Design and Simulation of Small Signal Model of a STATCOM for Reactive Power Compensation on Variation of DC link Voltage

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Abstract- In general, reactive power compensation for power factor and stability of the supply system can be improved. The STATCOM (STATic synchronous Compensator) is being increasingly popular in power system applications. A small signal model of the STATCOM has been proposed without controlling DC link voltage. This scheme controls the phase angle as well as modulation index of the switching pattern. With small perturbation of reference current (reactive current of load), the DC voltage remains nearly constant. On variation of DC link voltage, the spike and overshoot of the responses have been studied and suitable DC link voltage has been selected. The values of passive parameters which have been selected are used for designing of a STATCOM. All responses are obtained through MATLAB simulink tool box and presented here for clarity of the control strategy.

Index Terms—STATCOM, Small Signal Model, Jacobian Matrix.

I. INTRODUCTION

Power systems with long distance power transmission lines tend to become unstable when the power flow is heavy [1]-[6]. The interconnected grids tend to become unstable as the heavy loads vary dynamically in their magnitude and phase angle and hence power factor. The power system stability is a key issue to determine the available transmission power capacity in a power system especially in a deregulated market. The capacitor banks are used to improve power factor and hence stability of the power system [7]. Commissioning new transmission systems are extremely expensive and take considerable amount of time to build up. Therefore, in order to meet increasing power demands, utilities must rely on power export/import arrangements through the existing transmission systems. Power electronic devices are gaining popularity for applications in the field of power transmission and distribution systems. The reactive power (VAR) compensation and control have been recognized [8]-[11] as an efficient & economic means of increasing power system transmission capability and stability. The FACTS (Flexible AC Transmission Systems) devices, such as STATCOM has been introduced more recently which employs a VSI with a fixed DC link capacitor as a static replacement of the synchronous condenser. Large numbers of capacitor banks or any other passive elements are no more required. Only a fixed set of capacitor provides the required VAR control, with a rapid control of bus voltage and improvement of utility power factor. It offers several advantages over conventional thyristorised converters [12] in terms of speed of response. The penalty paid for this improvement is in terms of introduction of some harmonics, which requires separate handling using active filtration techniques. Moran et al [13] have shown in details how the utilization of SPWM techniques reduces harmonic distortion. It has also been shown that an increase of modulation index reduces the size of the link reactor and stress on switches which are significant issues in practical implementation. The modeling and analysis of STATCOM steady state and dynamic performance with conventional control method have been studied Schauder and Mehta [14]. In [15]-[23] the dynamic responses and steady state behavior of STATCOM with SVPWM has been studied and the advantages of introducing SVPWM inverter with higher values of MI are highlighted. Starting with an established steady state open loop model of the STATCOM, the present paper goes on to develop a closed loop model for investigating transient performance of the STATCOM by using small signal scheme. First, in Section III, a small signal model is developed to establish the effect of the variation of the modulation index ‘m’ and the phase angle ‘α’ (between the STATCOM terminal voltage and the voltage at point of common coupling (PCC)) on the STATCOM terminal current and the DC-link voltage. Thereafter, a closed loop control scheme has been proposed. In Section IV, the scheme has been simulated with variation DC-link voltage so that DC-link voltage may be kept unchanged. This scheme is both an extension and a significant improvement of the scheme suggested by Cho et al [17]. The results obtained by this method have been compared and appropriate conclusions have been drawn.

II. MODELING OF THE STATCOM AND ANALYSIS

A. Operating Principles

The fundamental phasor diagram of the STATCOM terminal voltage with the voltage at PCC for an inductive load in operation, neglecting the harmonic content in the STATCOM terminal voltage, is shown in Fig. 2. Ideally, increasing the amplitude of the STATCOM terminal voltage \( \vec{V}_{oa} \) above the amplitude of the utility voltage \( \vec{V}_{sa} \) causes leading (capacitive) current \( \vec{I}_c \) to be injected into the system at PCC. \( \vec{I}_{c,a} \), the real component of \( \vec{I}_c \), accounts for the losses in the resistance of the inductor coil and the power electronic converter. Ideally, if the system losses can
be minimized to zero, \( \bar{I}_{c-a} \) would become zero, and \( \bar{I}_c \)
would be leading at perfect quadrature. Then, \( \bar{V}_{oa} \), which is
lagging and greater than \( \bar{V}_{sa} \), would also be in phase with
\( \bar{V}_{sa} \). The STATCOM in such a case operates in capacitive
mode (when the load is inductive).

B. Modeling
The modeling is carried out with the following assumptions:
1) All switches are ideal
2) The source voltages are balanced
3) \( R_s \) represents the converter losses and the losses of the
coupling inductor
4) The harmonic contents caused by switching action
are negligible

The Park’s \( abc \) to \( dq \) transformation matrix is
\[
K = \frac{1}{\sqrt{3}} \begin{bmatrix}
\cos(\omega t) & \cos(\omega t - 2\pi/3) & \cos(\omega t + 2\pi/3) \\
\sin(\omega t) & \sin(\omega t - 2\pi/3) & \sin(\omega t + 2\pi/3) \\
1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{2}
\end{bmatrix}
\]
(1)
The actual proposed circuit is too complex to analyze as a
whole, so that it is partitioned into several basic subcircuits,
as shown in Fig.1. The 3-phase system voltage \( V_{s,abc} \)
lagging with the phase angle \( \alpha \) to the STATCOM output
voltage \( V_{o,abc} \) and differential form of the STATCOM
currents are defined in (2) and (3).

\[
V_{s,abc} = \begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix} = \frac{2}{\sqrt{3}} V_s \begin{bmatrix} \sin(\omega t - \alpha) \\ \sin(\omega t - \alpha - 2\pi/3) \\ \sin(\omega t + \alpha + 2\pi/3) \end{bmatrix}
\] (2)

\[
L_s \frac{d}{dt} \begin{bmatrix} i_{c,abc} \end{bmatrix} = -R_s i_{c,abc} + V_{s,abc} - V_{o,abc}
\] (3)

Where, \( V_s, \alpha, R_s \), and \( L_s \) have their usual connotations.
The above voltages and currents are transformed into \( dq \)
frame to give,
\[
V_{s,qdo} = V_s \begin{bmatrix} -\sin\alpha \\ \cos\alpha \end{bmatrix}^T
\] (4)

\[
L_s \frac{d}{dt} \begin{bmatrix} i_{c,q} \end{bmatrix} = -R_s i_{c,q} - \alpha L_s i_{cd} + V_{eq} - V_{eq}
\] (5a)

\[
L_s \frac{d}{dt} \begin{bmatrix} i_{cd} \end{bmatrix} = \alpha L_s i_{cq} - R_s i_{cd} + V_{id} - V_{id}
\] (5b)
The switching function \( S \) of the STATCOM can be defined as
follows
\[
S = \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \sqrt{2/3} \begin{bmatrix} \sin(\omega t) \\ \sin(\omega t - 2\pi/3) \\ \sin(\omega t + 2\pi/3) \end{bmatrix}
\] (6)
The modulation index, being constant for a programmed
PWM, is given by,
\[
MI = \frac{V_{o,peak}}{V_{dc}} = \frac{2}{\sqrt{3}} m
\] (7)
The STATCOM output voltages and \( dq \) transformation are
given by,
\[
V_{o,abc} = S_v V_{dc}
\] (8)

\[
V_{o,qdo} = m \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}^T V_{dc}
\] (9)
The dc side current in the capacitor and its \( dq \)
transformation may be written as,
\[
i_{dc} = S^T i_{c,abc}
\] (10)

\[
i_{dc} = m \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} i_{cq} \\ i_{cd} \\ i_{co} \end{bmatrix}
\] (11)
The voltage and current related in the dc side is given by
\[
i_{dc} = C \frac{dv_{dc}}{dt}
\] (12)
Now replacing (11) in (12) yields
\[
\frac{dv_{dc}}{dt} = \frac{m}{C} i_{cd}
\] (13)
The complete mathematical model of the STATCOM in \( dq \)
frame is obtained as given in (14)
The second order

$$ \begin{bmatrix} \frac{d}{dt} i_{cd} \\ \frac{d}{dt} v_{dc} \end{bmatrix} = \begin{bmatrix} \frac{R_s}{L_s} & -\omega & 0 \\ \omega & \frac{R_s}{L_s} - \frac{m}{L_S} & -\frac{m}{C} \\ 0 & \frac{m}{L_S} & \frac{C}{L_s} \end{bmatrix} \begin{bmatrix} i_{cd} \\ v_{dc} \end{bmatrix} + \begin{bmatrix} \frac{V_s}{L_s} \\ 0 \end{bmatrix} = \begin{bmatrix} -\sin \alpha \\ \cos \alpha \end{bmatrix} $$ (14)

C. Steady State Analysis

The steady state equations of $i_{eq}, i_{cd}, v_{dc}, P_c, Q_c$ are given in (15-16) and their responses with the parameters given in Table 1, are shown in Fig.3. The theoretical results of (15-16) are verified by the simulation results.

$$ i_{eq} = -\frac{V_s \sin \alpha}{R_s}, i_{cd} = 0 $$ (15)

$$ V_{dc} = \frac{V_s}{M} \left( \cos \alpha - \frac{\omega L_s}{R_s} \sin \alpha \right) $$

$$ P_c = \frac{V_s^2}{2R_s} \left[ 1 - \cos(2\alpha) \right] $$ (16)

$$ Q_c = \frac{V_s^2}{2R_s} \sin(2\alpha) $$

<table>
<thead>
<tr>
<th>Sl.</th>
<th>Parameters</th>
<th>Symbol</th>
<th>Values</th>
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<td>1</td>
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<td>2</td>
<td>Angular Frequency</td>
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<td>4</td>
<td>Coupling Resistance</td>
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<td>5</td>
<td>Coupling Inductance</td>
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<td>DClink capacitor</td>
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<tr>
<td>7</td>
<td>Modulation Index</td>
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<td>8</td>
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<td>$\alpha$</td>
<td>$\mp 5^o$</td>
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<tr>
<td>11</td>
<td>Load Power factor</td>
<td>$\phi$</td>
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</tr>
</tbody>
</table>

D. Transient Analysis

The transient responses of the system in (14) are simulated in MATLAB for an initial condition of $v_{dc} = 100V$ and the parameters given in Table.1 and are shown in Fig.4 to Fig.5.

Fig.4: Transient Responses: (a) $i_{eq}$ and (b) $i_{cd}$

Fig.5: Transient Response of $v_{dc}$

It can be seen that the transient responses take about one and half power cycle to reach at their steady state values.

III. SMALL SIGNAL MODEL

A. Modeling

For a given operating point, small signal equivalent circuit is derived based on the following assumption:

i) The disturbance is small,

ii) Hence, the second order terms (products of variations) are negligible,

iii) The phase nominal value of angle $\alpha$ is small.

With the above assumptions equations, (9) and (11) can be rewritten as:

$$ V_{od} + \tilde{V}_{od}(t) = MV_{dc} + \tilde{M}V_{dc} + \tilde{m}V_{dc} + \tilde{m}V_{dc} $$ (17)

$$ I_{dc} + \tilde{I}_{dc} = MI_{cd} + \tilde{M}I_{cd} $$ (18)

$$ \sin(\hat{\alpha}) = \hat{\alpha}, \cos(\hat{\alpha}) = 1 \text{ and } \alpha_0 = 0 $$ (19)

By using (5), (17), (18) and (19), and applying Laplace Transformation, we have,

$$ \begin{bmatrix} \tilde{V}_{eq}(s) \\ \tilde{V}_{eq}(s) \end{bmatrix} = \begin{bmatrix} sL_s + R_s & \omega L_s \\ 0 & -sM \end{bmatrix} \begin{bmatrix} \tilde{v}_{dc}(s) \\ \tilde{v}_{dc}(s) \end{bmatrix} = \begin{bmatrix} -V_s \hat{\alpha}(s) \\ -V_s \hat{\alpha}(s) \end{bmatrix} $$ (20)

The important transfer functions of the states of the STATCOM in small signal model can be derived as:

$$ \frac{i_{eq}(s)}{\tilde{m}(s)} = \frac{V_{dc}M_{c}s \alpha}{A(s)} $$ (21a)

$$ \frac{i_{eq}(s)}{\tilde{a}(s)} = \frac{-V_s (L_s C s^2 + R_s C + M^2)}{A(s)} $$ (21b)

$$ \frac{i_{cd}(s)}{\tilde{m}(s)} = \frac{-V_{dc} s (L_s + R_s) s}{A(s)} $$ (22a)

$$ \frac{i_{cd}(s)}{\tilde{a}(s)} = \frac{-V_{dc} M_{c} s \alpha}{A(s)} $$ (22b)

$$ \frac{\dot{v}_{dc}(s)}{\tilde{m}(s)} = \frac{-MV_{dc} (L_s + R_s) s}{A(s)} $$ (23a)

$$ \frac{\dot{v}_{dc}(s)}{\tilde{a}(s)} = \frac{-MV_s M_{c} s \alpha}{A(s)} $$ (23b)
\[ A(s) = CL_s^2 + 2L_sCR_s^2 + CL_s^2 + (oL_s^2) + M^2R_s^2 + \phi \]

**B. Open loop Responses**

The \( \dot{i}_{cq}, \dot{i}_{cd} \) and \( \dot{V}_{dc} \) in (21-23) are simulated in MATLAB with the same parameters given in Table.1 and also \( \dot{\alpha} = -5u(t) \) and \( m = 0.1u(t) \). The responses are shown in Figs.6 to 7.

**Fig.6: Step Response:** (a) \( \dot{i}_{cq} \) for small change of \( \dot{m} \) and \( \dot{\alpha} \) and (b) \( \dot{i}_{cd} \) for small change of \( \dot{m} \) and \( \dot{\alpha} \)

**Fig.7: Step response of \( V_{dc} \) for small change of \( \dot{m} \) and \( \dot{\alpha} \)**

**C. Closed loop Response**

To achieve fast dynamic response without using independent dc voltage source, it is required that the capacitor voltage \( V_{dc} \) be kept constant by controlling the phase angle. Simultaneously the load reactive power should also be compensated by controlling the modulation index of the converter. If the capacitor voltage is constant, \( \dot{V}_{dc}(s) \) should be zero giving,

\[ \dot{m}(s) = -\frac{V_c oL_s}{V_{dc}(sL_s + R_s)} \dot{\alpha}(s) \quad (24) \]

This principle though suggested by Cho et al [17], its implementation is not correctly presented by the authors there. The PI controller for controlling the reactive current of the STATCOM with independent control of DC link voltage is proposed as shown in Fig.8. The parameters of PI controller have been determined by root locus method and their values are \( K_{pi} = -0.02 \) and \( K_{ii} = -6.66 \). With the parameters given in Table.1 and reference reactive current as the reactive component of the load current, the whole system is simulated in SIMULINK tool box through Fig.9.

IV. SIMULATION RESULTS

A. Simulation of linear load

The grid phase A voltage and linear load phase A current is given in Fig.10. It is depicted that power factor of the load is 0.79 and hence at grid also. This power factor will be improved by using the proposed control strategy and the effect of variation of DC link voltage.

**Fig.8: Proposed \( \dot{i}_{cq} \) Control Block**

**Fig.9: Proposed Implementing Block**

**Fig.10: Grid Phases a Voltage and Phases a Load Current**

B. DC: link capacitor charged to 100V:

The controller works and STATCOM functions at the initial DC link voltage of capacitor 100V with a large spike at the starting position as shown in Fig.11 of grid phase A voltage and current. It is seen that the power factor of the load is 0.79 as per the \( R-L \) load taken to unity after one power cycle as given in Fig.11 (for phase A) as compared to Fig.10 with a spike of 128A. Fig.12 shows the dynamics of grid phase A voltage and the STATCOM phase A current with a spike of 115A.
C. DC-link capacitor charged to 550V:

The same model is simulated with initial DC link voltage of 550V. It is seen that the power factor improves from 0.79 as per the R-L load taken to unity at the starting position with a current spike of 18A as given in Fig.13 (for phase A) as compared to Fig.10. Fig.14 shows the dynamics of the STATCOM current without current spike at the starting position. The active and reactive power generated by the STATCOM is shown in Fig.17 and its transient nature is shown in Fig.18 respectively. Fig.19 shows that the maximum overshoot of DC link voltage is to 610 volts only. Figs.20 and 21 show the corresponding dynamics of incremental phase angle $\dot{\alpha}$ and incremental modulation index $\dot{m}$ respectively. The grid phase A voltage and STATCOM phase A output voltage is shown in Fig.22 which shows the STATCOM output voltage is greater than the grid phase A voltage and lagging with very small angle which is shown in Fig.23. With the small perturbation of reference current as shown in Fig.24 (reactive current of load), very small change in DC link voltage is obtained and when the reference current goes to its original value the DC link voltage also returns to its steady state value as shown in Fig.25.
Fig. 19: DC Link Voltage of STATCOM

Fig. 20: Variation of Incremental of Phase Angle $\alpha$

Fig. 21: Variation of Incremental of Modulation Index $m$

Fig. 22: Grid Phase a Voltage and STATCOM Phase a Output Voltage

Fig. 23: Grid Phase a Voltage and STATCOM Phase a Output Voltage in Expansion

Fig. 24: With Change of Load Reactive Current and STATCOM Reactive Current

Fig. 25: DC Link Voltage of STATCOM with Change of Reference

D. DC- link capacitor charged to 600V:
Once again the proposed model is simulated with initial DC link voltage of 600V. The relevant responses is obtained without any spike at the starting position as shown Fig. 26 of grid phase a voltage and phase a current.

Fig. 26: Grid Phase a Voltage and Phase a Current

V CONCLUSION
In this paper, the STATCOM model has been linearized by small signal scheme with nominal value of $'\alpha'$ and $'m'$. The system with controller operates transient to steady state value on 0.03 sec. in case of small signal scheme. The
transient overshoot nearly 525V in case of small signal scheme. With the small perturbation of reference current (reactive current of load), very small change in DC link voltage is obtained as shown in Fig.25 and when the reference current goes to its original value, the DC link voltage immediate jumps to its steady state value. The variation of DC link voltage is applied to the STATCOM model and higher the DC link voltage lesser is the spike of the responses. The suitable and appropriate DC link voltage has been selected as 600V. On overall, the small signal scheme is better for the operation of the STATCOM in compensating mode. It must be pointed out that all the results present here are on the basis of the assumption that the DC link voltage has stabilized to some significant value before the control can be applied. However, in practice the issue of the charging the DC-link voltage to the required value is quite significant.

REFERENCES


AUTHOR’S PROFILE

Sarat Kumar Das was born on 20th Feb 1982 in patanaipur, Remuna, Balasore, Odisha, India. He obtained his 10th as J.N.V. Bagudi, and Balasore in 1998 and +2 SC FROM S.C.C, Mitapur, Balasore in 2000. He got his Bachelor’s degree (B.Tech) in Electrical Engineering from CET, Bhubaneswar in 2007. He completed his Master’s degree in Power Electronics & Drives from Bengal Engineering and Science University, Shibpur, Howrah, West Bengal in 2010. His research interests include Power Electronics and its application to high performance of FACTS devices, especially in the STATCOM.

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