OF THE PROBLEM

By applying finite Marchi-Fasulo transform, Fourier sine and cosine transform w.r.t x, y and z respectively and using their inverses to the equations (4) to (11), one obtains the expression for temperature distribution as

$$T(x,y,z,t) = \frac{4k}{bh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l \frac{P_n(x)}{\mu_n}\sin(qy)\cos(\lambda_l z) \times \int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\mu_n^2+q^2+\lambda_l^2\right](t-t')}dt' \qquad (12)$$

where l, m, n are the positive integers and $\lambda_l = \frac{l\pi}{h}$,

$$\bar{\bar{f}}(m,n,t) = \int_{-a}^{a}\int_0^b f(x,y,t)\sin(qy)\frac{P_n(x)}{\mu_n}dxdy$$

IV. THERMAL DEFLECTION

Substituting the value of $T(x,y,z,t)$ from equation (12) in equation (2) one obtains

$$M_T(x,y,t) = \frac{4k\alpha E}{bh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l\left(\int_0^h z\cos(\lambda_l z)dz\right)\frac{P_n(x)}{\mu_n}\sin(qy) \times \int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\mu_n^2+q^2+\lambda_l^2\right](t-t')}dt' \qquad (13)$$

We assume that the solution of equation (1) satisfying equation (3) as

$$\omega(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\omega_{mn}(t)\frac{P_n(x)}{\mu_n}\sin(qy) \qquad (14)$$

Using the equations (13) and (14) in (1), one obtains

$$\omega_{mn}(t) = \frac{4k\alpha E}{D(1-\nu)bh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l\left(\frac{\lambda_l h\sin(\lambda_l h)+\cos(\lambda_l h)-1}{\lambda_l^2(\mu_n^2+q^2)}\right) \times \int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\mu_n^2+q^2+\lambda_l^2\right](t-t')}dt' \qquad (15)$$

Substituting the value of $W_{mn}(t)$ in equation (14), one obtains the expression for thermal deflection as

$$\omega(x,y,t) = \frac{4k\alpha E}{D(1-\nu)bh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l \times\left(\frac{\lambda_l h\sin(\lambda_l h)+\cos(\lambda_l h)-1}{\lambda_l^2(\mu_n^2+q^2)}\right)P_n(x)\sin(qy) \times \int_0^t \bar{\bar{f}}(m,n,t')e^{-k\left[\mu_n^2+q^2+\lambda_l^2\right](t-t')}dt' \qquad (16)$$

V. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(x,y,t) = (1-e^{-t})(x^2-ax)(y^2-by)$,

$\beta = \frac{32k}{abh}$, $\gamma = \frac{16k\alpha E}{D(1-\nu)abh}$ $a = 1, b = 2,$

h =2, t = 1sec and k= 0.86 in equations (3.1) and (4.4) one obtains

$$\frac{T(x,y,z,t)}{\beta} = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l\left[(-1)^m-1\right]\left[(-1)^n-1\right]\left(\frac{1}{\mu_n^3 q^3}\right) \times\cos(\lambda_l z)\,P_n(x)\,\sin(qy) \times\int_0^1(1-e^{-t'})e^{-k\left[\mu_n^2+q^2+\lambda_l^2\right](1-t')}dt' \qquad (17)$$

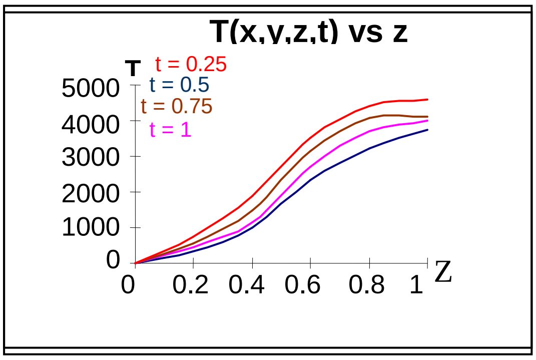

Fig.1 Temperature distribution versus z for different values of t

Fig 1 shows that as the value of z increase, the temperature goes on increasing for different values of t.

$$\frac{\omega(x,y,t)}{\gamma} = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l\left[(-1)^m-1\right]\left[(-1)^n-1\right] \times\left(\frac{\lambda_l h\sin(\lambda_l h)+\cos(\lambda_l h)-1}{\lambda_l^2\mu_n^3q^3(\mu_n^2+q^2)}\right)P_n(x)\sin(qy) \times\int_0^1(1-e^{-t'})e^{-k\left[\mu_n^2+q^2+\lambda_l^2\right](1-t')}dt' \qquad (18)$$

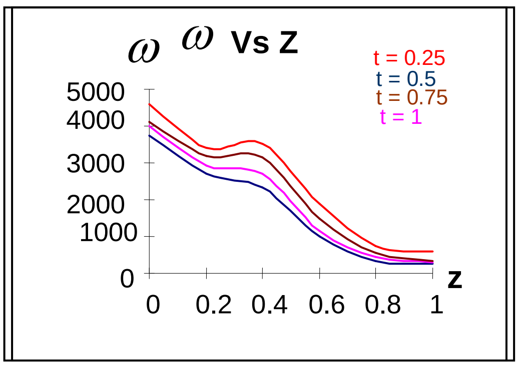

Fig.2 Deflection versus z for different values of t

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Fig 2 shows that as the value of z increase, the temperature goes on decreasing for different values of t.

VI. CONCLUSION

The temperature distribution and thermal deflection of a thick rectangular plate have been obtained, with the aid of finite Marchi-Fasulo transform; Fourier sine transform and cosine transform techniques when the stated boundary conditions are known. The results are obtained in the form of infinite series. The series solutions converge provided we take sufficient number of terms in the series. The expressions are represented graphically. The temperature distribution and deflection that are obtained can be applied to the design of useful structures or machines in engineering applications.

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