Improving Data Rate by Using Space-Time Trellis Codes Based On Coset Partitioning

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Abstract: Space-time coding has been introduced as an effective means to achieve high data rates in such wireless communication environment. In this paper, we propose a new and fast method to design the best space-time trellis codes (STTCs) for 2-PSK modulations. This method is an extension of the set partitioning proposed by Ungerboeck applied for multiple inputs multiple output (MIMO) systems [1].

I. INTRODUCTION

The MIMO systems use antenna arrays at both sides of a radio link to drastically improve the reliability and the data rate of the wireless communication. In order to exploit the performance of the MIMO systems, several channel coding techniques have been proposed. For example, the STTCs combine the spatial diversity and time diversity in order to combat efficiently the signal fading. The STTCs have been proposed by Tarokh et al. in [2], where the analytic bound and the first design criteria have been obtained. The rank and the determinant criteria have been proposed to design good codes in the case of slow Rayleigh fading channels. For fast Rayleigh fading channels, the product distance and the Hamming distance criteria have been proposed. In [2], Vucetic et al. proposed the trace criterion for a slow Rayleigh fading channel. Based on the Euclidean distance, this criterion governs the performance for a great product of the number of transmits and receives antennas.

Thanks to these criteria, many 4-PSK and bit and frame error rate [1–5]. However, a systematic search has been required to find these codes. The aim of this paper is to propose a new method to design the best 2n-PSK codes with n transmit antennas. This method is an extension of the set partitioning proposed by Ungerboeck [6, 8]. In fact, he states the following rules to design a trellis-coded modulation (TCM):

1) Each point of the constellation has the same number of occurrences.

2) In the trellis, transitions originating from the same state or merging into the same state should be assigned subsets which contain signal points separated by the largest Euclidean distance.

3) Parallel paths should be assigned signal points separated by the largest Euclidean distance. Since there are no parallel paths in the STTCs, this rule is not important for the new design of the STTCs.

Thanks to this new method called 'coset partitioning', it is possible to design the best STTCs without an exhaustive search. The paper is organised as follows. Section II describes the representations of space-time codes and the existing design criteria. In section III, the new method is presented. In the last section, new 4-PSK and 8-PSK are given and their performance is compared to the best known codes.

II. SYSTEM MODEL

A. Space-Time Trellis Encoder

Here we consider a 2n-PSK space-time trellis encoder with n transmit antennas and nR receive antennas. For n = 2, the encoder is shown in Fig. 1:

![Space-time trellis encoder for 4-PSK and nT transmit antennas](image)

The encoder is composed of one input block of n bits and v memory blocks of n bits. The state is defined by the binary values of the memory blocks of n bits. At each time t ∈ Z, all the bits of a block are replaced by the n bits of the previous block. The ith bit of the jth block, x_{i,j}, with i = 1…n and j = 1…v+1, is associated to n multiplier coefficients g_{i,j} ∈ Z_{2n}, k = 1,…,nT, where nT is the number of transmit antennas. With these nT(v+1) coefficients, we define a generator matrix G with nT lines and v+1 blocks of n columns:

\[ G = [G_1G_2…G_{nT}] = [G_11G_12…G_{nT}] \]

With

\[ G^j_i = \begin{bmatrix} g_{1(i,j)} & g_{2(i,j)} & \cdots & g_{nT(i,j)} \end{bmatrix}^T \in \mathbb{Z}_{2nT}, \]

at each time t, the encoder output \( y_{xt} = [x_{1T}^T x_{2T}^T \cdots y_{vT}^T] \in \mathbb{Z}_{2nT} \)

\[ y_{xt} = Gx^t \quad \text{(2)} \]

Where \( x^t = [x_{1,1}^t \ x_{1,2}^t \ \cdots \ x_{v,1,n}^t] \) is the extended form of the L=n(v+1) length shift register realized by the input block followed by the v blocks of n bits.
Each encoder output $y_k^t$ is mapped onto a $2^n$-PSK signal given by $s_k^t = \exp\left(i\frac{\pi}{2^{2^{n-1}}}y_k^t\right)$. Each output signal $s_k^t$ is sent to the $k^{th}$ transmit antenna. At each time $t$, the set of symbols transmitted simultaneously over the fading MIMO channel is given by, $s^t = [s_1^t s_2^t \ldots s_n^t]^T$.

An encoder can be represented by a trellis, as shown in Fig. 2 for 4-PSK 4 states STTC.

![Fig. 2: 4-PSK 4 states STTC [1]](image)

In the trellis, the states are described by the points and the transitions between the states by the lines. Each transition corresponds to an extended-state. The vector $y_1 \in 2^RT$ represents the MIMO symbol associated to an extended-state. The index $i$ is computed as the decimal value of this extended state, where $x_1^t$ is the least significant bit. In this example, the trellis representation $y_1^t y_2^t$ corresponds to the generator matrix $G = [y_1 y_2]$. In the general case, for a $2^n$-PSK STTC, there are $2^n$ transitions originating from the same state or merging into the same state. Each MIMO symbol belongs to $Z_{2^m}^T$.

**B. Design Criteria**

The main design criteria have been established in [1, 2] in order to decrease the bit and frame error rate. In this paper, only the case of slow fading channels is considered. Hence, the fading coefficients are constant during one frame of $L_f$ symbols. Besides, we assume that the decoder uses a maximum likelihood to estimate the transmitted symbols.

The main goal of this method is to reduce the pair-wise error probability (PEP) which is the probability that the decoder selects an erroneous sequence. It is possible to represent the $L_f$ length transmitted frame beginning at time $t = 0$ by a $n_T \times L_f$ dimension matrix $S = [s_0 s_1 \ldots s_{L_f-1}]$. An error occurs if the decoder decides that another frame $E = [s_0 e_1 \ldots e_{L_f-1}]$ is transmitted. Let us define the $n_T \times n_T$ difference matrix $B = E-S$. The $n_T \times n_T$ product matrix $A = BB^*$ is introduced, where $B^*$ denotes the hermitian of $B$. We define $r = \min(\text{rank}(B))$ where $B$ is computed for all pairs of different coded frames ($E, S$). The design criteria depend on the value of the product $rn_R$.

The product $rn_R \leq 3$: two criteria have been proposed [1, 10] to reduce the PEP. A has to be a full rank for any pair ($E, S$) and the coding gain given by $n = \sum_{d} N(d) \text{det}(A)^{-n_R}$ must be minimized. $N(d)$ is defined as the average number of error events with determinant $d = \text{det}(A)$.

The product $rn_R \geq 4$: it has been shown in [2] that the PEP is minimized if the minimum trace of a computed for all pairs ($E, S$) with $E \neq S$ is maximized.

In this paper we consider the case $rn_R \geq 4$ which is obtained if the rank of the generator matrix $G$ is greater than 1 and if there are at least 2 receive antennas.

**III. COSET PARTITIONING FOR STTC**

**A. Coset Decomposition**

The MIMO symbol belongs to the group $Z^T_{2^m}$. This group can be decomposed in cosets as in [7]:

$$z^T_{2^m} = \bigcup_{g \in G} g + C_0$$

Where $C_0 = 2^{n-1} Z^T_{2^n}$ is a normal subgroup of $Z^T_{2^m}$ such as $n = 2^n$. $\left[\square Z^T_{2^n}\{2^n\}^nT/C_0\}ight]$ denotes a set of cosets representatives, one for each coset. In this paper, the coset representatives are $\left[\square Z^T_{2^n}\{2^n\}^nT/C_0\} = Z^T_{4\{n-1\}}\right]$ mod 2.

So, the group $Z^T_{2^m}$ is decomposed in $2^{n}(n-1)$ cosets.

Besides, it is possible to create $n$ sets of these cosets. For $2^n$-PSK modulation and $n$ transmit antennas; the first set of cosets is $E_0 = C_0 = 2^{n-1} Z^T_{2^n}$ and the others sets $E_i$ of cosets with $1 \leq i \leq n-1$ are defined by,

$$E_i = \bigcup_{g_i} [g_i + C_0], \text{ with } g_i \in 2^{n_i-1} Z^T_{2^{n_i}}$$

Each coset $C_p = p + C_0 \in E_i$, with $0 \leq i \leq n-2$ is called relative to $q = 2p \in E_{i+1}$.

**B. Coset partitioning**

Calderbank et al. give an alternative to the set partitioning. It is based on the observation that the points of the constellation of a convolutional encoder can be regarded as a finite set of points from a group [9]. The group is partitioned into cosets. The Euclidean distances between the elements of the cosets are maximized. The coder selects at each time one coset. In each coset, it selects one element. It is possible to adapt the works of Ungerboeck and Calderbank et al. for the STTCs. In fact, the MIMO constellation can be seen as a finite subgroup of $Z$. The goal of this method is that all the MIMO symbols originating from or merging to the same state belong to the same coset. The elements of these cosets must be separated by the largest Euclidean distances. Thus, for a 2-PSK STTC with 2 states, the generator
matrix $G$ is constituted by $v+1$ blocks of $n$ columns. To design the STTCs with the coset partitioning, each block $i$ must generate a subgroup. Thus, the following rules must be respected:

- For each block $k$, its first column $G_{k1}$ must belong to $C_0$ with $1 \leq k \leq v+1$.
- The columns $G_{k2}$ with $2 \leq i \leq n$ and $1 \leq k \leq v+1$ must belong to the cosets relative to an element generated by the first $(i-1)$ columns or in $C_0$.

After the choice of all columns of $G$, it is possible to permute the columns and the lines of each block to obtain codes which respect the coset partitioning. Many subgroups exist and we show in the next section how to select the optimal elements to create the best generator matrices $G$.

### IV. NEW CODES AND PERFORMANCE ANALYSIS

In Table 1, Chen’s 4-PSK codes and new corresponding codes are shown for 3 transmit antennas/32 states and 4 transmit antennas/64 states. The new code with 4 transmit antennas has a higher value of the minimum trace than that of the Chen’s equivalent code. The new code with 3 transmit antennas has a trace equal to the trace of the Chen’s corresponding code, but, the minimum Euclidean distance between two different elements generated by each block of the new code are higher. In Table 2, Chen’s 8-PSK 8 states codes and new corresponding codes are shown for 3 and 4 transmit antennas. The new code with 3 transmit antennas has a trace equal to the trace of the Chen’s corresponding code and the code with 4 transmit antennas has a trace lower than the Chen’s corresponding code. We compute the minimum Euclidean distance between the elements generated by each block. Their minimum value is higher than that of the Chen’s corresponding code.

![Table 1: New 4-PSK code based on the Euclidean distance criteria [1]](image1)

The performance of each code is evaluated by simulation in a slow Rayleigh fading channel. The channel fading coefficients are independent samples of a complex Gaussian process with zero mean and variance 0.5 per dimension. These channel coefficients are assumed to be known at the decoder. Each frame consists of 130 4-PSK or 8-PSK symbols. For the simulation, there are 2 receive antennas. The decoding is performed by the Viterbi’s algorithm. Fig. 3 shows the performance of the 4-PSK 64 states codes for 3 and 4 transmit antennas given in Table 1.

![Table 2: New 8-PSK codes based on the Euclidean distance criterion [1]](image2)

Similarly, Fig. 4 shows the performance of the new 8-PSK codes and the corresponding Chen’s codes of Table 2.

![Fig. 3: Performance of 4-PSK 64/32 states STTCs [2]](image3)

![Fig. 4: Performance of 8-PSK 8 states STTCs [1]](image4)
V. CONCLUSION

A fast method to design easily the best 2\textsuperscript{n}-PSK STTCs with \(n_T\) transmit antennas has been presented in this paper. This method reduces significantly the number of possible codes which must be analyzed with an exhaustive search. The proposed method is based on the set partitioning proposed by Ungerboeck and an extension of the work proposed by Calderbank. This method is called 'coset partitioning'. Thanks to this method, new 4-PSK and 8-PSK codes are obtained. It is shown that these new codes outperform the best known codes.

REFERENCES


