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N x 2 Flow Shop Scheduling Model Using Branch and Bound Technique, Set up Times are Separated from Processing Times, With Job Block Criterion And Interval Of Non Availability Of Machines

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***Abstract:** This paper presents a branch and bound technique that solves the two stage flowshop scheduling problem in which processing times and set up times both are associated with their respective probabilities, an equivalent job for a group job block, setup times are separated from processing times of jobs and the interval of non availability of machines is being considered, to minimize the make span. The proposed method is very easy to understand. The algorithm is demonstrated by a numerical example.*

Keywords: **Branch and Bound, Flow shop, Make Span, Equivalent Job block, unavailability of Machine**

I. INTRODUCTION

Scheduling is the process of assigning activities to resources over time. In the general case, scheduling is a decision making process. It decides when an activity should start and which resources should be used for each activity. In scheduling theory the basic model assumes that all machines are continuously available for processing throughout the planning horizon. This assumption might be justified in some cases but it does not apply if certain maintenance requirements, breakdowns or other reasons that cause the machines not to be available for processing have to be considered. In flow shop scheduling problems, the objective is to obtain a sequence of jobs which when processed on the machines will optimize some well defined criterion. Every job will go on these machines in a fixed order of machines. Johnson[5] gave the algorithm for obtaining an optimal schedule which minimizes makespan for n-jobs, two machines problem and also for particular cases of n-jobs, three machines problem in which the minimum processing time of the jobs on the first machine is greater than or equal to the maximum processing time of the jobs on the second machine or the minimum processing time of the jobs on the third machine is greater than or equal to the maximum processing time of the jobs on the second machine. The general nx3 problem was solved by Lomnicki [6], who provided the Branch and Bound technique for makespan minimization. Adiri[1] studied Single machine Flow-Shop scheduling with a single Breakdown. Narain [7] studied the flow shop problem with the objective being minimum total rental cost. Narain and Bagga [8] provided an optimal solution for a three machine flow shop problem under a no-idle constraint with the objective being minimum total elapsed time. Gupta D. and Bala S.[3], studied two stage specially structured flow shop scheduling problem.

Ignall and Scharge [4], solved the nx2 problem by Branch and Bound technique, for finding an optimal sequence which minimizes mean flowtime. Chandrasekharan [2] has given a technique based on the branch and bound method and satisfaction of certain conditions to obtain a sequence which minimizes total flow time subject to minimum makespan, in two stage flow shop problem.

The present paper deals with general n-job,2-machine flow shop scheduling problem in which set up times are separated from processing time of jobs, both are being associated with their respective probabilities involving interval of non availability of machines and the concept of an equivalent job block is taken into consideration. Thus the problem discussed here is wider and practically more applicable and has significant results in the process industry.

II. PRACTICAL SITUATION

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. In centralized maintenance system where sets of tasks may be scheduled on equipments distributed on different geographic sites, the travelling time between two sites is considered as a setup time. In several real life situations set up times should be considered apart from processing times of the jobs, which allows having a more accurate solution. Sometimes the priority of one job over the other is preferred. It may be because of urgency or demand of its relative importance, the job block criteria becomes important. The practical situation may be taken in a production industry; manufacturing industry etc. when the machines are not continuously available due to electric current, breakage of machine etc., the study of break down interval becomes significant.

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Notations

S : Sequence of jobs 1, 2, 3,….,n
a_i : Processing time for job i^{th} on machine A
b_i : Processing time for job i^{th} on machine B
p_i : Probability associated to the processing time a_i.
q_i : Probability associated to the processing time b_i.
A_i : Expected Processing time for i^{th} job on machine A
B_i : Expected Processing time for i^{th} job on machine B.
s_i^A : Set up time of i^{th} job on machine A
s_i^B : Set up time of i^{th} job on machine B
p_i^A : Probability associated to the set up time s_i^A
q_i^B : Probability associated to the set up time s_i^B
S_i^A : Expected set up time for i^{th} job on machine A
S_i^B : Expected set up time for i^{th} job on machine B
Jr : Partial schedule of r scheduled jobs
Jr′ : The set of remaining (n-r) free jobs
(s_j, t_j) : M_j is unavailable from s_j to t_j, where $0 \le s_j \le t_j$
L_{fj} : Starting time of the i th job on Machine M_j.
U_{fj} : Completion time of the i th job on Machine M_j.
$t(J_r, j)$: The time when the last job of the assigned schedule Jr is completed on machine j , where j = 1,2.

III. PROBLEM FORMULATION

Consider n jobs say i=1, 2, 3 … n are processed on two machines A & B in the order AB. Let a_i and b_i be the processing times of ith job on machine A and machine B with probability p_i and q_i respectively. Let s_i^A & s_i^B be the set up times of ith job on machine A and machine B with probabilities p_i^A and p_i^B respectively. A_i and B_i be the expected processing times of i^{th} job with S_i^A & S_i^B be the expected set up times on each machine respectively. Let an equivalent job β is defined as (k, m) where k, m are any jobs among the given n jobs such that k occurs before job m in the order of job block (k , m). Let (s_j, t_j) be the interval of unavailability of machines. The mathematical model of the problem in matrix form can be stated as:

Our objective is to obtain optimal or near optimal sequence S_0 of the jobs which minimize the total elapsed time, using branch and bound technique.

Step 1: Calculate

(i) $A_i = a_i \times p_i$ (ii)
$B_i = b_i \times q_i$

Step 2: Calculate

(i) $S_i^A = a_i^A \times p_i^A$ (ii)
$S_i^B = b_i^B \times q_i^B$

Step 3: Calculate

(i) $A'_i = A_i - S_i^B$ (ii)
$B'_i = B_i - S_i^A$

Step 4 Find the expected processing time of job block β = (k, m) on machines A and B using equivalent job block criterion given by Maggu and Das (1977). Find A'_β and B'_β using $A'_\beta = A'_k + A'_m - \min(A'_m, B'_k)$

$$B'_\beta = B'_k + B'_m - \min(A'_m, B'_k)$$

Step 5 Define a new reduced problem with the processing time A'_i and B'_i as defined in step 3 and replacing job block β = (k, m) by a single equivalent job β with processing time A'_β and B'_β as defined in step 4

Step 6: Calculate Completion time

$$t(J_r,1) = \begin{cases} A'_1 + t_1 - s_1 & \text{if } (L_{f1}, U_{f1}) \cap (t_1, s_1) \ne \phi \\ A'_1 & \text{otherwise} \end{cases}$$

and

$$t(J_r,2) = \begin{cases} B'_i + t_2 - s_2 & \text{if } (L_{f2}, U_{f2}) \cap (t_2, s_2) \ne \phi \\ B'_i & \text{otherwise} \end{cases}$$

Step 7: Calculate

(i) $l_1 = t(J_r,1) + \sum_{i \in J'_r} A'_i + \min_{i \in J'_r}(B'_i)$

(ii) $l_2 = t(J_r,2) + \sum_{i \in j'_r} B'_i$

Step 8: Calculate $l = \max(l_1, l_2)$

Evaluate l for the n classes of permutations, i.e., for these starting with 1,2,…n respectively. Explore the lowest lower bound vertex for (n-1) subclasses and again concentrate on the lowest label vertex. Thus we get the optimal/near optimal sequence.

Step 9 : Prepare in-out table for the optimal sequence obtained in step 8 and get the minimum total elapsed time.

Numerical Example:

Consider 5 jobs 2 machine flow shop scheduling problem whose processing time and set up time of the jobs on each machine is given in Table 2. Our objective is to obtain optimal schedule for above said problem in which jobs 2,4 are to be processed as a group job (2,4) and the break down effect on machine A and on machine B is (35,40) & (60-65) respectively ,which minimizes the makespan.

Solution: As per Step1& step 2 Expected processing time and expected set up time:

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Table 3

As per Step 3:

Node	LB(Jr)
1	125.7
β	126
3	113.9
5	111.9
51	128.4
5 β	123.7
53	111.9
531	135.1
53 β	130.3

Jobs	Machine A		Machine B	
i	A_i	S_i^A	B_i	S_i^B
1	30	0.6	25	0.8
2	22	1.5	36	1.4
3	18	1.2	12	0.6
4	39	1.4	45	0.6
5	16	0.3	20	0.6

Table-4

$A'_\beta = A'_2 + A'_4 - \min(A'_4, B'_2)$ = 20.6 + 38.4 – min(38.4,34.5) = 24.5

$B'_\beta = B'_2 + B'_4 - \min(A'_4, B'_2)$ = 43.6

Reduced problem is as follows:

Jobs	Machine A	Machine B
i	A'_i	B'_i
1	29.2	24.4
β	24.5	43.6
3	17.4	10.8
5	15.4	17.7

Table 5

As per step 7 and step 8 the calculated Lower bounds are shown in the tableau-6

Jobs	Machine A	Machine B
i	A'_i	B'_i
1	29.2	24.4
2	20.6	34.5
3	17.4	10.8
4	38.4	43.6
5	15.4	17.7

Table-6

Thus, the optimal sequence is 5 – 3 - β - 1. i.e., 5 – 3 – 2- 4 - 1 and the In- Out table is as in Table-7

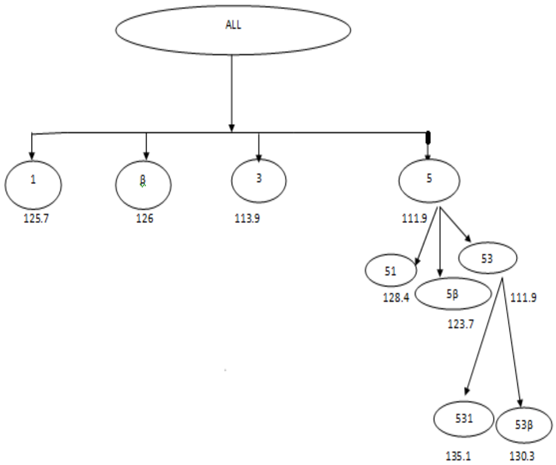
Table-7

Job i	Machine A In-out	Machine B In-out
5	0 -16	16 – 36
3	16.3 – 34.3	36.6 – 48.6
2	35.5– 57.5	57.5–93.5
4	59.0–98.0	98.0–143.0
1	99.4–129.4	143.6–168.6

Minimum elapsed time is 168.6 units.

IV. BRANCHING TREE

Problem is showed by using the tree structure where each node is a partial schedule. To determine the best partial schedule node to the branch, the lower bound value for completion times are calculated and the node having the least value is chosen. The procedure is repeated at each time by branching from lowest bound. After obtaining an order where all jobs are scheduled, the nodes having upper of the lower-bounds then the completion time of this schedule are fathomed. The tree is fathomed when no more branching is possible.



V. CONCLUSION

Heuristic methods do not guarantee the optimality of solution found. While the branch and bound method is the exact method to find optimal solution of the problem and confirms its optimality. Work can be further extended by considering transportation time, job weightage etc.

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APPENDIX

Table 1

Jobs	Machine A				Machine B			
i	a_i	p_i	s_i^A	p_i^A	b_i	q_i	s_i^B	q_i^B
1	a_1	p_1	s_1^A	p_1^A	b_1	q_1	s_1^B	q_1^B
2	a_2	p_2	s_2^A	p_2^A	b_2	q_2	s_2^B	q_2^B
3	a_3	p_3	s_3^A	p_3^A	b_3	q_3	s_3^B	q_3^B
-	-	-	-	-	-	-	-	-
n	a_n	p_n	s_n^A	p_n^A	b_n	q_n	s_n^B	q_n^B

Table 2

Jobs	Machine A				Machine B			
i	a_i	p_i	s_i^A	p_i^A	b_i	q_i	s_i^B	q_i^B
1	300	0.1	3	0.2	250	0.1	8	0.1
2	110	0.2	5	0.3	180	0.2	7	0.2
3	60	0.3	6	0.2	120	0.1	3	0.2
4	195	0.2	7	0.2	150	0.3	2	0.3
5	80	0.2	3	0.1	100	0.2	3	0.2