Construction of Mixed Sampling Plans Indexed Through Six Sigma Quality Levels with QSS-1(n, mn;c0) Plan as Attribute Plan

R. Radhakrishnan, J. Glorypersial

Abstract—Six Sigma is a concept, a process, a measurement, a tool, a quality philosophy, a culture and a management strategy for the improvement in the system of an organization, in order to reduce wastages and increase the profit to the management and satisfaction to the customers. Motorola [24] first adopted the concept of six sigma in their organization and established that it can produce less than 3.4 defects per million opportunities. Focusing on reduction of defects will result in more profit to the producer and enhanced satisfaction for the consumer. The concept of Six Sigma can be applied in the process of quality control in general and Acceptance sampling in particular. In this paper a new procedure for the construction and selection of Mixed Sampling Plan indexed through Six Sigma Quality level having the QSS-1(n, mn;c0) Plan as attribute plan is presented. The plans are constructed using SSQL-1 and SSQL-2 as indexing parameters. Tables are constructed for easy selection of the plan.

Index Terms— Six Sigma Quality Level, Mixed Sampling Plan, Quick Switching System-I, OC Curve

I. INTRODUCTION

Mixed sampling plans consist of two stages of different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables are used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise, the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, the plan is said to be independent otherwise dependent. The principal advantage of a mixed sampling plan over pure attribute sampling plan is a reduction in sample size for a similar amount of protection. The second stage attribute inspection becomes more important to discriminate the lot if the first stage variable inspection fails to accept the lot. If rejection occurs during the normal inspection then tightened inspection is recommended in the mixed system and vice versa in the second stage. Hence Quick Switching System is imposed in the second stage to sharpen the sampling situation and to insist the producer to manufacture goods within the Indifference Quality Level. Dodge [1] proposed a sampling system called a ‘Quick Switching System’ (QSS) consisting of pairs of normal and tightened plans. Romboski [21] introduced a system designated as QSS-1(n, mn; c0) refers to Quick Switching System, where in the normal and tightened plan have the same acceptance number but on tightened inspection the sample size is a multiple of m, m>1 of the sample size on normal inspection. If m = 1, then the system reduces to single sampling plan. Schilling [22] proposed a method for determining the operating characteristics of mixed variables – attributes sampling plans, single sided specification and standard deviation known using the normal approximation. QSSs were originally proposed by Dodge [1] and investigated by Romboski [21] and Govindaraju [2]. Romboski [21] has developed QSS by attributes is a reduction in the sample size required to achieve approximately the same operating characteristic curve. Radhakrishnan and Sampath Kumar [3-6] have made contributions to mixed sampling plans for independent case. Radhakrishnan and Sivakumaran [7] introduced SSQL in the construction of sampling plans. Radhakrishnan and Sivakumaran [8] constructed Quick Switching Systems indexed through Six Sigma Quality Levels. Radhakrishnan [9] constructed Six Sigma based sampling plan using Weighted Poisson Distribution and Intervened Random Effect Poisson Distribution as the base line distributions. Radhakrishnan and Saravanan [10] constructed dependent mixed sampling plan with single sampling and chain sampling plan as attribute plan. Radhakrishnan and Glorypersial [11-20] designed mixed sampling plans indexed through six sigma quality levels with Double Sampling Plan, Conditional Double Sampling Plan, Chain Sampling Plan – (0,1), Chain Sampling Plan-1, Link Sampling Plan, TNT – (n; c1, c2) Plan, Conditional Repetitive Group Sampling, TNT – (n1, n2; c) Plan, DSP – (0,1) plan and Repetitive Group Sampling plan as Attribute Plan. This paper deals with the construction of mixed variables – attributes sampling plan (independent case) using QSS-1 (n; mn, c0) plan as attribute plan indexed through Six Sigma Quality levels. Tables are constructed for easy selection of the plan and illustrations are also provided.

II. GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

\[ p \] : submitted quality of lot or process

\[ P_1(p) \] : probability of acceptance for given quality \( p \)
n1: sample size for variable sampling plan
n2: sample size for attribute sampling plan
c0: Attribute acceptance number
m: multiplicity factor for sample size
βf: probability of acceptance for lot quality p0
βα: probability of acceptance assigned to first stage for percent defective p0
βs: probability of acceptance assigned to second stage for percent defective p)
d: observed number of nonconforming units in a sample of n units
k: variable factor such that a lot is accepted if $X \leq A = U - k\sigma$

### III. OPERATING PROCEDURE OF MIXED SAMPLING PLAN WITH QSS-1 AS ATTRIBUTE PLAN

Schilling [22] has given the following procedure for the independent mixed sampling plan with Upper specification limit (U) and known standard deviation (σ).

- Determine the parameters of the mixed sampling plan $n_1$, $n_2$, m, and c0.
- Take a random sample of size $n_1$ from the lot. Determine the sample average $X$.
- If a sample average $X \leq A = U - k\sigma$, accept the lot.
- If a sample average $X > A = U - k\sigma$, go to step 1.

**Step 1**: From a lot, take a random sample of size ‘$n_1$’ at the Normal level. Count the number of defectives ‘$d$’.

- If $d \leq c_0$, accept the lot and repeat step 1.
- If $d > c_0$, reject the lot and go to step 2.

**Step 2**: From the next lot, take a random sample of size ‘$mn_2$’ at the tightened level. Count the number of Defectives ‘$d$’.

- If $d \leq c_0$, accept the lot and use step 1 for the next lot.
- If $d > c_0$, reject the lot and repeat step 2 for the next lot.

This procedure is continued until all the lots are sentenced in a production process.

### IV. CONDITIONS FOR APPLICATIONS

(i) Production process should be steady and continuous
(ii) Lots are submitted sequentially in the order of their production
(iii) Inspection is by variable in the first stage and attribute in the second stage with quality defined as the fraction defective
(iv) Human involvement should be less in the manufacturing Process.

### V. DEFINITION OF QSS-1 AND QSS-2

The proportion defective corresponding to the probability of acceptance of the lot as $1-3.4 \times 10^{-6}$, the concept of six sigma quality suggested by Motorola [24] in the OC curve is termed as Six Sigma Quality Level-1 (SSQL-1). This new sampling plan is constructed with a point on the OC curve (SSQL-1: 1-α1), where $\alpha_1 = 3.4 \times 10^{-6}$ suggested by Radhakrishnan and Sivakumaran [7]. Further the proportion defective corresponding to the probability $2\alpha_1$ in the OC curve is termed as Six Sigma Quality Level-2 (SSQL-2). This new sampling plan is constructed with a point on the OC curve (SSQL-2: β1), where $\beta_1 = 2\alpha_1$ suggested by Radhakrishnan and Sivakumaran [7].

### VI. CONSTRUCTION OF MIXED SAMPLING PLAN HAVING QSS-1(N, mn; c0) AS ATTRIBUTE PLAN

The operation of mixed sampling plans can be properly assessed by the OC curve for values of the fraction defective. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit ‘U’. A parallel discussion can be made for lower specification limits. The procedure for the construction of mixed sampling plans is provided by Schilling [22] for a given SSQ-1, SSQ-2 and $n_1$ on the OC curve is given below.

- Assume that the mixed sampling plan is independent.
- Split the probability of acceptance (βl) determining the probability of acceptance that will be assigned to the first stage. Let it be β1.
- Decide the sample size $n_1$ (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as $A = U - k\sigma = U - [z(p_0) + \{z(\beta_1)/\sqrt{n_1}\}]\sigma$, where $z(t)$ is the standard normal variate corresponding to ‘t’
- Such that $t = \int_{z(\beta_1)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$

Determine the sample average $X$. If a sample average $X > A = U - k\sigma$, take a second stage sample of size ‘$n_2$’ using attribute sampling plan.

Now determine $\beta_1''$, the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_1'' = (\beta_1 - \beta_1')/(1-\beta_1')$.

Determine the appropriate second stage sample of size ‘$n_2$’ and ‘c’ from $P_0(p) = \beta_2''$ for $p=\text{SSQ-1}$.

Determine $\beta_2''$, the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_2'' = (\beta_2 - \beta_2')/(1-\beta_2')$.

Determine the appropriate second stage sample of size ‘$n_2$’ and ‘c’ from $P_0(p) = \beta_2''$ for $p=\text{SSQ-2}$.

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with QSS-1(n, mn; c0) plan as attribute plan indexed through SSQ-1, SSQ-2.
VII. OPERATING CHARACTERISTIC FUNCTION
According to Soundararajan and Arumainayagam [23], the operating characteristic function of QSS-1 is given below.

\[
P_Q(p) = \frac{b}{1 - \alpha + b} \left( \frac{e^{\alpha p} - 1}{\alpha} \right)^{n_2} = \frac{\beta}{1 - \beta + \beta p} \left( \frac{e^{\beta p} - 1}{\beta} \right)^{n_2}
\]

Where \( \alpha = \frac{c_0}{n_2} \) and \( \beta = \frac{n_2}{n_2} \left( \frac{e^{\beta p} - 1}{\beta} \right) \)

(For sample size tightening)

VIII. CONSTRUCTION OF MSP WITH QSS-1 (N, MN; c0) PLAN AS ATTRIBUTE PLAN INDEXED THROUGH SSQL-1

In this section the mixed sampling plan indexed through SSQL-1 is constructed. A point on the OC curve can be fixed such that the probability of acceptance of fraction defective \( \pi \) is \( \beta_1 \). The general procedure given by Schilling [22] is used for constructing the mixed sampling plan as attribute plan indexed through SSQL-1 for \( \beta_1 = (\beta_1 - \beta_1') / (1 - \beta_1') \). For the assumption \( \beta_1 = 0.0000066 \) and \( \beta_1' = 0.50 \), the \( n_2 \) and \( SSQL_1 \) values are calculated for different values of \( c_0 \) and \( m \) using Visual Basic program and is presented in Table 1. The sigma level of the process is calculated using the Process Sigma Calculator [25] by providing the sample size and acceptance number.

A. Selection of the plan

Table 1 is used to construct the plans when SSQL-1, \( c_0 \) and \( m \) are given. For any given values of SSQL-1, \( c_0 \) and \( m \), one can determine \( n_2 \) value using \( n_2 = n_2 \) SSQL-1/SSQL-1.

Example 1

Given SSQL-1=0.00008, \( c_0=2 \), \( m=1.50 \) and \( \beta_1'=0.50 \), the value of \( n_2 \) SSQL-1 is selected from Table 1 as 0.03472256 and the corresponding sample size \( n_2 \) is computed as \( n_2 = n_2 \) SSQL-1/SSQL-1=0.03472256/0.00008=4340 which is associated with 4.8 sigma level. For a fixed \( \beta_1=0.50 \), the Mixed Sampling Plan with QSS-1 (n, mn; c0) plan as attribute plan are \( n_2=4340 \), \( c_0=2 \) and \( m=1.50 \) for a specified SSQL-1=0.00008.

Practical Application

Suppose the plan with \( n_1=10 \), \( k=1.5 \), \( c_0=2 \) and \( m=1.50 \) to be applied to the lot-by-lot acceptance inspection of a metric cap screws. The characteristic to be inspected is the “diameter of a screw head of metric cap screws in mm” for which there is a specified upper limit of 3.8 mm with a known standard deviation (\( \sigma \)) of 0.02 mm. In this example,

\[ U = 3.8 \text{ mm}, \quad \sigma = 0.02 \text{ mm and } k = 1.5. \]

\[ A = U - k \sigma = 3.8 - (1.5)(0.02) = 3.77 \text{ mm} \]

Now, by applying the variable inspection first, take a random sample of size \( n_1=10 \) from the lot. Record the sample results and find \( \overline{X} \). If \( \overline{X} \leq A = U - k \sigma = 3.77 \text{ mm} \), accept the lot otherwise take a random sample of size \( n_2 = 4340 \) and apply attribute inspection. Under attribute inspection, the QSS-1 (n, mn; c0) plan as attribute plan, if the manufacturer fixes the values \( \beta_1'=0.50 \), SSQL-1=0.00008 (8 non-conformities out of 1 lakh metric cap screws) then select a sample of 4340 metric cap screws (Normal Inspection), and count the number of non-conformities (d). If the diameter of any metric cap screw is greater than 3.77 mm, then it is termed as defective. If d \( \leq 2 \) accept the lot and if d > 2 reject the lot and inform the management for quality improvement. From the next lot take a sample of 6510 metric cap screws (Tightened inspection) and count the number of non-conforming metric cap screws (d). If d \( \leq 2 \) then the lot is accepted and if d > 2 in the sample, then reject the lot and inform the management for corrective action. The OC curve of the plan in Example 1 is presented in the Figure 1.

![OC curve for the plan](image)

IX. CONSTRUCTION OF MSP WITH QSS-1 (N, MN; c0) PLAN AS ATTRIBUTE PLAN INDEXED THROUGH SSQL-2

In this section the mixed sampling plan indexed through SSQL-2 is constructed. A point on the OC curve can be fixed such that the probability of acceptance of fraction defective SSQL-2 is \( \beta_2 \). The general procedure given by Schilling (1967) is used for constructing the mixed sampling plan as attribute plan indexed through SSQL-2 for \( \beta_2 = (\beta_2 - \beta_2') / (1 - \beta_2') \). For the assumption \( \beta_2 = 0.0000068 \) and \( \beta_2' = 0.0000034 \), the \( n_2 \) and SSQL-2 values are calculated for different values \( c_0 \) and \( m \) using Visual Basic program and is presented in Table 2.

Example 2

Given SSQL-2=0.007, \( c_0=1 \), \( m=1.75 \) and \( \beta_2' = 0.000034 \), the value of \( n_2 \) SSQL-2 is selected from Table 1 as 10.19529 and the corresponding sample size \( n_2 \) is computed as \( n_2 = n_2 \) SSQL-2/SSQL-2=10.19529/0.007=1457 which is associated with 4.7 sigma level. For a fixed \( \beta_2 = 0.0000068 \), the Mixed Sampling Plan with QSS-1 (n, mn; c0) plan as attribute plan are \( n_2=1457 \), \( c_0=1 \) and \( m=1.75 \) for a specified SSQL-2=0.007.

Practical Application

Suppose the plan with \( n_1=10 \), \( k=1.5 \), \( c_0=1 \) and \( m=1.75 \) to be applied to the lot-by-lot acceptance inspection of a metric cap screws. The characteristic to be inspected is the “diameter of a counter bore of a metric cap screws in mm” for which there is a specified upper limit of 4.40 mm with a known standard deviation (\( \sigma \)) of 0.02 mm. In this example,

\[ U = 4.40 \text{ mm}, \quad \sigma = 0.02 \text{ mm and } k = 1.5. \]

\[ A = U - k \sigma = 4.40 - (1.5)(0.02) = 4.37 \text{ mm} \]
Now, by applying the variable inspection first, take a random sample of size \( n_1 = 10 \) from the lot. Record the sample results and find \( \bar{X} \). If \( \bar{X} \leq \mu - U \cdot k \sigma = 4.37 \) mm, accept the lot otherwise take a random sample of size \( n_2 = 1457 \) and apply attribute inspection. Under attribute inspection, the QSS-1(\( n_2, m; c_0 \)) plan as attribute plan, if the distributor fixes the values \( \beta_2^* = 0.0000034 \), SSQL-1=0.007 (7 non-conformities out of 1 thousand metric cap screws) then select a sample of 1457 metric cap screws(Normal inspection), and count the number of non-conformities (d). If the diameter of any metric cap screw is greater than 4.37 mm, then it is termed as defective. If \( d \leq 1 \) accept the lot and if \( d > 1 \) reject the lot and inform the management for corrective action. From the next lot take a sample of 2550 metric cap screws(Tightened inspection) and count the number of non-conformities (d). If \( d \leq 1 \) then the lot is accepted and if \( d > 1 \) in the sample, then reject the lot and inform the management for corrective action. The OC curve of the plan in Example 2 is presented in the Figure 2.

![OC curve for the plan n2=1457, c_e=1 and m=1.75](image)

**Fig 2. OC curve for the plan n2=1457, c_e=1 and m=1.75**

### Table 1: Various characteristics of the MSP when (SSQL-1, \( \beta_1 \)) is known with \( \beta_1^* = 0.9999966 \), \( \beta_1^* = 0.50 \).

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<th>( n_2\text{SSQL-1} )</th>
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### Table 2: Various characteristics of the MSP when (SSQL-2, \( \beta_2 \)) is known with \( \beta_2^* = 0.0000068 \) and \( \beta_2^* = 0.0000034 \).

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### X. Conclusion

This paper provides a procedure to engineers for the selection of Mixed Sampling Plan through Six Sigma Quality Levels having QSS-1(\( n, m; c_0 \)) plan as attribute plan. These plans are very effective in place of classical plans indexed through SSQ-1 and SSQ-2 and these plans are useful for the companies in developed and developing countries which are practicing six sigma quality initiatives in their process. The procedure outlined in this paper can be used for other plans also.


AUTHOE BIOGRAPHY

Dr. R. Radhakrishnan is having a bachelor degree, post graduate degree in Statistics and also possessing research degrees such as MPhil and PhD and also has additional qualification of post graduate degree in Business Administration. He has 34 years of teaching experience in teaching theoretical and applied Statistics, presented more than 160 papers in National and International seminars & conferences and published more than 119 articles in reputed National and International journals. He is also a quality auditor for ISO certifications and certified Six Sigma Black Belt also. He gains sufficient knowledge in Six Sigma methodologies and he is conducting training programme for Six Sigma black belts. He has visited countries like Sri Lanka and China for presenting his research contributions.

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