An Investigation into Differential Torque Based Strategies for Electronic Stability Control in an In-Wheel Electric Vehicle

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Abstract—This paper presents a differential torque based electronic stability control (ESC) for in-wheel electric vehicle (EV). To find a suitable and effective differential torque based ESC, several torque based strategies has been investigated. Analysis has been done here considering parameters used in control, control law, vehicle drive train and feasibility of implementation. After analysing several strategies of ESC for independent wheel control, suitable differential torque based ESC system architecture is presented with the simulation. A four in-wheel motors EV has been developed primarily as test bench for practical realization of this proposed differential torque based ESC and this EV can be used for further real-time tests. Simulation results show a promising possibility of implementation of this suitable differential torque based ESC in an in-wheel EV.

Index Terms—Differential Torque, Electronic Stability Control, Sliding Mode, In-Wheel.

I. INTRODUCTION

Pollution is of major concern due to increasing number of vehicles on the road. Vehicles are using fossil fuel as main source of energy but this gasoline is troubled with serious environmental hazards, economic and political risk. Electric vehicles are potential candidates towards sustainability if improvements continue in EV technology and infrastructure developments [1]. Increase in passenger vehicles is also increasing the incident of vehicle crash. Electronic Stability Control (ESC) is an active safety control system to assist the driver to maintain directional control of the ground vehicle in critical manoeuvrings. ESC improves controllability and prevents accident due to loss of control. Many studies have been done from different perspectives to realize the effectiveness of ESC in reducing loss of control and in accident prevention. A decreases of 24.6% is found in loss of control of vehicle when ESC is present [2]. ESC reduces 30-50% fatal crash of passenger car and it also reduces 50-70% fatal crash of Sports Utility Vehicle (SUV) [3]. If focus is given on the differential torque based ESC for electric vehicles, several modern techniques can be found where a centralized electric motor or multiple electric motors are being used. A number of techinics for differential torque based ESC are being followed by researchers for EVs in different research works. In recent development of electric vehicles, the drive trains are consists of centralized electric motors or two electric motors as rear wheel or front wheel drive. But most popular among the researcher and industries is the four independent electric motors in the hub of four wheels, they are known as in-wheel motors [4]. An advanced EV with in-wheel motors has a drive train consists of 4 in-wheel motors. To make the differential torque based ESC viable focus should be given on fully independent control of drive torque distribution. A recent development in EV is focusing fully independent control of wheel torque using in-wheel motors. In-wheel motors allow regulating the torque and braking with high precision and available built-in sensor inside the motors provides the rotational information of the wheel. This kind of flexibility can offer greater controllability in vehicle handling than a conventional EV. Currently there is no such type of vehicle in production. Swinburne University of Technology has developed a four in-wheel EV as shown in fig 1 which is used as a test bench in this research.

II. PROCEDURE FOR PAPER SUBMISSION

Electronic Stability Control (ESC) based on differential braking is popular and mostly available in the vehicles. In differential braking type ESC, differential brake force in each wheel is applied to control vehicle stability. Similar stability enhancement can be achieved by using independent torque control in each wheel. Differential braking based ESC has the demerit of slowing down the vehicle and drivers do not get the expected longitudinal response. To avoid this demerit, differential torque based ESC can be used for the same purpose. The key point of differential torque ESC is finding

![Fig.1. A four in-wheel EV](image-url)
clutch and gear are giving a higher degree of freedom for torque distribution but may not provide the precise required torque always. Investigation of different methods used in several research can provide know how to find a proper method suitable for a four in-wheel independently control electric vehicle. In this research, investigation is done into several differential torque based ESC research works considering the system parameters, control strategy, control law and implementation complexity.

A. Parameters and estimations

Several researches have been conducted on differential torque based stability controller considering independent wheel control using in-wheel EV. Vehicle stability can be improved by controlling driving force of the wheels which is the general concept of differential torque based ESC. It is essential to know the way and the required methods involved to determine the corrective wheel torque. Vehicle body dynamic forces are important parameters in ESC design and value of these parameters is achieved using a vehicle model. Similarly to determine the tire forces a tire model is required to use. All these parameters like vehicle body forces, tire forces, vehicle slip, yaw rate and coefficient of frictions are required for ESC. How these parameters are determined or estimated is important to discuss based on the previous research works. Discussion on these parameters and their estimation methods are given below.

1. Vehicle dynamics from vehicle model

Though different methods of control systems adopted in different research for ESC, vehicle model is similar in most of the research works. For the simulation of ESC and in real-time operation, a vehicle model is required to observe the responses of the vehicle body. Several research works are taken under review to observe the vehicle models used. In [5]-[7] vehicle models are considered for a four-wheel-drive electric vehicle with the equation of motion of vehicle dynamics, later on simulation result is presented using Carmaker software [6]. After designing ESC controller, simulation can be presented using numerical simulation software like CarSim or ADAMS/Car for detail observation. Vehicle models available in different simulating software tools may not be usable entirely for a simulation considered here in this research paper. Also a controller designed to control such vehicles may not be available in these software systems. Specific software with built in reconfigurable models and controller from other software tools show some constrains in model definition, defining input and output variables, inputandsoutputwithin the required structure. A simple vehicle model is used here for designing ESC controller which can provide the vehicle responses in planner motion like longitudinal, lateral and yaw. So to keep the model simple some of these research works have neglected influence aerodynamic drag, rolling resistance and suspension systems. As vehicle dynamics are much slower compared to the control loop [8], a simple vehicle model would be easier to implement for controller to get expected vehicle response. On the other hand a reference vehicle model is required to be hosted by an embedded controller for real-time. This model is also suitable computer simulation using MATLAB/Simulink. Equations of motion are used here to find the velocity of the vehicle and its yaw rate. Let $\dot{x}$ is the longitudinal velocity of vehicle, Let $\dot{y}$ is the lateral velocity of vehicle and $\dot{\psi}$ is yaw moment of inertia of vehicle then equations of motions of a vehicle body are:

$$m\ddot{x} = (F_{xfl} + F_{xfr})\cos(\delta) + F_{zrl} + F_{zrr} - (F_{yfl} + F_{yfr})\sin(\delta) + m\Psi \dot{y}$$

$$m\ddot{y} = F_{yrl} + F_{yrr} + (F_{xfl} + F_{xfr})\sin(\delta) + (F_{yfl} + F_{yfr})\cos(\delta) - m\Psi \dot{x}$$

$$I_{x} \ddot{\psi} = l_{f}(F_{xfl} + F_{xfr})\sin(\delta) + l_{r}(F_{xfr} + F_{xfr})\cos(\delta) - l_{f}(F_{yfl} + F_{yfr}) + \frac{L}{2}(F_{xfr} - F_{xfr})\cos(\delta) - \frac{L}{2}(F_{xfr} + F_{xfr})\sin(\delta)$$

Here, $F_{xfl}$, $F_{xfr}$, $F_{yfl}$, $F_{yfr}$ are the longitudinal wheel forces, $F_{zrl}$, $F_{zrr}$ are the lateral forces, $l_{f}$ is the steering angle, $\Psi$ is vehicle mass, $L$ is the distance of front axle from center of gravity (CoG), $W$ is the width between the wheels and $\dot{\psi}$ is the yaw rate of the vehicle.

2. Tire forces from tire model

Complex and non-linear tire models like Pacejka’s Magic formula [9] or Dugoff tire models [10] are used in different vehicle simulation for tire force calculation. For accuracy in simulation results tire model should accurate. Most famous tire Pacejka’s Magic Formula tire model has been used in [5] where the model provides calculations for longitudinal and lateral force relation, aligning moment. Pacejka’s Magic Formula tire model used for the same purpose also in [11], [12] for a perfect vehicle simulation result. This Magic formula is a kind of experimental method which can derive tire road surface forces. This is a semi-empirical model developed using experimental data fitted into it mathematically. Using this model in vehicle simulation is difficult as it requires many experimental coefficients. Dugoff’s tire model is also a popular tire model and it is used in [13], [6] for vehicle simulation and tire force calculation. Dugoff’s model provides calculation of forces for combined longitudinal and lateral tire force using friction circle concept. The model offers a significant advantage of using independent values of tire cornering stiffness and longitudinal stiffness.According to Dugoff’s tire model:

Longitudinal tire force $F_{x} = C_{e} \frac{\tau_{eff} \omega_{w}}{q}\sin(\delta)$

(4)

Lateral tire force is $F_{y} = C_{e} \frac{\tau_{eff} \omega_{w}}{q}\tan(\delta)$

(5)

Where, $\tau_{eff}$ is effect radius of the tire, $\omega_{w}$ is the wheel angular velocity,

Longitudinal slip ratios for each wheels for braking is

$$\sigma = \frac{\tau_{eff} \omega_{w} - \dot{x}}{\dot{x}}$$

and for acceleration is

$$\sigma = \frac{\tau_{eff} \omega_{w} - \dot{x}}{\dot{x}}$$
Slip angles at the front and rear tires are $\alpha_f = \delta - \frac{v_y}{v_x}$ and $\alpha_r = -\frac{v_y}{v_x}$.

Function $\lambda = \frac{\mu F_v(1+\delta)}{2(C_s \tan \alpha)^2 + (C_s \tan \alpha)^2}$, here $\mu$ is tire road friction coefficient and $F_v$ is the vertical force on tire.

If value of $\lambda < 1$ then function, $f(\lambda) = (2 - \lambda)\lambda$. If value of $\lambda \geq 1$ then function, $f(\lambda) = 1$.

In case of Dugoff’s tire model, it requires less number of coefficients compared to Magic model to calculate longitudinal and lateral forces. A practical approach has taken into account for a differential torque based ESC where calculation complexity can be avoided by using less number of coefficients. Though both of these tire models are very popular and accurate, preferences given to adopt Dugoff’s tire model for being less complex for creating a reference tire model in computer simulation and as well as for hosting by an embedded controller.

3. Control law parameters estimations

It is important to discuss to identify what are the required control parameters for a yaw stability controller. If the controller is designed using only yaw velocity as the control variable to maintain yaw rate equal to or close to the nominal value it would fail to stabilize the vehicle on the road where tire road adhesion is very low or in general on a slippery road. If the tire road coefficient is very low then vehicle slip angle might increase rapidly without increasing or decreasing lateral force because on the slippery road, yaw velocity and lateral acceleration do not correspond to each other [14]. Steer-ability of the vehicle depends on the side slip angle of the vehicle [15], steering angle input can hardly change the direction of a vehicle if the side slip angle increases. From the analysis in the paper[15] shows that on dry surface vehicle steer-ability vanishes when the vehicle slip angle is larger than 10 degrees and on icy surface vehicle steer-ability vanishes when the vehicle slip angle is less than 4 degrees. So by controlling yaw velocity or yaw rate only may not achieve stability of the vehicle in critical situation. To keep the yaw velocity and slip angle of the vehicle limited to the values that correspond to coefficient of friction of the road, the controller should take both of the yaw velocity and slip angle as controller parameters [14], [16]. Majority of the researcher have used similar control parameters like yaw rate and vehicle slip in control laws. Researchers in [5], [6], [7],[11] and [17] have used both the yaw rate and vehicle slip angle as their control parameters in ESC controller, regardless of the types of the control law. This concept of minimizing the error of yaw rate and slip angle simultaneously in the control law is more logical approach as they have to be limited to values correspond to the tire road coefficient friction.

a) Yaw rate estimation:

Primary objective of yaw stability controller is to maintain yaw rate close to the nominal value when the vehicle is instable. So actual yaw rate and nominal yaw rate determination is required to discuss. For a real-time controller an actual yaw rate of a vehicle which can be measured directly using electronic yaw rate sensors. But for simulation the actual yaw rate can be found from the vehicle model. In the controller, difference between the actual yaw rate and desired yaw rate is used in most of the cases. Desired yaw rate has to be determined from steady state relation between steering angle and its generated radius of the vehicle’s trajectory. The desired yaw rate can be determined from steering angle, radius, vehicle speed, vehicle mass, tire stiffness and vehicle’s physical measurements. Desired yaw rate is calculated here to use them in the control law for simulation.

Desired yaw rate $\Psi_{desired} = \frac{x}{r}$ (6)

Where, $R$ is the radius of the circular road calculated using the steering angle, $x$ is the longitudinal velocity.

Relation between steady state steering angle $\delta_{ss}$ for a circular path and radius $R$ of the path is

$\delta_{ss} = \frac{1}{R} \left( \frac{\tan \alpha_f - \tan \alpha_r}{C_s f_c p_y (1+\delta)} \right) R$ (7)

Where, $C_{af}$ and $C_{ar}$ are the cornering stiffness of front tire and rear tire respectively. Vehicle mass is $m$.

b) Vehicle Side Slip angle:

Vehicle body side slip angle is one of the most significant parameters in vehicle stability. This is the angle between vehicle’s longitudinal axis and the direction of the velocity at the CoG is called the slip angle of the vehicle. So, vehicle slip angle can be determined as:

$\beta = \frac{v_x}{v_x}$ (9)

Here, $v_x$ is the lateral velocity and $v_y$ is the longitudinal velocity of the vehicle. This side slip angle cannot be measured using any commercially available sensor. It can be possible to measure using some optical sensors but developing this kind of system is not cost effective, this can be used in development and test purpose rather than using in the vehicle. In real time, using lateral accelerometer along with a yaw rate sensor can be used to estimate the side slip angle of the vehicle. Desired side slip angle $\beta_{desired}$ can be obtained from steady state steering angle $\delta_{ss}$ as given below.

$\beta_{desired} = \frac{1}{2 \pi} \left( \frac{m v_x^2}{2 C_{af} f_c p_y (1+\delta)} \right) \delta_{ss}$ (10)

The actual or measured side slip angle need to compare with the nominal or desired side slip angle and the difference between them is used in the control law of ESC.

4. Coefficient of friction estimation

Tire road friction coefficient is complex phenomenon of tire road interaction. In vehicle dynamics it is an important parameter that helps to calculate the tire force and also helps to describe the condition of the road. Due to this adhesive capability vehicle moves forward when there is any tractive effort on the wheel. The maximum tractive effort that can be
supported by the tire-ground contact patch is the product of vehicle load and coefficient of road adhesion or tire-road frictional coefficient denoted as $\mu$. For further increase in tractive effort will cause sliding of the wheel. It is difficult to determine the value of tire road friction coefficient online as this coefficient is affected by different factor like material used in tire, surface condition, tire pressure and vehicle load. Tire road friction coefficient is mostly estimated as the ratio of normal force of the vehicle and longitudinal or lateral force of the vehicle.

$$\mu = \frac{F_n}{F_x}$$

Considering the longitudinal force of the vehicle (17) can be written for maximum value or the peak value of tire road friction coefficient as

$$\mu_{\text{max}} = \frac{F_{\text{l}}}{F_x}$$

Where, $F_{\text{l}}$ is the longitudinal tire force and $F_x$ is the normal load. A few research works related to ESC development or ESC simulation has discussed the estimation of tire road friction coefficient with accuracy by following any special method. Piezoelectric wireless tire sensor is used inside the tire on the contact patch in paper [18] to measure force, moment and slip angle variables and then using brush tire model estimation can be done for tire road friction and slip angle of the vehicle. Slip based estimation of tire road friction coefficient is widely used in several literatures. In normal driving using only wheel slip or from the relative difference from wheel velocity, tire road friction coefficient is estimated in the [19]. It is based on the difference of driven and non-driven wheel velocity. In [20] tire road friction is estimated only using angular wheel velocity. Slip based estimation of maximum tire force is also done using the brake force during braking in [21]. More generalized methods are related to vehicle dynamics where controller can estimate this without complexity. Simplifying (17) the friction can be written as

$$\mu = \frac{\frac{F_{\text{y}}}{F_{\text{y}}}}{g}$$

Where, “$a$’’ is the acceleration and $g$ is the gravitational force. For simulation of the ESC system discussed here, this tire-road friction coefficient is an input which can be varied to observe the effect on the vehicle.

### B. Control law and Strategies

The key point in the strategy of yaw stability control law is to find the required corrective yaw moment. Control laws, defined by the researchers in previous research works, have the likeness of using similar control parameters like yaw rate and vehicle slip angle. But dis-similarities are there in the use of control parameters in a single controller or multiple controllers. Strategies in different research works are addressed below to understand the requirements so that a simple strategy can be adopted.

A fuzzy logic based direct yaw control for all wheel drive EV is proposed in [12], [22] which can be taken as case 1 for discussion. A novel strategy is adopted where yaw rate and wheel slip ratio is controlled in the control loop. Separate fuzzy logic based controllers for yaw rate and wheels’ slip ratio are used in this yaw motion control loop, where the yaw rate controller is determining the required torque. Figure 2 shows the concept of control strategy of case 1.

![Fig.2 Control Structure of Case 1](image)

Applied corrective torque from yaw controller can saturate the tire force, to avoid this problem a slip controller is used to keep the slip ratio within the stable region by generating weakening torque for the wheels. In yaw rate controller, reference yaw rate is generated by a neural-network. This neural network learned to generate the reference yaw based on the vehicle speed and steering angle and it required to get trained on the road. Result shows that yaw-controller along with slip controller can bring the vehicle to alignment without blocking the wheels. In case 2, the discussion is done based on hierarchical vehicle stability control systems using fuzzy logic based controller is designed in the paper [5], [6]. Here controller is designed to calculate the desire yaw moment from yaw rate error and vehicle slip angle error. Desired wheels’ slip ratios are calculated based on desired yaw moment using fuzzy logic based control allocation. It is assumed that ABS/TCS is available to manipulate braking or wheel traction to achieve the yaw moment. The key points in this paper realized are, the controller has used yaw rate and slip angle to find yaw moment and it is responsible for controlling the individual wheel spins. Figure 3 depicts the control strategy of this case.

![Fig.3 Control Structure of Case 2](image)

Both the yaw rate error and slip angle errors as control parameters used in [23] which is taken as case 3. A fuzzy controller is used to determine the corrective yaw moment for more robustness an active front steering controller based on sliding mode controller is used which provides the corrective steering angle. Figure 4 shows the control strategy of this case. Use of steering control along with the fuzzy controller is a dissimilar noticeable strategy.
Another strategy is followed for yaw rate control using independent in-wheel motors in [24] is taken as case 4. A PID controller for lateral acceleration error, a sliding mode controller for yaw rate error and another PID controller for wheel sleep error is used in this research work to generate the corrective wheel torque. Corrective wheel torques, generated by these three controllers, is then combined and used as total corrective torque to control the yaw rate of the vehicle. Figure 5 shows the details structure of control strategy considered for case 4.

**C. Calculation for wheel torque control**

Differential torque based ESC is in the focus of the researchers and industries currently as advanced EVs are having the feasibility of individual wheel control. In differential torque based control for stability, for actuation, differential torque is required to generate by controlling the wheel rotation. Actuation or control of the wheel rotation is described at best in various publications. One noticeable reason can be mentioned about this shortcoming is considering simulation of the system mainly and less considered to discuss the wheel rotation control approaches. Only algebraic calculation of corrective torque and force is done and fed into the plant for stabilization as a result the details of wheel rotation control or controlling the electric motors are kept very brief. As a part of investigation into differential torque based ESC, several research works are addressed here to realize the techniques are adopted and coverage is made on the wheel torque calculation and control. A permanent magnet motor model and motor torque dynamics model is used in [22] to provide the demanded required torque in the stability system of an all-wheel drive EV. A relatively similar motor torque dynamics model is also used in [5] for stability control of a four-wheel drive EV. The second order motor torque dynamics model is given below.

\[
C_\tau = \frac{T_m}{T_{m, \text{desired}}} \tag{14}
\]

Where, \(T_m\) is the motor torque, \(T_{m, \text{desired}}\) is the desired motor torque and \(C_\tau\) is the motor torque time constant. Both of these above mentioned research papers have addressed this torque control dynamics but more description might be needed to clarify their detail relation with the main control system specially, the motor control techniques if an electric motor is used.

A more clear idea of wheel control for demanded torque is depicted in [11] for vehicle stability control. Here tire model is used for force calculation and simplified motor model
constructed using the MAP of torque, PWM and rotational speed of the motor. The torque model is given below:

\[ T_e = \frac{T_{s2}}{r_m} \]  

(15)

Here, \( T_e \) is motor torque, \( T_{s2} \) is motor steady torque and \( r_m \) is time constant. Though an innovative concept of yaw control system presented by determining the torque of the wheel in [24] the torque and wheel rotational dynamics are unnoticed due to the use of simulation only for stability. Some more research works on vehicle stability using differential torque are considered to realize the wheel torque control where only wheel angular velocity is mentioned in [12] and overlooked in [23] and [17]. It is necessary to discuss the wheel torque and rotation control as a four-in-wheel motors EV is under consideration for ESC. This kind of EV provides flexibilities and resources to determine wheels’ rotations, forces and electrical data. Individual motor controllers in four-in-wheel motors EV, offers the benefit of controlling the wheel torque. A built-in sensor of in-wheel motor provides the rotational speed of the wheel which can be used to compare the expected rotation and actual rotation. From the estimation of vehicle states using models, force of the wheel and angular velocity can be determined. Desired torque or required mechanical torque is achievable from control law and numerical calculations. \( \omega \) is the wheel angular velocity is mentioned in [12] and overlooked in [23] and [17].

D. Overall discussion in the investigation

From the above discussions, a generalized view is depicted here in figure 7 on ESC systems developed or proposed in the previous research works for four in-wheel motors EV. This self-explanatory figure includes the basic levels of the ESC systems. In each level several variables and parameters are included and several researchers have focused on these. From the discussion in the previous sections on limited numbers of research works related to ESC systems, six speculations are pointed in numbers for each level of the system as shown in the figure 7.

1. Wide range of simulation software used in vehicle modelling for more accuracy and these software use experimental data and empirical equations in finding vehicle states.

2. Different types of sensors and critical estimations are done for several parameters. Dedicated researches on different parameter estimations and later on adopted as parameter in the ESC systems offered more precise outcome overwhelming the system.

3. A number of reference values introduced in different systems, more than two types of references are used sometimes.

4. Innovative and novel ideas used in control laws with multiple variables. Several strategies have been adopted combining a number of control parameters and to some extent, overlooking the complexity for the real-time.

Overall discussion

Fig. 7 Overall Coverage Done On Differential Torque Based ESC in Previous Works

5. Mostly numerical calculations for actuations are done on the wheels for driving torque or sometimes brake-torque and driving-torque both assuming pre-existing ABS or TCS. Sometimes steering-angle is also considered for actuation along with the wheels. In every case of actuations a more detail steps could make more suitable outcomes. This part of the system opens the scope for more research.

6. Real-time controller hosting reference value generators and capable of controlling wheels or steering are slightly discussed and at best in some research works. It is essentially important for more coverage which creates the opportunity of further research of a more simplified system than conventional systems.
III. ENTIRE ESC SYSTEM MODELING

A necessity of simplified ESC system model with data and control signal flow, parameter estimations, control law and actuations is observed from the previous discussions. A four in-wheel EV is the focus for its ESC system development. The entire ESC system modeling and simulation is presented here with a modular approach focusing the vehicle. The detail schematic diagram is given in fig. 8 for ESC controller with vehicle components model. As discussion is done in the previous sections to explain the reasons for selecting or choosing any approach, in this part of the research work they are accumulated with brief discussion.

![Schematic Diagram](image)

**Fig. 8** Entire ESC System with Vehicle Components for Simulation.

**A. Plant vehicle model**

If the safety feature like Electronic Stability Control (ESC) is concern for intelligent control then control analysis require a vehicle model that includes all the components of vehicle dynamics those affecting on vehicle stability. It indicates the need of a detailed and comprehensive vehicle model to reproduce the behavior of individual components as exactly as possible. Such a vehicle modeling requires equations of motions and interactions between subsystems which are in the form of mathematical equation. Using mathematical equations (1), (2) and (3), a MATLAB based computer model is made to analyze the controller before prototyping. Figure 8 (a) shows the detail of the modular model vehicle.

**B. Tire model**

Simulation of Electronic Stability Control (ESC) for 4 in-wheel electric vehicle, require details of the involved forces acting on the wheel. Considering ESC, a wheel model is presented here that can reproduce the exact behavior of forces as much as possible. To meet the purpose of simulating ESC, Dugoff’s tire model has been chosen. Equation (4) and (5) along with their associated parameters is used to model the tires. Figure 8 (b) shows the tire model with its necessary input and output.

**C. Reference vehicle model**

From the reference vehicle model, the desired yaw rate desired and side-slip angle can be achieved. To develop the reference vehicle model for nominal value of yaw rate, (7) and (8) are used here. On the other hand (10) is used to find the nominal value of side-slip angle.
D. Road friction coefficient \( \mu \)

In this wheel model different values of this coefficient \( \mu \) will be used as constant input for different simulations. The idea is to vary the value of tire-road friction coefficient ‘\( \mu \)’ in the simulation to see the effect on the vehicle motion. Value of \( \mu \) will be set from 1.0 to 0.75 for resembling an asphalt road and it will be set from 0.2 to 0.15 for a snow packed road.

E. Controller

A sliding mode based control law has been chosen with the weighted combination of yaw rate error and slip angle error. The control law is described below, where \( S \) is the sliding surface, \( \dot{\psi} \) is the yaw rate, \( \beta \) is the controller variable and \( \delta \) is the actual side-slip of the vehicle.

\[
S = \dot{\psi} - \Psi_{\text{target}} + \epsilon(\beta - \beta_{\text{target}})
\]

By differentiating (26),

\[
\dot{S} = \dot{\psi} - \Psi_{\text{target}} + \epsilon(\beta - \beta_{\text{target}})
\]

As each wheel is a driving wheel with identical electric motors inside the hub and they have identical longitudinal wheel driving force, wheel forces can be written as \( F_{xxr} = \mu_x F_r \) and \( F_{xrr} = \rho_t F_r \) and set \( \rho_t = \frac{1}{2} \).

From (3) we get:

\[
\dot{\psi} = \frac{1}{I_2} \left[ I_f (F_{yfr} + F_{yfr}) \cos \delta - I_r (F_{yrr} + F_{yrr}) + \left( \cos \delta + \frac{\mu_t}{2} \right) \left( F_{yfr} - F_{yfr} \right) \right]
\]

Using (6) and (10) desired yaw torque \( M_{\psi_{\text{des}}} \) determined by the control law as

\[
M_{\psi_{\text{des}}} = \frac{I_2}{\eta} \times \left[ \frac{1}{2} \left[ l_f (F_{yfr} + F_{yfr}) \cos \delta + l_r (F_{yrr} + F_{yrr}) - \Psi_{\text{target}} - \xi (\beta - \beta_{\text{target}}) \right] \right]
\]

Here, \( \eta \) is another control variable. Detail of this calculation can be found in [25] for more clarifications.

In differential driving torque ESC the In-Wheel ESC controller determines amount of required torque at each wheels to generate a corrective yaw moment to meet the targeted yaw rate determined desired value generator. The extra differential longitudinal tire force \( F_{xfr} - F_{xfl} = \Delta F_{xfr} \) can be calculated as \( \Delta F_{xfr} = \frac{\Delta M_{\psi_{\text{des}}}}{l_2} \).

Using (24) and (19) the rotational velocity of the wheel or tires can be determined and then a lower controller can track the wheels’ rotations. Simulation has been performed with the popular Sine with Dwell [26] maneuver which has been used by transport authorities around the world for stability testing. Fig. 9 shows the steering angle input for a Sine with Dwell test. Differential wheel angular velocity between right and left wheels is created based on the required differential torque between the wheels.

![Fig.9. Steering angle input for Sine with Dwell test.](image)

In the first step, simulation is performed without the differential torque based ESC system. Figure 10(a) shows the yaw rate not returning to zero, Figure 10(b) shows the wheel velocity, figure 10 (c) shows vehicle longitudinal velocity kept varying for the steering angle input. Without differential torque ESC, vehicle is not following the desired values.
A second simulation is performed with differential torque ESC controller and the output is given in figure 11. Figure 11 (a) shows the yaw rate is crossing zero and following the desired yaw rate, figure 11 (b) shows differential wheel rotations and figure 11 (c) shows the vehicle velocity which is almost constant.

Vehicle is following the steering angle and yaw rate is returning to zero which shows the directional stability. The noticeable behavior of the vehicle here is the longitudinal velocity which is almost constant; this indicates the overcoming the slowing down problem of vehicle while braking based ESC is in operation.

**IV. CONCLUSION**

Investigation on different research works is done to understand the differential torque based ESC for independent control wheels EV, which have been sketchy in most of the research works. A limited research works are available with a complete control algorithm and description of necessary parameter estimations for differential torque based ESC for in-wheel EVs. This research work would be helpful for realizing a complete ESC system for an EV with independently control wheels. A more logical and realistic method is proposed considering simulation as well as further real-time implementation of ESC in different steps of analysis. As, simulation showed a promising outcome, a real-time ESC controller for the four in-wheels EV is undertaken as future works.

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