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Stability and Convergence of Jungck Modified S-iterative Procedures

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Abstract— ***In this paper, we introduce Jungck Modified S-iterative procedure to establish weak stability, weak convergence and strong convergence results for a pair of Jungck asymptotically no expansive mappings. Our results extend and improve many existing results in literature. An illustrative example is also discussed to show that Jungck Modified S iterative procedure converges faster than Jungck Modified Picard, Jungck Modified Mann and Jungck Modified Ishikawa iterative procedures.***

Index Terms**—** **Iterative procedure, Jungck asymptotically nonexpansive mappings, fixed point, weak and strong convergence, weak stability.**

I. INTRODUCTION AND PRELIMINARIES

Throughout this paper N will denote the set of natural numbers. Let C be a nonempty subset of a Banach space E and S, T: $C \to C$ be two mappings. Then, we denote the set of all fixed points of T by F(T) and the set of common fixed points of S and T by F.

We know that the Picard iterative procedure is defined by the sequence $\{x_n\}$:

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = Tx_n, n \in N. \end{cases} \quad (1.1.1)$$

In 1953, Mann [15] defined the following iterative procedure:

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-a_n)x_n + a_n Tx_n, n \in N, \end{cases} \quad (1.1.2)$$

where $\{a_n\}$ is in $[0,1]$. If we take a_n =1 for all $n \in N$, then Mann iterative procedure (1.1.2) reduces to Picard iteration (1.1.1).

The sequence $\{x_n\}$ defined by

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-a_n)x_n + a_n Ty_n, \\ y_n = (1-b_n)x_n + b_n Tx_n, n \in N, \end{cases} \quad (1.1.3)$$

where $\{a_n\}$ and $\{b_n\}$ are in $[0,1]$, is known as the Ishikawa[12] iterative procedure. It reduces to the Mann iterative procedure if b_n =0 for all $n \in N$.

In 2007, Agarwal et. al.[10] introduced S- iterative procedure as follows:

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-a_n)Tx_n + a_n Ty_n, \\ y_n = (1-b_n)x_n + b_n Tx_n, n \in N, \end{cases} \quad (1.1.4)$$

where $\{a_n\}$ and $\{b_n\}$ are in $[0,1]$.

They showed that S- iterative procedure is independent of Ishikawa (1.1.3) (and hence of Mann (1.1.2)) but converges faster than both of these iterative procedures.

Schu[6] defined the modified Mann iterative procedure which is a generalization of Mann iterative procedure,

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-a_n)x_n + a_n T^n x_n, n \in N, \end{cases} \quad (1.1.5)$$

Where $\{a_n\}$ is in $[0,1]$. If a_n =1 for all $n \in N$, then it reduces to modified Picard iteration defined as

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = T^n x_n, n \in N. \end{cases} \quad (1.1.6)$$

Tan and Xu[7] generalized Ishikawa iteration procedure and studied modified Ishikawa iteration procedure,

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-a_n)x_n + a_n T^n y_n, \\ y_n = (1-b_n)x_n + b_n T^n x_n, n \in N, \end{cases} \quad (1.1.7)$$

where $\{a_n\}$ and $\{b_n\}$ are in $[0,1]$. By taking b_n =0 for all $n \in N$ in (1.1.7), we obtain modified Mann iterative procedure (1.1.5).

In 2007, Agarwal, O' Regan and Sahu[10] defined the modified S-iterative procedure as follows:

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-a_n)T^n x_n + a_n T^n y_n, \\ y_n = (1-b_n)x_n + b_n T^n x_n, n \in N, \end{cases} \quad (1.1.8)$$

where $\{a_n\}$ and $\{b_n\}$ are in $[0,1]$. We note that (1.1.8) is independent of (1.1.7) (and hence of (1.1.5)). If we take n=1, modified Picard (1.1.6), modified Mann (1.1.5), modified Ishikawa (1.1.7) and modified S (1.1.8) iterative procedures become Picard (1.1.1), Mann (1.1.2), Ishikawa (1.1.3) and S (1.1.4) iterative procedures respectively.

Jungck [2], in 1976, introduced the Jungck-Picard iterative procedure to approximate the common fixed points of two mappings S and T such that $T(C) \subseteq S(C)$ as follows:

$$\begin{cases} x_1 = x \in C, \\ Sx_{n+1} = Tx_n, n \in N. \end{cases} \quad (1.1.9)$$

Singh et. al. [13] studied Jungck Mann iterative procedure as:

$$\begin{cases} x_1 = x \in C, \\ Sx_{n+1} = (1-a_n)Sx_n + a_n Tx_n, n \in N, \end{cases} \quad (1.1.10)$$

where $\{a_n\}$ is in $[0,1]$ and it reduces to Jungck Picard iterative procedure (1.1.9) when a_n =1 for all $n \in N$.

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In 2008, Olatinwo[8]defined Jungck-Ishikawa iterative procedure

$$\begin{cases} x_1 = x \in C, \\ Sx_{n+1} = (1-a_n)Sx_n + a_nTy_n, \\ Sy_n = (1-b_n)Sx_n + b_nTx_n, n \in N, \end{cases} \quad (1.1.11)$$

where $\{a_n\}$ and $\{b_n\}$are in $[0,1]$. This iterative procedure reduces to the Jungck Mann iterative procedure (1.1.10) if b_n =0 for all $n \in N$.

Recently, Chugh and Kumar[9] introduced Jungck- S iterative procedure as follows:

$$\begin{cases} x_1 = x \in C, \\ Sx_{n+1} = (1-a_n)Tx_n + a_nTy_n, \\ Sy_n = (1-b_n)Sx_n + b_nTx_n, n \in N, \end{cases} \quad (1.1.12)$$

where $\{a_n\}$ and $\{b_n\}$are in $[0,1]$.

For S=I (identity mapping), the Jungck -Picard(1.1.9), Jungck- Mann(1.1.10), Jungck-Ishikawa(1.1.11) and Jungck-S (1.1.12) iterative procedures reduce to the Picard(1.1.1), Mann(1.1.2), Ishikawa(1.1.3) and S (1.1.4) iterative procedures respectively.

Motivated by the above facts, we introduce the following Jungck modified type iterative procedures:

Let $T(C) \subseteq S(C)$ and $\{a_n\}$ and $\{b_n\}$ are in $[0,1]$ then

Jungck modified Picard Iterative procedure:

$$\begin{cases} x_1 = x \in C, \\ Sx_{n+1} = T^n x_n, n \in N. \end{cases} \quad (1.1.13)$$

Jungck modified Mann Iterative procedure:

$$\begin{cases} x_1 = x \in C, \\ Sx_{n+1} = (1-a_n)Sx_n + a_nT^n x_n, n \in N. \end{cases} \quad (1.1.14)$$

Jungck modified Ishikawa Iterative procedure:

$$\begin{cases} x_1 = x \in C, \\ Sx_{n+1} = (1-a_n)Sx_n + a_nT^n y_n, \\ Sy_n = (1-b_n)Sx_n + b_nT^n x_n, n \in N. \end{cases} \quad (1.1.15)$$

Jungck modified S-Iterative procedure:

$$\begin{cases} x_1 = x \in C, \\ Sx_{n+1} = (1-a_n)T^n x_n + a_nT^n y_n, \\ Sy_n = (1-b_n)Sx_n + b_nT^n x_n, n \in N. \end{cases} \quad (1.1.16)$$

The iterative procedure (1.1.16) is independent of (1.1.15) (and hence of (1.1.14)).

Remarks.

(i) If a_n =1 for all $n \in N$, then Jungck modified Mann Iterative procedure (1.1.14) reduces to Jungck Modified Picard iteration (1.1.13).

(ii) If b_n =0 for all $n \in N$, then Jungck modified Ishikawa Iterative procedure (1.1.15) reduces to Jungck Modified Mann iteration (1.1.14).

(iii) If we take S=I (identity mapping) in the Jungck modified Picard(1.1.13), Jungck modified Mann(1.1.14), Jungck modified Ishikawa(1.1.15) and Jungck modified S (1.1.16) iterative procedures then they reduce to the modified Picard(1.1.6), modified Mann (1.1.5), modified Ishikawa (1.1.7) and modified S (1.1.8) iterative procedures respectively.

(iv) The Jungck modified Picard(1.1.13), Jungck modified Mann(1.1.14), Jungck modified Ishikawa(1.1.15) and Jungck modified S (1.1.16) iterative procedures reduce to the Picard(1.1.1), Mann(1.1.2), Ishikawa(1.1.3) and S (1.1.4) iterative procedures respectively when *n=1* and *S=I* (identity mapping).

Now, we recall some well known concepts and results.

Definition.1.1. A sequence $\{x_n\}$ in a normed linear space E is said to be

(i) **strongly convergent** if there exists an element $x \in X$ such that $\|x_n - x\| \to 0$ as $n \to \infty$. The element x is called the strong limit of the sequence $\{x_n\}$.

(ii) **Weakly convergent** if there exists an element $x \in X$ such that $\lim_{n\to\infty} f(x_n) = f(x)$ for every bounded linear functional *f* on *X*. The element x is called the weak limit of the sequence $\{x_n\}$.

It should be noted that strong convergence implies weak convergence but converse may not be true.

Definition.1.2. Let E be a Banach space with its dimension greater than or equal to 2. The modulus of E is the function $\delta_E(\varepsilon):(0,2] \to [0,1]$ defined by

$$\delta_E(\varepsilon) = \inf\left\{1 - \frac{1}{2}\|x+y\| : \|x\| \le 1, \|y\| \le 1 \|x-y\| \ge r\right\}.$$

A Banach space E is **uniformly convex** if and only if $\delta_E(\varepsilon) > 0$ for all $\varepsilon \in (0,2]$.

It is known that a uniformly convex Banach space is reflexive and strictly convex.

Definition.1.3.[7] Let $S = \{x \in E : \|x\| = 1\}$ and let E^* be the dual of *E*, that is, the space of all continuous linear functional on *E*. Then the value of $x^* \in E^*$ at $x \in E$ will be denoted by $\langle x, x^* \rangle$ and we associate the set $J(x) = \left\{x^* \in X^* : \langle x, x^* \rangle = \|x\|^2 = \|x^*\|^2\right\}$ where J is the normalized duality map from E to E^*. The norm of E is said to be **Frechet differentiable** if, for each $x \in S$ the limit $\lim_{t\to 0} \frac{\|x+ty\| - \|x\|}{t}$ exists and is attained uniformly for y in *S*. In this case, there exists an increasing function

b: [*0*, ∞)→[*0*, ∞) with $\lim_{t\to 0} \frac{b(t)}{t} = 0$ such that

$$\frac{1}{2}\|x\|^2 + \langle h, J(x)\rangle \le \frac{1}{2}\|x+h\|^2 \le \frac{1}{2}\|x\|^2 + \langle h, J(x)\rangle + b(\|h\|)$$

for all $x, h \in E$.

Definition.1.4.[17] A Banach space E is said to satisfy **Opial's condition** if $x_n \to x$ weakly as $n \to \infty$ and $x \ne y$ implying that

$$\limsup_{n\to\infty} \|x_n - x\| < \limsup_{n\to\infty} \|x_n - y\|.$$

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Examples of Banach spaces satisfying Opial's condition are Hilbert spaces and all l^p $(1<p<\infty)$ spaces. On the other hand, $L^p[0,2\pi]$ with $1< p \neq 2$ fail to satisfy Opial's condition.

Definition.1.5. A Banach space E has **Kadec-Klee** property if for every sequence $\{x_n\}$ in E, x_n converges weakly to x and $\|x_n\|$ converges strongly to $\|x\|$ together imply x_n converges strongly to x.

Definition.1.6. Let C be a nonempty subset of a Banach space E. A mapping $T:C\to E$ is said to be **demiclosed** at $y\in E$, if for each sequence $\{x_n\}$ in C and each $x\in E$, the condition x_n converges weakly to x and Tx_n converges strongly to y imply that $x\in C$ and $Tx = y$.

Definition.1.7. Let C be a nonempty subset of a normed linear space E. Then $T:C\to C$ is called

a) **contraction** if there exists a $k\in(0,1)$ such that

$$\|Tx-Ty\|\le k\|x-y\| \quad \text{for all } x,y\in C.$$

b) **nonexpansive** if $\|Tx-Ty\|\le\|x-y\|$ for all $x,y\in C.$

c) **asymptotically nonexpansive** if for a sequence $\{k_n\}\subset[1,\infty)$ with $\lim_{n\to\infty}k_n=1$, we have

$$\|T^n x-T^n y\|\le k_n\|x-y\| \quad \text{for all } x,y\in C$$

Every contraction mapping in nonexpansive and every nonexpansive mapping is asymptotically nonexpansive mapping. But the converse of each may not be true.

Definition.1.8.[2] Let C be a nonempty subset of a normed linear space E. Then $T,S:C\to C$ are called **Jungck contraction** if there exists a $k\in(0,1)$ such that

$$\|Tx-Ty\|\le k\|Sx-Sy\| \quad \text{for all } x,y\in C.$$

Motivated by the concept of Jungck contraction, nonexpansive mappings and asymptotically nonexpansive mappings, we introduce the concept of **Jungck-nonexpansive mappings and Jungck- asymptotically nonexpansive mappings** as follows:

Definition.1.9. Let C be a nonempty subset of a normed linear space E. Then $T,S:C\to C$ are called

a) **Jungck- nonexpansive mappings** if $\|Tx-Ty\|\le\|Sx-Sy\|$ for all $x,y\in C.$

b) **Jungck- asymptotically nonexpansive mappings** if for a sequence $\{k_n\}\subset[1,\infty)$ with $\lim_{n\to\infty}k_n=1$, we have

$$\|T^n x-T^n y\|\le k_n\|Sx-Sy\| \text{ for all } x,y\in C\ .$$

It is clear from the definition that the class of Jungck-asymptotically nonexpansive mappings includes the class of Jungck-nonexpansive mappings, asymptotically nonexpansive maps, nonexpansive maps, Jungck contraction and contraction maps. However converse of each need not be true.

Senter and Doston[3] introduced the concept of condition (A) as follows:

Definition.1.10.[3]. A mappings T: $C\to C$ is said to satisfy the condition(A) if there exists a nondecreasing function $f:[0,\infty)\to[0,\infty)$ with $f(0)=0$, $f(r)>0$ for all $r\in(0,\infty)$, such that $f(d(x,F))\le\|x-Tx\|$ for al $x\in C$, where $d(x,F)=\|x-p\|: p\in F$ }.

Definition.1.11.[15] Let E be a normed linear space and $\{x_n\}\subset E$ be any given sequence. Then $\{y_n\}\subset E$ is said to be approximate sequence of $\{x_n\}$ if for any $k\in N$, there exists $\eta=\eta(k)$ such that $\|x_n-y_n\|\le\eta$ for all $n\ge k$.

Definition.1.12.[14] Two sequences $\{x_n\}$ and $\{y_n\}$ are said to be equivalent if $\|x_n-y_n\|\to 0$ as $n\to\infty$.

Harder and Hicks[1] introduced the following concept of T-stability :

Definition.1.13.[1] Let E be a normed linear space and T: $E\to E$ be a map. Let $\{x_n\}$ be an iterative procedure defined by $x_0\in E$ and $x_{n+1}=f(T,x_n), n\ge 0.$ (1.13.1)

Suppose that $\{x_n\}$ converges to a fixed point p of T. Let $\{y_n\}\subset E$ be any arbitrary sequence and set $\varepsilon_n=\|y_{n+1}-f(T,y_n)\|$ for n=0,1,2,….We say that the iteration procedure (1.13.1) is T-stable or stable if $\lim_{n\to\infty}\varepsilon_n=0$ implies that $\lim_{n\to\infty}y_n=p.$

Later, Berinde[15] introduced the concept of weak stability by choosing $\{y_n\}$ to be an approximate sequence of $\{x_n\}$. With the help of some examples, he proved the importance of choosing approximate sequence in place of arbitrary sequence and showed that every stable iteration is also weakly stable but reverse may not be true .

Definition.1.14.[15] Let E be a normed linear space and T: $E\to E$ be a map. Let $\{x_n\}$ be an iterative procedure defined by $x_0\in E$ and $x_{n+1}=f(T,x_n), n\ge 0.$ Suppose that $\{x_n\}$ converges to a fixed point p of T. If for any approximate sequence $\{y_n\}\subset E$ of $\{x_n\}$, $\lim_{n\to\infty}\|y_{n+1}-f(T,y_n)\|=0$ implies $\lim_{n\to\infty}y_n=p$, then we say that the iteration procedure is weakly T- stable or weakly stable with respect to T.

In 2010, Timis [4] introduced the concept of weak w^2-stability with respect to T by using equivalent sequence in place of approximate sequence as follows:

Definition.1.15.[4] Let E be a normed linear space and T: $E\to E$ be a map. Let $\{x_n\}$ be an iterative procedure defined by $x_0\in E$ and $x_{n+1}=f(T,x_n), n\ge 0.$ Suppose that $\{x_n\}$ converges to a fixed point p of T. If for any equivalent sequence $\{y_n\}\subset E$ of $\{x_n\}$, $\lim_{n\to\infty}\|y_{n+1}-f(T,y_n)\|=0$ implies $\lim_{n\to\infty}y_n=p$, then we say that the iteration procedure is weak w^2-stable with respect to T.

He remarked that any equivalent sequence is an approximate sequence but converse may not be true. Thus this concept generalizes the concept of weak T-stability and T-stability.

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Recently, Timis[5] extend it to the case of two mappings and defined the following concept of weak w^2-stablity with respect to (S,T) :

Definition.1.16.[5] Two sequences $\{Sx_n\}$ and $\{Sy_n\}$ are said to be S-equivalent if $\|Sx_n - Sy_n\| \to 0$ as $n \to \infty$.

Definition.1.17.[5] Let E be a normed linear space and T, S: $E \to E$ be two maps such that $T(E) \subseteq S(E)$ and z is a coincidence point of S and T, that is $Sz = Tz = p \in E$. Let $\{Sx_n\}$ be an iterative procedure defined by $x_0 \in E$ and $Sx_{n+1} = f(T, x_n), n \ge 0$. Suppose that $\{Sx_n\}$ converges to p. If for any S-equivalent sequence $\{Sy_n\} \subset E$ of $\{Sx_n\}$

$$\lim_{n\to\infty} \|Sy_{n+1} - f(T, y_n)\| = 0 \text{ implies } \lim_{n\to\infty} Sy_n = p,$$

then the iteration procedure is weak w^2-stable with respect to (S,T).

This definition reduces to the definition of weak w^2-stable with respect to T if we take S=I. Also weak w^2-stable with respect to (S,T) generalized the concept of (S,T)-Stability and weakly (S,T)-stability.

We need the following useful known results to prove our main results.

Lemma 1.18.[6] Let E be a uniformly convex Banach space and $0 < p \le t_n \le q < 1$ for all $n \in N$.
Suppose that $\{x_n\}$ and $\{y_n\}$ are two sequences of E such that

$$\lim_{n\to\infty} \sup \|x_n\| \le r, \ \lim_{n\to\infty} \sup \|y_n\| \le r,$$

$$\text{and } \lim_{n\to\infty} \|t_n x_n + (1-t_n) y_n\| = r$$

holds for some $r \ge 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0.$

Lemma 1.19. Let $\{r_n\}$ and $\{t_n\}$ be two sequences of nonnegative real numbers such that $r_{n+1} \le (1+t_n) r_n$, for all $n \in N$. If $\sum_{n=1}^{\infty} t_n < \infty$, then $\lim_{n\to\infty} r_n$ exists.

Lemma 1.20.[11] Let C be a nonempty closed convex subset of a uniformly convex Banach space and let T:C→C be an asymptotically nonexpansive map. Then I-T is demiclosed at 0.

In the next two sections, we establish our main results. Section 2 deals with weak stability of Jungck Modified S iterative procedure (1.1.16). Weak and strong convergence results for (1.1.16) are obtained in section 3. Moreover, in this section, we give an example to show that the Jungck Modified S iterative procedure converges faster than Jungck Modified Picard, Jungck Modified Mann and Jungck Modified Ishikawa iterative procedures.

I. WEAK STABILITY RESULTS

Now, we prove that the iterative procedure (1.1.16) is weakly w^2-stable.

Theorem 2.1. Let C be a nonempty subset of a Banach space E. Let S, $T: C \to C$ be Jungck- asymptotically nonexpansive mappings such that $T(C) \subseteq S(C)$ and $F \ne \phi$. For $x_0 \in C$, let $\{Sx_n\}$ generated by (1.1.16) converges to $p \in F$. Suppose that $\{Sy_n\} \subset C$ is an equivalent sequence of $\{Sx_n\}$ and define

$$\varepsilon_n = \|Sy_{n+1} - (1-a_n) T^n y_n - a_n T^n s_n\|,$$

$$Ss_n = (1-b_n) Sy_n + b_n T^n y_n, \ n \in N,$$

where $\{a_n\}, \{b_n\}$ are sequences in [0,1]. Then the Jungck modified S-iterative procedure (1.1.16) is weak w^2-stable with respect to (S,T).

Proof. Let $\{Sy_n\}$ be an equivalent sequence of $\{Sx_n\}$. Suppose that

$$\lim_{n\to\infty} \|Sy_{n+1} - (1-a_n) T^n y_n - a_n T^n s_n\| = 0.$$

Then for all $\varepsilon > 0$, there exists $n_0 = n(\varepsilon)$ such that

$$\|Sy_{n+1} - (1-a_n) T^n y_n - a_n T^n s_n\| < \varepsilon \text{ for all } n \ge n_0.$$

Now to prove that Jungck modified S-iterative procedure (1.1.16) is w^2-stable, we prove that $\lim_{n\to\infty} Sy_n = p$.

So,

$$\begin{aligned}
&\|Sy_{n+1} - p\| \\
&\le \|Sy_{n+1} - Sx_{n+1}\| + \|Sx_{n+1} - p\| \\
&\le \|Sy_{n+1} - (1-a_n) T^n y_n - a_n T^n s_n + (1-a_n) T^n y_n + a_n T^n s_n - Sx_{n+1}\| \\
&\quad + \|Sx_{n+1} - p\| \\
&\le \|Sy_{n+1} - (1-a_n) T^n y_n - a_n T^n s_n\| \\
&+ \|(1-a_n) T^n y_n + a_n T^n s_n - \left((1-a_n) T^n x_n + a_n T^n y_n\right)\| + \|Sx_{n+1} - p\| \\
&\le \varepsilon_n + (1-a_n)\|T^n y_n - T^n x_n\| + a_n \|T^n s_n - T^n y_n\| + \|Sx_{n+1} - p\| \\
&\le \varepsilon_n + (1-a_n) k_n \|Sy_n - Sx_n\| + a_n k_n \|Ss_n - Sy_n\| + \|Sx_{n+1} - p\|.
\end{aligned} \tag{2.1.1}$$

Also

$$\begin{aligned}
\|Ss_n - Sy_n\| &= \|(1-b_n) Sy_n + b_n T^n y_n - (1-b_n) Sx_n - b_n T^n x_n\| \\
&\le (1-b_n)\|Sy_n - Sx_n\| + b_n \|T^n y_n - T^n x_n\| \\
&\le (1-b_n)\|Sy_n - Sx_n\| + b_n k_n \|Sy_n - Sx_n\|.
\end{aligned} \tag{2.1.2}$$

Using (2.1.2) in (2.1.1), we get

$$\begin{aligned}
&\|Sy_{n+1} - p\| \\
&\le \varepsilon_n + k_n [(1-a_n) + a_n (1-b_n) + a_n b_n k_n] \|Sy_n - Sx_n\| \\
&\quad + \|Sx_{n+1} - p\| \\
&\le \varepsilon_n + k_n [1 + (k_n - 1) a_n b_n] \|Sy_n - Sx_n\| + \|Sx_{n+1} - p\| \\
&\le \varepsilon_n + k_n [1 + k_n - 1] \|Sy_n - Sx_n\| + \|Sx_{n+1} - p\| \\
&\le \varepsilon_n + k_n^2 \|Sy_n - Sx_n\| + \|Sx_{n+1} - p\|.
\end{aligned}$$

Since $\{Sx_n\}$ and $\{Sy_n\}$ are S-equivalent sequences, so $\lim_{n\to\infty} \|Sy_n - Sx_n\| = 0$. Also $\{Sx_n\}$ converges to p, thus $\lim_{n\to\infty} \|Sx_n - p\| = 0$. Hence $\lim_{n\to\infty} \|Sy_{n+1} - p\| = 0$. Therefore the iterative procedure (1.1.16) is weak w^2-stable with respect to (S,T).

Corollary 2.2. Let C be a nonempty subset of a Banach space E. Let S, $T: C \to C$ be Jungck- nonexpansive mappings such

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that $T(C)\subseteq S(C)$ and $F\neq\phi$. For $x_0\in C$, let $\{Sx_n\}$ generated by (1.1.12) converges to $p\in F$. Suppose that $\{Sy_n\}\subset C$ is an equivalent sequence of $\{Sx_n\}$ and define

$$\varepsilon_n=\|Sy_{n+1}-(1-a_n)Ty_n-a_nTs_n\|,$$
$$Ss_n=(1-b_n)Sy_n+b_nTy_n,\ n\in N,$$

where $\{a_n\},\{b_n\}$ are sequences in [0,1]. Then the Jungck S-iterative procedure (1.1.12) is weak w^2-stable with respect to (S,T).

Proof. The result follows by taking n=1 in Theorem 2.1.

Corollary 2.3. Let C be a nonempty subset of a Banach space E. Let $T:C\to C$ be an asymptotically nonexpansive mapping such that $F(T)\neq\phi$. For $x_0\in C$, let $\{x_n\}$ generated by (1.1.8) converges to $p\in F$. Suppose that $\{y_n\}\subset C$ is an equivalent sequence of $\{x_n\}$ and define

$$\varepsilon_n=\left\|y_{n+1}-(1-a_n)T^ny_n-a_nT^ns_n\right\|,$$
$$s_n=(1-b_n)y_n+b_nT^ny_n,\ n\in N,$$

where $\{a_n\},\{b_n\}$ are sequences in [0,1]. Then the modified S-iterative procedure (1.1.8) is weak w^2-stable with respect to T.

Proof. The result follows by taking S=I(the identity mapping) in Theorem 2.1.

Corollary 2.4. Let C be a nonempty subset of a Banach space E. Let $T:C\to C$ be a nonexpansive mapping such that $F(T)\neq\phi$. For $x_0\in C$, let $\{x_n\}$ generated by (1.1.4) converges to $p\in F$. Suppose that $\{y_n\}\subset C$ is an equivalent sequence of $\{x_n\}$ and define

$$\varepsilon_n=\|y_{n+1}-(1-a_n)Ty_n-a_nTs_n\|,$$
$$s_n=(1-b_n)y_n+b_nTy_n,\ n\in N,$$

where $\{a_n\},\{b_n\}$ are sequences in [0,1]. Then the S-iterative procedure (1.1.4) is weak w^2-stable with respect to T.

Proof. The result follows by taking n=1 and S=I(the identity mapping) in Theorem 2.1.

II. CONVERGENCE RESULTS

In this section, we establish weak and strong convergence results for iterative procedure (1.1.16)

Lemma 3.1. Let C be a nonempty closed convex subset of a uniformly convex Banach space E. Let $T,S:C\to C$ be Jungck- asymptotically nonexpansive mappings with $\sum_{n=1}^{\infty}(k_n^2-1)<\infty$. Let $\{Sx_n\}$ be defined by the iterative procedure (1.1.16) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n\in N$. If $F\neq\phi$, then

(i) $\lim_{n\to\infty}\|Sx_n-p\|$ exists for all $p\in F$.

(ii) $\lim_{n\to\infty}\left\|T^nx_n-Sx_n\right\|=0.$

Proof. Let $p\in F$. Then

$$\|Sx_{n+1}-p\|=\left\|(1-a_n)T^nx_n+a_nT^ny_n-(1-a_n+a_n)p\right\|$$
$$\le(1-a_n)\left\|T^nx_n-p\right\|+a_n\left\|T^ny_n-p\right\|$$
$$\le(1-a_n)k_n\|Sx_n-p\|+a_nk_n\|Sy_n-p\|.\qquad(3.1.1)$$

Now, $\|Sy_n-p\|=\left\|(1-b_n)Sx_n+b_nT^nx_n-(1-b_n+b_n)p\right\|$

$$\le(1-b_n)\|Sx_n-p\|+b_n\left\|T^nx_n-p\right\|$$
$$\le(1-b_n)\|Sx_n-p\|+b_nk_n\|Sx_n-p\|$$
$$\le(1-b_n+b_nk_n)\|Sx_n-p\|.\qquad(3.1.2)$$

Substituting (3.1.2) in (3.1.1), we have

$$\|Sx_{n+1}-p\|\le(1-a_n)k_n\|Sx_n-p\|$$
$$+a_nk_n\left[(1-b_n+b_nk_n)\|Sx_n-p\|\right]$$
$$\le k_n\left[1-a_n+a_n-a_nb_n+a_nb_nk_n\right]\|Sx_n-p\|$$
$$\le k_n\left[1+(k_n-1)a_nb_n\right]\|Sx_n-p\|$$
$$\le k_n\left[1+k_n-1\right]\|Sx_n-p\|$$
$$\le k_n^2\|Sx_n-p\|$$
$$\le\left[1+\left(k_n^2-1\right)\right]\|Sx_n-p\|.$$

By Lemma 1.19, $\lim_{n\to\infty}\|Sx_n-p\|$ exists for all $p\in F$.

Let $\lim_{n\to\infty}\|Sx_n-p\|=c$. (3.1.3)

Thus, $\{Sx_n\}$is bounded. Now, using (3.1.3) in (3.1.2), we get

$$\lim_{n\to\infty}\sup\|Sy_n-p\|\le c.\qquad(3.1.4)$$

Now, $\left\|T^nx_n-p\right\|\le k_n\|Sx_n-p\|$ for all $n\in N$.

From (3.1.3), we get

$$\lim_{n\to\infty}\sup\left\|T^nx_n-p\right\|\le c.\qquad(3.1.5)$$

Again, $\left\|T^ny_n-p\right\|\le k_n\|Sy_n-p\|$

By using (3.1.3), we obtain

$$\lim_{n\to\infty}\sup\left\|T^ny_n-p\right\|\le c.\qquad(3.1.6)$$

Moreover,

$$c=\lim_{n\to\infty}\|Sx_{n+1}-p\|=\left\|(1-a_n)(T^nx_n-p)+a_n(T^ny_n-p)\right\|,$$

then by Lemma 1.18, we get

$$\lim_{n\to\infty}\left\|T^nx_n-T^ny_n\right\|=0.\qquad(3.1.7)$$

Now, $\|Sx_{n+1}-p\|=\left\|(1-a_n)T^nx_n+a_nT^ny_n-p\right\|$

$$=\left\|(T^nx_n-p)+a_n(T^ny_n-T^nx_n)\right\|$$
$$\le\left\|T^nx_n-p\right\|+a_n\left\|T^ny_n-T^nx_n\right\|,$$

It follows from (3.1.3), and (3.1.7) that

$$c\le\lim_{n\to\infty}\inf\left\|T^nx_n-p\right\|.$$

By (3.1.5), we obtain

$$\lim_{n\to\infty}\left\|T^nx_n-p\right\|=c.\qquad(3.1.8)$$

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Again, $\|T^n x_n - p\| \le \|T^n x_n - T^n y_n\| + \|T^n y_n - p\|$

$$\le \|T^n x_n - T^n y_n\| + k_n \|Sy_n - p\|$$

From (3.1.7) and (3.1.8), we have

$c \le \liminf_{n\to\infty} \|Sy_n - p\|$.

Thus, (3.1.4) gives

$$\lim_{n\to\infty} \|Sy_n - p\| = c. \quad (3.1.9)$$

Now,

$$c = \lim_{n\to\infty} \|Sy_n - p\| = \lim_{n\to\infty} \|(1-b_n)(Sx_n - p) + b_n(T^n x_n - p)\|$$

and by Lemma 1.18, we get

$$\lim_{n\to\infty} \|T^n x_n - Sx_n\| = 0. \quad (3.1.10)$$

In the following result, we prove the weak convergence of iterative procedure (1.1.16) without using any of the Opial's condition, Kadec-Klee property and Frechet differentiable norm.

Theorem 3.2. Let E be a uniformly convex Banach space and let C, T, S and $\{Sx_n\}$ be defined as in lemma 3.1. If $F \ne \phi$ then $\{Sx_n\}$ converges weakly to a point of F.

Proof. Let $q \in F$. By lemma 3.1, $\lim_{n\to\infty} \|Sx_n - p\|$ exists for all $p \in F$ and so $\{Sx_n\}$ is bounded. Also E is a uniformly convex Banach space, so E is reflexive and thus every bounded sequence in E has a weakly convergent subsequence. So, there exists a subsequence $\{Sx_{n_j}\}$ of $\{Sx_n\}$ such that Sx_{n_j} converges weakly to $q \in \omega_w(Sx_n)$, where $\omega_w(Sx_n)$ denotes the set of all weak subsequential limits of $\{Sx_n\}$. Thus $\omega_w(Sx_n) \ne \phi$. Also I-T and I-S are demiclosed with respect to zero by Lemma 1.20, therefore $Tq = Sq = q$ and so $q \in F$. Thus $\omega_w(Sx_n) \subset F$.

Now, for any $q \in \omega_w(Sx_n)$, there exists a subsequence $\{Sx_{n_i}\}$ of $\{Sx_n\}$ such that

Sx_{n_i} converges weakly to q as $i \to \infty$. (3.2.1)

Using (3.2.1) and (3.1.10), we get

$T^{n_i} x_{n_i} = (T^{n_i} x_{n_i} - Sx_{n_i}) + Sx_{n_i}$ converges weakly to q as $i \to \infty$. (3.2.2)

From (1.1.16), (3.2.1) and (3.2.2), we have

$Sy_{n_i} = (1-b_{n_i})Sx_{n_i} + b_{n_i} T^{n_i} x_{n_i}$ converges weakly to q as $i \to \infty$. (3.2.3)

By using (3.1.7) and (3.2.2), we obtain

$T^{n_i} y_{n_i} = (T^{n_i} y_{n_i} - T^{n_i} x_{n_i}) + T^{n_i} x_{n_i}$ converges weakly to q as $i \to \infty$. (3.2.4)

It follows from (3.2.2) and (3.2.4) that

$Sx_{n_{i+1}} = (1-a_{n_i}) T^{n_i} x_{n_i} + a_{n_i} T^{n_i} y_{n_i}$ converges weakly to q as $i \to \infty$. (3.2.5)

Continuing in this way, by mathematical induction, we can prove that for any $m \ge 0$

$Sx_{n_{i+m}}$ converges weakly to q as $i \to \infty$.

Again, by applying mathematical induction, we can obtain

$\bigcup_{m=0}^{\infty} \{Sx_{n_{i+m}}\}_{j=1}^{\infty}$ converges weakly to q as $i \to \infty$.

Since $\{Sx_n\}_{n=n_1}^{\infty} = \bigcup_{m=0}^{\infty} \{Sx_{n_{i+m}}\}_{j=1}^{\infty}$, therefore $S\,x_n$ converges weakly to q as $i \to \infty$.

Hence, $\{Sx_n\}$ converges weakly to a point of F.

Corollary 3.4. Let C be a nonempty closed convex subset of a uniformly convex Banach space E. Let $T, S : C \to C$ be Jungck- nonexpansive maps. Let $\{Sx_n\}$ be defined by the iterative procedure (1.1.12) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If $F \ne \phi$ then $\{Sx_n\}$ converges weakly to a point of F.

Proof. In Theorem 3.2, choose n=1 to obtain Corollary 3.4.

Corollary 3.5. Let C be a nonempty closed convex subset of a uniformly convex Banach space E. Let $T : C \to C$ be an asymptotically nonexpansive mapping with $\sum_{n=1}^{\infty}(k_n^2 - 1) < \infty$.

Let $\{x_n\}$ be defined by the iterative procedure (1.1.8) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If $F(T) \ne \phi$ then $\{x_n\}$ converges weakly to a point of $F(T)$.

Proof. In Theorem 3.2, choose S=I(the identity mapping) to obtain Corollary 3.5.

Corollary 3.6. Let C be a nonempty closed convex subset of a uniformly convex Banach space E. Let $T : C \to C$ be a nonexpansive mapping . Let $\{x_n\}$ be defined by the iterative procedure (1.1.4) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If $F(T) \ne \phi$ then $\{x_n\}$ converges weakly to a point of $F(T)$.

Proof. In Theorem 3.2, choose n=1 and S=I(the identity mapping) to obtain Corollary 3.6.

Now, we obtain our strong convergence results for iterative procedure (1.1.16)

Theorem 3.7. Let all the hypothesis of Lemma 3.1 are satisfied. If $\liminf_{n\to\infty} d(Sx_n, F) = 0$, or $\limsup_{n\to\infty} d(Sx_n, F) = 0$, where $d(Sx,F)$ = $\inf\{\|Sx - p\| : p \in F\}$, then $\{Sx_n\}$ converges strongly to a point of F.

Proof. By Lemma 3.1, $\lim_{n\to\infty} \|Sx_n - p\|$ exists for all $p \in F$. Therefore $\lim_{n\to\infty} d(Sx_n, F)$ exists. But by hypothesis, $\liminf_{n\to\infty} d(x_n, F) = \limsup_{n\to\infty} d(x_n, F) = 0$, thus we have $\lim_{n\to\infty} d(Sx_n, F)$ =0. Now, we show that $\{Sx_n\}$ is a Cauchy

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sequence in *C*. Let $\varepsilon > 0$ be arbitrary chosen. Since $\lim_{n\to\infty} d(Sx_n, F) = 0$, there exists a positive integer n_0 such that $d(Sx_n, F) < \frac{\varepsilon}{4}$, for all $n \ge n_0$.

In particular, $\inf\left\{\left\|Sx_{n_0} - p\right\| : p \in F\right\} < \frac{\varepsilon}{4}$ }. Thus there must exist $p^* \in F$ such that $\left\|Sx_{n_0} - p^*\right\| < \frac{\varepsilon}{2}$.

Now, for all $m, n \ge n_0$, we have

$$\left\|Sx_{n+m} - Sx_n\right\| \le \left\|Sx_{n+m} - p^*\right\| + \left\|Sx_n - p^*\right\|$$

$$\le 2\left\|Sx_{n_0} - p^*\right\| < 2\frac{\varepsilon}{2} = \varepsilon.$$

Hence $\{Sx_n\}$ is a Cauchy sequence in a closed subset *C* of a Banach space *E* and so it must converge to a point *q* in *C*. Now $\lim_{n\to\infty} d(Sx_n, F) = 0$ gives that $d(q, F) = 0$. Thus $q \in F$.

Therefore $\{Sx_n\}$ converges strongly to a point of *F*.

Corollary 3.8. Let *C* be a nonempty closed convex subset of a uniformly convex Banach space *E*. Let $T, S : C \to C$ be Jungck- nonexpansive maps and $F \ne \phi$.. Let $\{Sx_n\}$ be defined by the iterative procedure (1.1.12) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If $\liminf_{n\to\infty} d(Sx_n, F) = 0$, or $\limsup_{n\to\infty} d(Sx_n, F) = 0$, where *d*(S*x*,*F*) = inf{ $\|Sx - p\| : p \in F$ }, then $\{Sx_n\}$ converges weakly to a point of *F*.

Proof. In Theorem 3.7, set n=1 to obtain Corollary 3.8.

Corollary 3.9. Let *C* be a nonempty closed convex subset of a uniformly convex Banach space *E*. Let $T : C \to C$ be an asymptotically nonexpansive mapping with $\sum_{n=1}^{\infty}\left(k_n^2 - 1\right) < \infty$ and *F(T)* $\ne \phi$. Let $\{x_n\}$ be defined by the iterative procedure (1.1.8) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If $\liminf_{n\to\infty} d(x_n, F) = 0$, or $\limsup_{n\to\infty} d(x_n, F) = 0$, where *d*(*x*,*F*) = inf{ $\|x - p\| : p \in F(T)$ }, then $\{x_n\}$ converges strongly to a point of *F(T)*.

Proof. In Theorem 3.7, set S=I(the identity mapping) to obtain Corollary 3.9.

Corollary 3.10. Let *C* be a nonempty closed convex subset of a uniformly convex Banach space *E*. Let $T : C \to C$ be a nonexpansive mapping and *F(T)* $\ne \phi$. . Let $\{x_n\}$ be defined by the iterative procedure (1.1.4) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If $\liminf_{n\to\infty} d(x_n, F) = 0$, or $\limsup_{n\to\infty} d(x_n, F) = 0$, where *d*(*x*,*F*) = inf{ $\|x - p\| : p \in F(T)$ }, then $\{x_n\}$ converges strongly to a point of *F(T)*.

Proof. In Theorem 3.7, set n=1 and S=I(the identity mapping) to obtain Corollary 3.10.

Theorem 3.11. Let all the hypothesis of Lemma 3.1 are satisfied. If $\lim_{n\to\infty}\left\|Sx_n - Tx_n\right\| = 0$ and there exists a nondecreasing function $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0, f(r) > 0$ for all $r \in (0,\infty)$, such that $f(d(Sx, F)) \le \|Sx - Tx\|$ for all $x \in C$,
where *d*(S*x*,*F*) = inf{ $\|Sx - p\| : p \in F$ },

Then $\{Sx_n\}$ converges strongly to a point of *F*.

Proof. By hypothesis we have

$$\lim_{n\to\infty} f(d(Sx_n, F)) \le \lim_{n\to\infty}\left\|Sx_n - Tx_n\right\| = 0.$$

Hence $\lim_{n\to\infty} f(d(x_n, F)) = 0.$

Since $f : [0,\infty) \to [0,\infty)$ is a non decreasing function satisfying $f(0) = 0, f(r) > 0$ for all $r \in (0,\infty)$, so we have

$$\lim_{n\to\infty} d(x_n, F) = 0.$$

Thus $\liminf_{n\to\infty} d(Sx_n, F) = \limsup_{n\to\infty} d(Sx_n, F) = 0.$

Therefore by Theorem 3.7 $\{Sx_n\}$ converges strongly to a point of *F*.

Corollary 3.12. Let *C* be a nonempty closed convex subset of a uniformly convex Banach space *E*. Let $T, S : C \to C$ be Jungck- nonexpansive maps and $F \ne \phi$.. Let $\{Sx_n\}$ be defined by the iterative procedure (1.1.12) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If there exists a nondecreasing function $f : [0,\infty) \to [0,\infty)$ with $f(0) = 0, f(r) > 0$ for all $r \in (0,\infty)$, such that $f(d(Sx, F)) \le \|Sx - Tx\|$ for all $x \in C$, where *d*(S*x*,*F*) = inf{ $\|Sx - p\| : p \in F$ }, then $\{Sx_n\}$ converges strongly to a point of *F*.

Proof. In Theorem 3.11, take n=1 to obtain Corollary 3.12.

Corollary 3.13. Let *C* be a nonempty closed convex subset of a uniformly convex Banach space *E*. Let $T : C \to C$ be an asymptotically nonexpansive mapping with $\sum_{n=1}^{\infty}\left(k_n^2 - 1\right) < \infty$ and *F(T)* $\ne \phi$ Let $\{x_n\}$ be defined by the iterative procedure (1.1.8) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If T satisfies Condition(A) and $\lim_{n\to\infty}\left\|x_n - Tx_n\right\| = 0$ then $\{x_n\}$ converges strongly to a point of *F(T)*.

Proof. In Theorem 3.11, choose S=I(the identity mapping) to obtain Corollary 3.13.

Corollary 3.14. Let *C* be a nonempty closed convex subset of a uniformly convex Banach space *E*. Let $T : C \to C$ be a nonexpansive mapping and *F(T)* $\ne \phi$. Let $\{x_n\}$ be defined by the iterative procedure (1.1.4) where $\{a_n\},\{b_n\}$ are in [0,1] for all $n \in N$. If T satisfies Condition (A) then $\{x_n\}$ converges strongly to a point of *F(T)*.

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Proof. In Theorem 3.11, choose n=1 and S=I (the identity mapping) to obtain Corollary 3.14.

Finally, we give an example for comparing the rate of convergence of Jungck Modified Picard, Jungck Modified Mann, Jungck Modified Ishikawa and Jungck Modified S-Iterative procedure.

Example 3.15. Let $E = R$, $C = [0,1]$ and $S, T : C \to C$ are defined as $Sx = 1 - x$ and $Tx = \frac{2x+1}{4}$. Then T and S are Jungck- asymptotically nonexpansive mappings and 0.5 is the unique common fixed point.

The comparison of the rate of convergence of the Jungck-Modified Picard, Jungck-Modified Mann, Jungck-Modified Ishikawa and Jungck-Modified S Iterative procedure to a common fixed point of S and T is shown in the following table, with $x_1 = 0.3$ and $a_n = b_n = 0.1$ for all iterative procedures.

No. of Iterations	**Jungck Modified S Iterative Procedure**			**Jungck Modified Picard Iterative Procedure**			**Jungck Modified Mann Iterative Procedure**			**Jungck Modified Ishikawa Iterative Procedure**		
n	$\mathbf{Sx_{n+1}}$	$\mathbf{Tx_n}$	$\mathbf{x_{n+1}}$	$\mathbf{Sx_{n+1}}$	$\mathbf{Tx_n}$	$\mathbf{x_{n+1}}$	$\mathbf{Sx_{n+1}}$	$\mathbf{Tx_n}$	$\mathbf{x_{n+1}}$	$\mathbf{Sx_{n+1}}$	$\mathbf{Tx_n}$	$\mathbf{x_{n+1}}$
1	0.6	0.4	0.4	0.6	0.4	0.3	0.33	0.4	0.67	0.69	0.4	0.31
2	0.55	0.45	0.45	0.55	0.45	0.4	0.6445	0.585	0.3555	0.6805	0.405	0.3195
3	0.525	0.475	0.475	0.525	0.475	0.45	0.377175	0.42775	0.622825	0.671475	0.40975	0.328525
4	0.5125	0.4875	0.4875	0.5125	0.4875	0.475	0.604401	0.561412	0.395599	0.662901	0.414262	0.337099
5	0.50625	0.49375	0.49375	0.50625	0.49375	0.4875	0.411259	0.447799	0.588741	0.654756	0.418549	0.345244
6	0.503125	0.496875	0.496875	0.503125	0.496875	0.49375	0.57543	0.544371	0.42457	0.647018	0.422622	0.352982
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18	0.500001	0.499999	0.499999	0.500001	0.499999	0.499998	0.510729	0.506311	0.489271	0.579443	0.458188	0.420557
19	**0.5**	**0.5**	**0.5**	0.5	0.5	0.499999	0.49088	0.494635	0.50912	0.575471	0.460279	0.424529
20	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	0.507752	0.50456	0.492248	0.571697	0.462265	0.428303
21	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	**0.5**	0.493411	0.496124	0.506589	0.568112	0.464151	0.431888
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79							0.499999	0.5	0.500001	0.503477	0.49817	0.496523
80							**0.5**	**0.5**	**0.5**	0.503303	0.498262	0.496697
81							**0.5**	**0.5**	**0.5**	0.503138	0.498348	0.496862
---										---	------	----
250										0.500001	0.5	0.499999
251										0.500001	0.5	0.499999
252										**0.5**	**0.5**	**0.5**
253										**0.5**	**0.5**	**0.5**

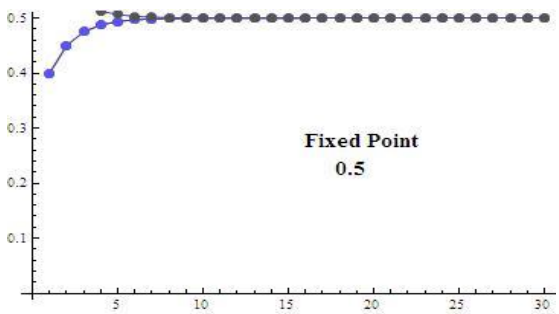

Fig 1. Graphical observations of Jungck modified S iterative procedure .The merging point with value 0.5 is fixed point.

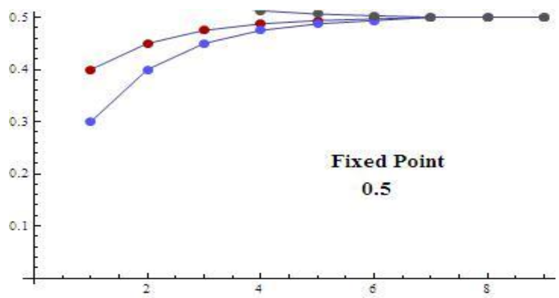

Fig 2. Graphical observations of Jungck modified Picard iterative procedure. The merging point with value 0.5 is fixed point.

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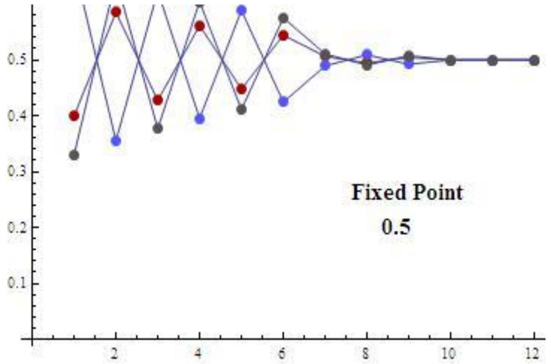

Fig 3. Graphical observations of Jungck modified Mann iterative procedure. The merging point with value 0.5 is fixed point.

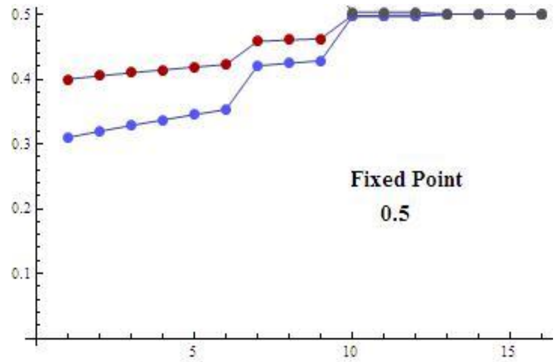

Fig.4. Graphical observations of Jungck modified Ishikawa iterative procedure. The merging point with value 0.5 is fixed point.

III. CONCLUSION

By observing the above table and graphs, we conclude that the decreasing rate of convergence of iterative schemes is as follows: Jungck Modified S, Jungck Modified Picard, Jungck Modified Mann, Jungck Modified Ishikawa Iterative procedure. Thus Jungck Modified S iterative procedure converges faster than Jungck Modified Picard, Jungck Modified Mann and Jungck Modified Ishikawa iterative procedures.

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