Selection of Mixed Sampling Plan Indexed Through AOQcc with Conditional Double Sampling Plan as Attribute Plan

R. Radhakrishnan, K. Esther Jenitha

Abstract - In this paper a procedure for the selection of Mixed Sampling Plan (MSP) indexed through AOQcc, which is a convex combination of consumer oriented concept, AOQL (Average Outgoing Quality Limit) and producer oriented concept, MAAOQ (Maximum Allowable Average Outgoing Quality) with Conditional Double Sampling Plan (CDSP) as attribute plan is given. This plan may safeguard the interests of both producer as well as consumer by properly choosing a right combination using the gain parameter $\lambda(0<\lambda<1)$. Tables are also constructed for the easy selection of the plan.

I. INTRODUCTION

The AOQL is the average quality that the consumer will receive in the long run when the defective items are replaced by non-defective items. The construction of sampling plans based on AOQL is largely consumer oriented and MAAOQ is the average outgoing quality at the inflection point is a producer oriented, which is the average outgoing quality at MAPD. The advantage of using MAAOQ for designing a sampling plan instead of AOQL is that it reduces the sample size to be inspected which reduces the cost of inspection and results reduction in total cost. The use of MAAOQ for deriving sampling plans was justified by Suresh and Ram Kumar (1996). Radhakrishnan (2002) studied various sampling plans indexed through MAPD and MAAOQ. Radhakrishnan and Mallika (2008) constructed single sampling plans indexed through AOQcc. Radhakrishnan and Esther Jenitha (2011a, 2011b, 2011c, 2011d) constructed continuous sampling plan of the type CSP, CSP-3, CSP V (i-2x), Continuous Sampling Plan of the Type T CSP-3 plan indexed through AOQcc which is the convex combination of AOQL and MAAOQ. In this paper an attempt is made to introduce AOQcc in MSP using CDSP as an attribute plan. This plan may safeguard the interests of both producer as well as consumer by choosing a right combination using the gain parameter $\lambda$.

Sampling plans indexed through $p^*$ (MAPD) which is the quality level corresponding to the inflection point of the operating characteristic (OC) curve has been explained by Mandelson (1962), Mayer (1967) and further studied by Soundararajan (1975). The MAPD is the value of fraction defective ($p^*$) at which $d^2P_d(p)/dp^2 = 0$, for $p = p^*$.

$\frac{d^2P_d(p)}{dp^2} < 0$, for $p < p^*$.

A variety of plans and procedures have been developed for special sampling situations involving both measurements and attributes. Each is tailored to do a specific job under prescribed circumstances. They range from a simplified variables approach to a more technically complicated combination of variables and attributes sampling in a so-called mixed sampling plans. Mixed sampling plan is a two stage sampling procedure involving variables inspection in the first stage and attributes inspection in the second stage if the variables inspection of the first sample does not lead to acceptance. Use of variables on the first sample with attributes on the second sample combines the economy of variables for quick acceptance on the first sample with the broad non-parametric protection of attributes sampling when a questionable lot requires a second sample.

The mixed sampling plans are initially introduced by Dodge (1932) and later developed by Bowker and Goode (1952), Schilling (1967) has given a method for determining the operating characteristics for mixed variables-attributes sampling plans. Using Schilling’s procedure, Devaarul (2003) has constructed mixed sampling plan. Sampath Kumar (2007) constructed mixed sampling plan with DSP plan as attribute plans indexed through the parameters MAPD, AOQL and MAAOQ. Radhakrishnan and Saravanan (2011) Constructed Dependent Mixed Sampling Plan using Double Sampling Plan of the type DSP $(0, 1)$ indexed through IQL.

II. GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

- $p_j$: submitted quality of lot or process
- $P_a(p_j)$: probability of acceptance for given quality $p_j$
- $p^*$: maximum allowable percent defective (MAPD)
- $P_a$: the product quality at which AOQ is maximum
- $h$: relative slope at $p^*$
- $c$: attributes acceptance number
- $c_1$: first attributes acceptance number
- $c_2$: second attributes acceptance number
- $c_3$: third attributes acceptance number
- $d$: number of defectives in the sample
- $d_j$: number of defectives in the $j^{th}$ sample ($j = 1, 2, 3, \ldots$)
- $n_1$: sample size for variable sampling plan
- $n_{1,2}$: first sample size for attribute sampling plan
- $n_{2,2}$: second sample size for attribute sampling plan
- $\beta_j$: probability of acceptance for lot quality $p_j$
βj: probability of acceptance assigned to first stage for percent defective pj
βj*: probability of acceptance assigned to second stage for percent defective pj
z (j) : "z" value for the jth ordered observation
z(p) : standard normal deviate
k: variable factor such that a lot is accepted if X ≤ A = U- kσ

III. FORMULATION OF MSP WITH CDSP

Procedure: Independent Plan

- Determine the parameters of the mixed plan n1, n1,2, n2, k, c1, c2 and c1
- Take a random sample of n1 from the lot.
- If the sample average X ≤ A = U - kσ, accept the lot.
- If the sample average X > A = U - kσ, take another sample of size n1,2 and count the number of defectives d1 therein.
- If the number of defectives d1 ≤ c1, accept the lot.
- If the number of defectives d1 > c1, reject the lot.
- If c1+1 ≤ d1 ≤ c1, take a second sample of size n2,2 from the remaining lot and find the number of defectives 'd2'.
- If d2 ≤ c2 or d2 > c2, reject the lot. Otherwise accept the lot.

IV. CONDITIONS FOR APPLICATIONS

(i) Production process should be steady and continuous.
(ii) Lots are submitted substantially in the order of their production.
(iii) Inspection is by variable in the first stage and attribute in the second stage with quality defined as the fraction defective.

V. CONSTRUCTION OF MSP WITH CDSP AS ATTRIBUTE PLAN INDEXED THROUGH MAPD

The procedure for the construction of mixed variables – attributes sampling plans is provided by Schilling (1967) for a given ‘n1’ and a point ‘p1’ on the OC curve. A modified procedure for the construction of mixed variables – attributes sampling plan for a given MAPD and ‘n1’ is given below.

- Assume that the mixed sampling plan is independent.
- Split the probability of acceptance (βj) determining the probability of acceptance that will be assigned to the first stage. Let it be βj;
- Decide the sample size n1 (for variable sampling plan) to be used;
- Calculate the acceptance limit for the variable sampling plan as
  A = U - kσ = U - [z (p1) + z (βj)/√n1]σ, where z (t) is the standard normal

VI. CONSTRUCTION OF TABLES

The probability of acceptance for CDSP under Poisson model is given by

Pd(p) = \sum_{r=0}^{c1} \frac{e^{-n1p} (n1p)^r}{r!} \frac{1}{1-\beta_1} + \left\{ \sum_{k=c1+1}^{c2} \frac{e^{-n1p} (n1p)^k}{k!} \left( \sum_{r=0}^{c2} \frac{e^{-n2p} (n2p)^r}{r!} \right) \right\} = \frac{\beta_1}{\text{for } p = p_1}

For n1 = n2 = n (say), the inflection point (p*) is obtained by using [dP(p)/dp] = 0 and [d^2P(p)/dp^2] ≠ 0. The relative slope of the OC curve h_β = \left[ -\frac{np}{Pa(p)} \right] \frac{dPa(p)}{dp}

at p = p_1. The inflection tangent of the OC curve cuts the ‘p’ axis at p = p_1 + (p/h_β). The values of h_β, np, np_1 and R = p_1/p_2 are calculated for a specified β_1 = 0.35 using visual basic program and presented in Table 1.

The general procedure for designing a CDSP indexed through a parameter which is a convex combination of AOQL and MAAOQ using Poisson distribution as base line distribution is given below.

Step 1. Determine n2MAAOQ and n2AOQL for CDSP for various combinations of c1, c2, c3 and n2p. Find R1 = n2AOQL/n2p and R2 = n2MAAOQ/n2p. Find R3 = n3AOQc/n2p for selected values of λ.

Step 2. Present the results of Step1 and Step 2 in Table 1.

VII. SELECTION OF THE PLAN

Table 1 is used to construct the plans when MAPD (p1) and tangent intercept (p2) are given. For any given values of c1, c2, c3, p1 and p2, one can find the ratio R = p_2/p_1. Corresponding to the value of c1, c2 and c3, find the value of R in Table 1. From this c1, c2 and c3 values one can determine the value of ‘n’ using n=np/R/p_1.

Example:
(i) For a specified AOQL = 0.00325 and p_1 = 0.0055 compute the ratio R1 = AOQL/p_1 = 0.5909 which is
associated with \( c_1=9, c_2=13 \) and \( c_2=23 \) in Table 1 and \( n_1=n_{p/p} = 13.069/0.0055 = 2376 \). Thus \( n_{i2}=2376, n_2=2376, c_1=9, c_2=13 \) and \( c_2=23 \) are the parameters selected for the MSP with CDSP as attribute plan for a specified \( p=0.0055, \text{AQL}=0.00325 \).

(ii) For a specified MAACL = 0.0021 and \( p=0.0055 \) compute the ratio \( R= \text{MAACL} / p_c = 0.3818 \) which is associated with \( c_1=2, c_2=6 \) and \( c_2=8 \) in Table 1 and \( n_1=n_{p/p} = 4.867 / 0.0055 = 885 \). Thus \( n_{i1}=885, n_{i2}=885, c_1=2, c_2=6 \) and \( c_2=8 \) are the parameters selected for the MSP with CDSP as attribute plan for a specified \( p=0.0055, \text{AQL}=0.00325 \).

(iii) For a specified value of \( \text{AOQL} = 0.00325 \), MAACL = 0.0021 and \( p=0.0055 \) and \( \lambda=0.2 \), \( \text {AOQ}_{cc} = 0.0023 \) compute \( R= \text{AOQ}_{cc} / p_c = 0.4236 \) with \( c_1=5, c_2=8 \) and \( c_2=12 \) in Table 1 and \( n_1=n_{p/p} / p_c = 7.3230 / 0.0055 = 1331 \). Thus \( n_{i1}=1331, n_{i2}=1331, c_1=5, c_2=8 \) and \( c_2=12 \) are the parameters selected for the MSP with CDSP as attribute plan for a specified \( p=0.0055, \text{AQL}=0.00325 \), MAACL=0.0021.

**Practical application:**

Suppose the plan with \( n_1 = 10, k = 1.5 \) is to be applied to the lot-by-lot acceptance inspection of shafts from a production line, the characteristic to be inspected is the "shafts diameters in mm" for which there is a specified upper limit (U) of 45mm with a known standard deviation (\( \sigma \)) of 0.066 mm. In this example, \( U=45 \), \( \sigma = 0.066 \) mm and \( k = 1.5 \). Now, in applying the variable inspection first, take a random sample of size \( n_1=10 \) from the lot. Record the sample results and find \( \bar{X} \). If \( \bar{X} \leq U - k \sigma = 44.991 \) mm, accept the lot otherwise take a random sample of size \( n_2 = 1331 \) and apply attribute inspection.

**REFERENCES**


[7]. R. Radhakrishnan, and M. Mallika, “Designing of Sampling plans indexed through the convex combination of AOQL and MAAOQ,” Published as a proceedings of the National level Conference on IT and Business Intelligence organized by Institute of Management Technology, Nagpur, India Excel publishers pp : 45-52, 2008.


[12]. R. Radhakrishnan, and P.G. Saravanan, “Construction & Selection of Dependent Mixed Sampling Plan using Double Sampling Plan of the type DSP(0,1) indexed through IQL,” Published as a proceedings of the International Conference on Mathematics & its Applications organized by Department of Mathematics, Avinashilingam University, Coimbatore, 2011.


AUTHOR BIOGRAPHY

**Dr. Radhakrishnan** is having a bachelor degree, post graduate degree in Statistics and also possessing research degrees such as MPhil and PhD and also has additional qualification of post graduate degree in Business Administration. He has 34 years of teaching experience in teaching theoretical and applied Statistics, presented more than 160 papers in National and International seminars & conferences and published more than 119 articles in reputed National and International journals. He is also a quality auditor for ISO certifications and certified Six Sigma Black Belt also. He gains sufficient knowledge in Six Sigma methodologies and he is conducting training programme for Six Sigma black belts. He has visited countries like Sri Lanka and China for presenting his research contributions.

**Esther Jenitha** is having is having postgraduate and MPhil degrees in Statistics. She is an Assistant Professor in Statistics with more than five years of teaching experience and doing research under the guidance of Dr. R. Radhakrishnan. She has presented 4 papers in the National and International seminars & conferences and published 8 articles in reputed National and International journals.
<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>( n_2p^a )</th>
<th>nAOQL</th>
<th>R₁</th>
<th>nMAAQL</th>
<th>R₂</th>
<th>( \lambda = 0.2 )</th>
<th>( \lambda = 0.4 )</th>
<th>( \lambda = 0.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.7850</td>
<td>0.4042</td>
<td>0.5149</td>
<td>0.4866</td>
<td>0.6198</td>
<td>0.5017</td>
<td>0.5016</td>
<td>0.4867</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1.9010</td>
<td>0.8754</td>
<td>0.4605</td>
<td>0.8867</td>
<td>0.4665</td>
<td>0.8847</td>
<td>0.8825</td>
<td>0.8803</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>4</td>
<td>2.5180</td>
<td>1.1882</td>
<td>0.4719</td>
<td>1.0889</td>
<td>0.4325</td>
<td>1.1119</td>
<td>1.1333</td>
<td>1.1536</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>5</td>
<td>3.0120</td>
<td>1.4181</td>
<td>0.4708</td>
<td>1.2289</td>
<td>0.4080</td>
<td>1.2746</td>
<td>1.3167</td>
<td>1.3547</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>4</td>
<td>2.6140</td>
<td>1.2473</td>
<td>0.4772</td>
<td>1.1799</td>
<td>0.4514</td>
<td>1.1950</td>
<td>1.2091</td>
<td>1.2227</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>4</td>
<td>2.6500</td>
<td>1.2732</td>
<td>0.4805</td>
<td>1.1929</td>
<td>0.4502</td>
<td>1.2111</td>
<td>1.2283</td>
<td>1.2441</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>3.7970</td>
<td>1.8715</td>
<td>0.4929</td>
<td>1.5901</td>
<td>0.4188</td>
<td>1.6603</td>
<td>1.7234</td>
<td>1.7799</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6</td>
<td>3.8660</td>
<td>1.9331</td>
<td>0.5000</td>
<td>1.6207</td>
<td>0.4192</td>
<td>1.7943</td>
<td>1.9224</td>
<td>1.9997</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>8</td>
<td>4.8670</td>
<td>2.5162</td>
<td>0.5170</td>
<td>1.8569</td>
<td>0.3815</td>
<td>2.0335</td>
<td>2.1869</td>
<td>2.3191</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>8</td>
<td>5.0300</td>
<td>2.6062</td>
<td>0.5181</td>
<td>2.0143</td>
<td>0.4005</td>
<td>2.1693</td>
<td>2.3066</td>
<td>2.4262</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>11</td>
<td>6.3580</td>
<td>3.2815</td>
<td>0.5161</td>
<td>2.2136</td>
<td>0.3482</td>
<td>2.4986</td>
<td>2.7502</td>
<td>2.9669</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>8</td>
<td>5.0490</td>
<td>2.6255</td>
<td>0.5200</td>
<td>2.0227</td>
<td>0.4006</td>
<td>2.1806</td>
<td>2.3206</td>
<td>2.4425</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>11</td>
<td>6.4930</td>
<td>3.4746</td>
<td>0.5351</td>
<td>2.3115</td>
<td>0.3560</td>
<td>2.6263</td>
<td>2.8991</td>
<td>3.1361</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>12</td>
<td>7.3230</td>
<td>4.0048</td>
<td>0.5469</td>
<td>2.7591</td>
<td>0.3768</td>
<td>3.0924</td>
<td>3.3871</td>
<td>3.6402</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>12</td>
<td>7.3590</td>
<td>4.0562</td>
<td>0.5512</td>
<td>2.7806</td>
<td>0.3778</td>
<td>3.1223</td>
<td>3.4243</td>
<td>3.6842</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>15</td>
<td>8.7020</td>
<td>4.8663</td>
<td>0.5592</td>
<td>2.9533</td>
<td>0.3394</td>
<td>3.4677</td>
<td>3.9228</td>
<td>4.3151</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>15</td>
<td>8.8510</td>
<td>4.9427</td>
<td>0.5884</td>
<td>3.1251</td>
<td>0.3531</td>
<td>3.6129</td>
<td>4.0445</td>
<td>4.4167</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>18</td>
<td>10.3550</td>
<td>5.8471</td>
<td>0.5647</td>
<td>3.4442</td>
<td>0.3326</td>
<td>4.0886</td>
<td>4.6600</td>
<td>5.1532</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>18</td>
<td>10.4610</td>
<td>6.0606</td>
<td>0.5793</td>
<td>3.5847</td>
<td>0.3392</td>
<td>4.2219</td>
<td>4.8202</td>
<td>5.3377</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>21</td>
<td>11.3390</td>
<td>6.6764</td>
<td>0.5888</td>
<td>4.0960</td>
<td>0.3612</td>
<td>4.7796</td>
<td>5.3906</td>
<td>5.9219</td>
</tr>
<tr>
<td>9</td>
<td>13</td>
<td>23</td>
<td>13.0690</td>
<td>7.7618</td>
<td>0.5939</td>
<td>4.1801</td>
<td>0.3198</td>
<td>5.1337</td>
<td>5.9854</td>
<td>6.7269</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>26</td>
<td>14.8310</td>
<td>9.0429</td>
<td>0.6097</td>
<td>4.8052</td>
<td>0.3240</td>
<td>5.9278</td>
<td>6.9344</td>
<td>7.8139</td>
</tr>
<tr>
<td>11</td>
<td>16</td>
<td>26</td>
<td>14.8960</td>
<td>9.2080</td>
<td>0.6182</td>
<td>4.8882</td>
<td>0.3282</td>
<td>6.0311</td>
<td>7.0571</td>
<td>7.9545</td>
</tr>
<tr>
<td>11</td>
<td>16</td>
<td>29</td>
<td>16.0790</td>
<td>9.9485</td>
<td>0.6187</td>
<td>5.0587</td>
<td>0.3146</td>
<td>6.3464</td>
<td>7.5049</td>
<td>8.5220</td>
</tr>
<tr>
<td>15</td>
<td>19</td>
<td>34</td>
<td>19.1570</td>
<td>12.1047</td>
<td>0.6319</td>
<td>6.0153</td>
<td>0.3140</td>
<td>7.0697</td>
<td>9.0501</td>
<td>10.3201</td>
</tr>
<tr>
<td>17</td>
<td>21</td>
<td>36</td>
<td>20.5900</td>
<td>13.2551</td>
<td>0.6438</td>
<td>6.7884</td>
<td>0.3297</td>
<td>8.4781</td>
<td>10.0079</td>
<td>11.3584</td>
</tr>
<tr>
<td>23</td>
<td>28</td>
<td>52</td>
<td>28.5630</td>
<td>19.1107</td>
<td>0.6691</td>
<td>8.3052</td>
<td>0.2908</td>
<td>11.0698</td>
<td>13.5998</td>
<td>15.8663</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td>56</td>
<td>30.6840</td>
<td>20.7102</td>
<td>0.6749</td>
<td>8.8606</td>
<td>0.2888</td>
<td>11.8780</td>
<td>14.6461</td>
<td>17.1333</td>
</tr>
</tbody>
</table>

ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 2, Issue 1, July 2012

APPENDIX

Table 1: Parameters of Mixed Sampling plan (CDSP) for \( \beta' = 0.35, \beta_{aoq} = 0.35 \).