A Simple Comparative Evaluation of Adaptive Beam forming Algorithms

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Abstract- Adaptive Antennas can be used to increase the capacity, the link quality and the coverage of the existing and future mobile communication networks. Using beam forming algorithms the weight of antenna arrays can be adjusted to form certain amount of adaptive beam to track corresponding users automatically and at the same time to minimize interference arising from other users by introducing nulls in their directions. This paper presents a simulation test-bed of a smart antenna system for the comparative performance evaluation of various adaptive beam forming algorithms and the smart antenna itself. The adaptive beam forming algorithms simulated and analyzed in this work include the Least Mean Square (LMS), Direct Matrix Inverse (DMI), Recursive Least Square (RLS), Constant Modulus Algorithm (CMA), and Least Square-Constant Modulus Algorithm (LSCMA) algorithms. The results show that LMS algorithm requires about 53 iterations before it can finally converge. The DMI algorithm is computed for a number of samples; hence it does not require iterations in its calculations. It has faster convergence than LMS but characterized by numerical instability and increased computational complexity due to matrix inversions. The RLS algorithm requires less number of iterations (about 10 iterations before it converges). This simulation and performance evaluation is done using MATLAB platform as the simulator.

Keywords: Adaptive Antennas, Beam forming Algorithm, Signal Nulling, Performance Evaluation.

I. INTRODUCTION

Conventional base station antennas in existing operational systems are either omni-directional or sectorised. There is a waste of resources since the vast majority of transmitted signal power radiates in directions other than toward the desired user. In addition, signal power radiated throughout the cell area will be experienced as interference by any other user than the desired one. Concurrently the base station receives interference emanating from the individual users within the system Smart Antennas offer a relief by transmitting/receiving the power only to/from the desired directions. Smart Antennas can be used to achieve different benefits. The most important is higher network capacity. It increase network capacity [1], [2] by precise control of signal nulls quality and mitigation of interference combine to frequency reuse reduce distance (or cluster size), improving capacity. It provides better range or coverage by focusing the energy sent out into the cell, multi-path rejection by minimizing fading and other undesirable effects of multi-path propagation. The adaptive antenna system is a new technology and has been applied to the mobile communication system such as GSM and CDMA [3]. It will be used in 3G mobile communication system or IMT 2000 also. Adaptive antenna can be used to achieve different benefits. By providing higher network capacity, it increases revenues of network operators and gives customers less probability of blocked or dropped calls. A adaptive antenna consists of number of elements (referred to as antenna array), whose signals are processed adaptively in order to exploit the spatial dimension of the mobile radio channel. All elements of the adaptive antenna array [4], [5] have to be combined (weighted) in order to adapt to the current channel and user characteristics. This weight adaptation is the “smart” part of the adaptive antenna. The adaptive antenna systems approach communication between a user and base station in a different way, in effect adding a dimension of space. By adjusting to an RF environment as it changes, adaptive antenna technology can dynamically alter the signal patterns to near infinity to optimize the performance of the wireless system. Adaptive arrays utilize sophisticated signal-processing algorithms to continuously distinguish between desired signals, multipath, and interfering signals as well as calculate their directions of arrival. This approach continuously updates its transmit strategy based on changes in both the desired and interfering signal locations. Adaptive Beam forming [5] is a technique in which an array of antennas is exploited to achieve maximum reception in a specified direction by estimating the signal arrival from a desired direction (in the presence of noise) while signals of the same frequency from other directions are rejected. This is achieved by varying the weights of each of the sensors (antennas) used in the array. It basically uses the idea that, though the signals emanating from different transmitters occupy the same frequency channel, they still arrive from different directions. This spatial separation is exploited to separate the desired signal from the interfering signals. Rani, Subbaiah, and Reddy, in [6] discussed the adaptive beam forming approach for Smart antennas and adaptive algorithms used to compute the complex weights in a WCDMA mobile environment. Bahri and Bendimerad in [7] proposed a downlink multiple-input multiple-output multiple-carrier code division multiple access system with the Least Mean Square adaptive algorithm for Smart antennas. In [8], Shubair, Mahmoud, and Samhan developed a setup for the evaluation of the MUSIC and LMS algorithms for a Smart antenna system. The authors presented a practical design of a Smart antenna system based on direction-of-arrival estimation and adaptive beam forming. Susmita Das in his work [9] provides description, comparative analysis and utility of various
reference signal based algorithms as well as blind adaptive algorithms. Thomas Biedka [10] presented a framework for the development and analysis of blind adaptive beam forming algorithms for Smart antenna system. The authors give an exclusive summary of concepts, measurements, and parameters and validate results from research conducted within the scope of their work. This work extensively x-rayed the adaptive beam forming algorithms, in terms of their performance in the areas of forming main lobes, interference suppression, convergence rate and computational complexities. The evaluation and analysis is done using MATLAB as a simulator.

II. BEAMFORMING

Using multiple antennas in a receiver can reduce the effects of co-channel interference, multipath fading and background noise. An array forms an improved estimate of the desired signal by weighting and summing the signals received at multiple spatially separated antennas. By appropriately selecting the weights, high gain can be placed in the direction of a desired signal and low gain can be placed in the direction of interfering signals. This process is often referred to as beam forming or spatial filtering [11]. The weighting applied to the signals received at each antenna may be fixed or may be continuously adjusted to track changes in the signal environment. Beam forming creates the radiation pattern of the antenna array by adding the phases of signals in the desired direction and by nulling the pattern in the unwanted direction. The phases (the inter-element phases) and usually amplitudes are adjusted to optimize the received signal.

A. Fixed Weight Beamforming

The basic objective of a beam former is to adjust the complex weights at the output of each array element so as to produce a pattern that optimizes the reception of a target signal along the direction of interest, in some statistical sense.

B. Minimum Mean-Square Error

In this method of optimizing the array weights, the shape of the desired received waveform is known by the receiver. Complex weights are adjusted to minimize the Mean Square Error (MSE) between the beamformer output and the expected signal waveform. The output of the array is given as

\[ y = W^H X(t) \]  

The error signal is given as

\[ \epsilon(t) = d(t) - W^H X(t) \]  

Where \( d(t) \) the reference is signal, and \( W^H X(t) \) is the array output. The Mean Square Error (MSE) is given by

\[ \epsilon^2(t) = [d(t) - W^H X(t)]^2 \]  

Taking expectation on both sides of the above equation we obtain

\[ E[\epsilon^2(t)] = E[[d(t) - W^H X(t)]^2] \]  

\[ E[\epsilon^2(t)] = E[[d(t)]^2 - 2W^HE[d(t)X(t)] + E[W^H X(t)] \]  

\[ E[\epsilon^2(t)] = E[[d^2(t)]] - 2W^HR_{xx}X + W^HR_{xx}W \]  

\[ r = E[[d(t)X(t)]] \]  

\[ R_{xx} = E[XX^H] = R_{ss} + R_{uu} \]  

Where \( R \) is the cross correlation matrix between the desired signal and the received signal? \( R_{xx} \) is the auto correlation matrix of the received signal, and \( R_{ss} \) is the source (desired signal) correlation matrix, and \( R_{uu} \) is the undesired (noise) signal correlation matrix. The minimum MSE can be obtained by taking the gradient of the MSE with respect to the weight vectors and equating it to zero.

\[ \nabla_W(E[\epsilon^2(t)]) = 2R_{xx}W - 2R = 0 \]  

Therefore the optimum solution for the weight vector \( W \) is given by

\[ W_{MSE} = R_{xx}^{-1}R \]  

C. Maximum Signal-To-Interference Ratio

In this method of optimizing the array weights, the receiver can estimate the strength of the desired signal and of an interfering signal, and weights are adjusted to maximize the ratio. The weight array output power for the desired signal is given as
\[ \sigma_x^2 = E[[W^H X(t)]^2] = W^H R_{xx} W \]  
(11)

Also the weight array output power for the undesired signal is given as
\[ \sigma_u^2 = E[[W^H U(t)]^2] = W^H R_{uu} W \]  
(12)

The Signal-to-Interference Ratio (SIR) is defined as the ratio of the desired signal power to the undesired signal power
\[ SIR = \frac{\sigma_x^2}{\sigma_u^2} = \frac{W^H R_{xx} W}{W^H R_{uu} W} \]  
(13)

The maximum SIR can be obtained by taking the derivative with respect to \( W \) and setting the result equal to zero.
\[ R_{uu}^{-1} R_{ss} W = SIR W \]  
(14)

The maximum SIR is equal to the largest Eigen value for the Hermitian matrix \( R_{uu}^{-1} R_{ss} \). And the Eigen vectors associated with the largest Eigen value is the optimum weight vector \( W_{opt} \).
\[ R_{uu}^{-1} R_{ss} W_{SIR} = \lambda_{max} W_{opt} = SIR_{max} W_{SIR} \]  
(15)

\[ W_{SIR} = \beta R_{uu}^{-1} \alpha_0 \]  
(16)

Where
\[ \beta = \frac{E[[S]^2]}{SIR_{max} \alpha_0^H W_{SIR}} \]  
(17)

D. Minimum Variance

In this method of optimizing the array weights, the signal shape and source direction are both known, the weights are then selected to minimize the noise on the beam former output. From the weighted array output given as
\[ y = W^H X(t) = W^H \alpha_0 s + W^H U \]  
(18)

To ensure a distortion less response, then
\[ W^H \alpha_0 = 1 \]  
(19)

Therefore we have that
\[ y = S + W^H U \]  
(20)

Taking the expectation on both sides assuming that the unwanted signal has zero mean,
\[ E[y] = S \]  
(21)

The Variance of \( y \) is given as
\[ \sigma_{MV} = E[[W^H X]^2] = E[[S + W^H U]^2] = W^H R_{uu} W \]  
(22)

The Variance can be minimized by setting the gradient of a cost function equal to zero. The cost function is given as
\[ \nabla_W J(W) = R_{uu} W_{MV} - \lambda \alpha_0 = 0 \]  
(23)

Hence the minimum Variance optimum weights can be obtain by
\[ W_{MV} = \frac{R_{uu}^{-1} \alpha_0}{\alpha_0^H R_{uu} \alpha_0} \]  
(24)

Fixed weight beam forming systems are subject to degradation by various causes. The array SNR can be severely degraded by the presence of unwanted interfering signals, electronic countermeasures, clutter returns or multipath interference and fading. If the arrival angles of the emitters do not change with time, the optimum array weights would not need to be adjusted. However, if the desired arrival angles change with time, it is necessary to use adaptive algorithm that will update and compensate the array weight iteratively in order to track the desired user in the changing environment.

III. ADAPTIVE BEAMFORMING

In an ever-changing propagation environment, such as in a mobile cellular network, where the arrival angles of the emitters change continuously with time, fixed beam forming becomes ineffective. In such environment adaptive beam forming is used to overcome the problems of fixed beam forming [11]. Adaptive beam forming combines the inputs of multiple antennas (from an antenna array) to form very narrow beams toward individual users in a cell. An adaptive beam former is a device that is able to separate signals collocated in the frequency band but separated in the spatial domain. This provides a means for separating a desired signal from interfering signal. An adaptive beam former is able to automatically optimize the array pattern by adjusting the elements control weights until a prescribed objective function is satisfied. The means by which the optimization is achieved is specified by an algorithm designed for that purpose.

Fig2: Adaptive Beam Forming Block Diagram [1, 2]

The digital signal processor interprets the incoming data information, determines the complex weights (amplitude and phase information) and multiplies the weights to each
element output to optimize the array pattern. From the figure above the output response of the uniform linear array is given as,

\[ y(t) = W^H X(t) \]  \hspace{1cm} (25)

Where \( W \) is the complex weights vector and \( X \) is the received signal vector?

The complex weights vector is obtained using an adaptive beam forming algorithm. Adaptive beam forming algorithms are classified as Direction of Arrival (DOA)-based, temporal-reference-based or signal-structure-based. In DOA-based beam forming, the direction of arrival algorithm passes the DOA information to the beam former. The beam forming algorithm is then used to form radiation patterns, with the main beam directed towards the signal of interest and with nulls in the directions of the interferers. On the other hand, temporal-reference-based beam forming uses a known training sequence to adjust the weights and to form a radiation pattern with a maximum towards the signal of interest. If \( d(t) \) denotes the referenced sequence or the training symbol known a priori at the receiver at time \( t \), an error \( \varepsilon(t) \) is formed as

\[ \varepsilon(t) = d(t) - W^H X(t) \]  \hspace{1cm} (26)

This error signal is used by the beam former to adaptively adjust the complex weights vector, so that the Mean Square Error (MSE) is minimized. The choice of weights that minimize the MSE is such that the radiation pattern has a beam in the direction of the source that is transmitting the reference signal, and that there are nulls in the radiation pattern in the directions of the interferers.

**IV. ADAPTIVE BEAMFORMING ALGORITHMS**

Using the information supplied by the DOA, the adaptive algorithm computes the appropriate complex weights to direct the maximum radiation of the antenna pattern toward the desired user and places nulls toward the directions of the interferers. There are several adaptive algorithms used for Smart antenna system, they are typically characterized in terms of their convergence properties and computational complexity. The adaptive algorithms considered in this research work include;

1. **Direct Matrix Inversion (DMI) Algorithm**

In the Direct Matrix Inversion (DMI) algorithm, the adaptive beamformer uses a block of data to estimate the complex weights vector. These weights are computed from the estimate of the covariance matrix. The DMI weights can be calculated for the \( n \)th block of data of length \( k \) as

\[ W_{DMI}(n) = R_{xx}^{-1}(n)r(n) \]  \hspace{1cm} (27)

Where \( R_{xx} \) is the array correlation matrix given by

\[ R_{xx} = \frac{1}{k} X_k(n) X_k^H(n) \]  \hspace{1cm} (28)

Similarly \( r(n) \) is the correlation vector given as

\[ r(n) = \frac{1}{k} d(n) X_k(n) \]  \hspace{1cm} (29)

The accuracy of the estimate of this matrix inversion increases as the number of data samples received increases, because the DMI is a time average estimate of the array correlation matrix using k-time samples. The DMI algorithm although faster than the Least Mean Square algorithm has several drawbacks, which include the computational burden and the potential singularities can cause problems. The correlation matrix may be ill conditioned resulting in errors or singularities when inverted, also for large arrays; there is the challenge of inverting large matrices.

2. **Constant Modulus Algorithm (CMA)**

The configuration of the Constant Modulus Algorithm (CMA) adaptive beam forming is similar to that of the Direct Matrix Inversion (DMI), except that it does not require any reference signal. CMA is also called “blind” beam forming algorithm, since it does not make use of the reference signal. CMA is a gradient-based algorithm that works on the theory that the existence of interference causes changes in the amplitude of the transmitted signal, which otherwise has a constant envelope (modulus). This algorithm tries to restore the amplitude of the original signal, by updating the complex weights with the equation given as;

\[ W_{(n+1)} = W_{(n)} + \mu d(n)e(n) \]  \hspace{1cm} (30)

Where \( \mu \) is the step-size parameter and the error \( e(n) \) is given by

\[ e(n) = y(n)(R_2 - |y(n)|^2) \]  \hspace{1cm} (31)

With

\[ R_2 = \frac{E[|X(n)|^4]}{E[|X(n)|^2]^2} \]  \hspace{1cm} (32)

One of the attractive features of the CMA is that carrier synchronization is not required; furthermore it can be applied successfully to non-constant modulus signals if the Kurtosis of the beamformer output is less than two. This means that CMA can be applied to for example PSK signals that have non-rectangular pulse shape. This is an important because it implies that the CMA is also robust to symbol timing error when applied to pulse-shaped PSK signals. Pulse shaping typically is used to limit the occupied bandwidth of the transmitted signal.

3. **Least Square-Constant Modulus Algorithm (LS-CMA)**

One severe disadvantage of the Godard CMA algorithm is the slow convergence time. The slow convergence time
limits the usefulness of the algorithm in dynamic environment where the signal must be captured quickly. A faster converging CMA algorithm similar in form to the Recursive Least Square (RLS) method is the Orthogonalized-CMA. Another fast converging CMA is the Least Square CMA (LS-CMA) which is a block update iterative algorithm that is guaranteed to be stable and easily implemented. At the n-iteration, nth signal samples of the beamformer output are generated using the current weight vector \( W_n \). This gives

\[ y_k(n) = W^H X(n) \]  

The initial weight vector \( W_0 \) can be taken as

\[ W_0 = [1 \ 0 \ 0 \ldots \ 0]^T \]  

if no a priori information is available. The nth signal estimate is then hard limited to yield

\[ d_k(n) = \frac{y_k(n)}{|y_k(n)|} \]  

and a new weight vector is formed according to

\[ W_{n+1} = R_{xx}^{-1} r_{xd} \]  

Where,

\[ R_{xx} = (X(n)X^H(n))^N \] \hspace{1cm} (37)

\[ r_{xd} = (X(n)d_k * (n))^N \] \hspace{1cm} (38)

Equations (26) and (27) denote a time average over \( 0 \leq n \leq N - 1 \). The update weight vector \( W_{n+1} \) minimizes the mean square error. The iteration described above continues until either the change in the weight vector is smaller than some threshold or until the envelope variance of the output signal is deemed sufficiently small. When the iteration is performed using a new block of data it is known as dynamic LSCMA. But when it is re-applied to the same block of data it is known as static LSCMA.

4. Least Mean Square (LMS) Algorithm

The Least Mean Square (LMS) algorithm uses a gradient based method of steepest decent [12]. This algorithm uses the estimate of the gradient vector from the available data. This algorithm computes the complex weights vector recursively using the equation, given as;

\[ W(n + 1) = W(n) + \mu X(n)[d^*(n) - X(n)W^H(n)] \] \hspace{1cm} (39)

Where \( \mu \) is the step size parameter and controls the convergence characteristics of the LMS algorithm. The LMS algorithm is initiated with an arbitrary value of \( W(0) \) for the weight vector at \( n = 0 \). The successive corrections of the weight vector eventually leads to minimum value of the Mean Square Error (MSE). The LMS algorithm is important because of its simplicity and ease of computation, because it does not require off-line gradient estimations or repetition of data. One of the drawbacks of the LMS adaptive scheme is that the algorithm must go through many iterations before satisfactory convergence is achieved. If the signal characteristics are rapidly changing, the LMS algorithm may not allow the tracking of the desired signal in a satisfactory manner. The rate of convergence of the weight is dictated by the Eigen value spread of the correlation matrix, given as

\[ 0 < \mu < \frac{1}{\lambda_{\text{max}}} \] \hspace{1cm} (40)

Where \( \lambda_{\text{max}} \) is the largest eigenvalue of the correlation matrix \( R_{xx} \).

5 Recursive Least Square Algorithm (RLS)

The Recursive Least Square (RLS) algorithm was developed to solve the problem of slow convergence speed in an environment yielding an array correlation matrix with large Eigen value spread. This is achieved by making its convergence independent of the Eigen values distribution of the correlation matrix. In RLS algorithm, the weights are updated using the equation below,

\[ W(n) = W(n - 1) + k(n)e^*(n) \] \hspace{1cm} (41)

Where \( k(n) \) is referred to as the gain vector and \( e(n) \) is a prior estimation error which is given as

\[ e(n) = d(n) - W^H(n - 1)U(n) \] \hspace{1cm} (42)

The RLS algorithm does not require any matrix inversion computations as the inverse correlation matrix is computed directly. It requires reference signal and correlation matrix information. An important feature of the RLS is that its rate of convergence is typically an order of magnitude faster than that of the LMS algorithm, due to the fact the RLS algorithm convergence is independent of the Eigen values distribution of the correlation matrix. This improvement however is achieved at the expense of an increase in the computational complexity of the Recursive Least Square algorithm.

V. SIMULATION AND PERFORMANCE EVALUATION

For simulation purpose and analysis the uniform linear array with \( N = 8 \) number of elements is considered. The inter-element spacing is considered to be half wavelength. It is considered that the desired user is arriving at an angle of 30 degrees and an interferer at an angle of -60 degrees. The simulation is carried out on MATLAB platform. Figure 3 shows the beam pattern form with the DMI beamforming algorithm and figure 4 shows that according to equations (28) and (29) that the accuracy...
and the performance of the DMI algorithm increases as the number of sample data received increases.

Figure 3 shows the array factor plot and how the CMA algorithm has suppressed the multipath signals while directing maximum to the direct path signal. From this figure we can verify that the CMA algorithm has a slow convergence time, and to overcome this problem of the CMA a fast convergence algorithm LSCMA is introduced as shown in figure 6.

Figure 5 shows the array factor plot for the desired user with AOA 30° and interferer with AOA -60°, the spacing between the elements is 0.5λ and the block sample is K = 50.

Figure 4 shows the array factor plot for the desired user with AOA 30° and interferer with AOA -60°, the spacing between the elements is 0.5λ and the block sample is K = 100.

Figure 7 shows the polar plot of the LMS algorithm, while figure 8 shows the weighted LMS array plot. Figure 9 shows the array factor plot and how the LMS algorithm places deep null in the direction of the interfering signal and maximum in the direction of the desired signal. Figure 10 shows that according to the condition stated in (40) using a larger value for the LMS adaptive step size...
\( \mu = 0.02 \) yields better results when compared to a smaller step size \( \mu = 0.002 \).

**Fig 7** Polar Plot Of Beam Pattern Of The LMS Algorithm When The Desired User With AOA 30 Deg And Interfere With AOA -60 Deg, The Spacing Between The Elements Is \( 0.5\lambda \).

**Fig 10** Shows That According To Equation (40) That Using A Larger Value For The LMS Adaptive Step Size \( \mu = 0.02 \) Yields Better Result When Compared To A Smaller Step Size \( \mu = 0.002 \).

Figure 11 shows that the rate of convergence of RLS algorithm is typically an order of magnitude faster than that of the LMS algorithm, due to the fact the RLS algorithm convergence is independent of the Eigen values distribution of the correlation matrix, as shown by equation (41).

**VI. CONCLUSION**

The significance of LMS algorithm cannot be ruled out in generating better main lobe in a specified direction of user and nulls in the interfering signal, The LMS is important because of its simplicity and ease of computation, however, its slow convergence presents an acquisition and tracking problem for cellular system. Simulation results revealed that RLS algorithm involves more computations than LMS; it provides safe side towards main lobe and has better response towards co channel interference. It has been revealed as well that convergence rate of RLS is faster than LMS. The effect of changing step size for LMS algorithm has also been
studied. RLS Algorithm is found to have minimum BER and error signal magnitude, therefore it has been proved the best algorithm for implementation on Base Station. While Constant Modulus Algorithm (CMA) has satisfactory response towards beamforming and it gives better outcome for interference rejection, but one of its major drawbacks is the slow convergence which the LSCMA implementation tends to address.

REFERENCES


