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Improvement of Convergence Characteristics of FIR filter using Adaptive Techniques

USN Rao, Manish Garg, Prateek

Abstract*-- The most popular means of FIR filtering technique is to utilize NLMS algorithm. As the length of the filter and consequently the number of filter coefficients increase, the design of the filter becomes complex and therefore the popular Max NLMS algorithm has been introduced. As a consequence the filter design becomes very easy but at the cost of its performance in terms of convergence characteristics i.e., convergence occurs at a later stage taking too long computational time for the processing of the signal. In this paper, a proposal of improving the convergence characteristics is made which does not affect the performance of the design without compromising the tap-selection process of the MMax NLMS algorithm. A concept of variable step-size for the filter coefficients is applied so that loss in performance due to MMax NLMS algorithm is effectively reduced and the convergence is better achieved for the given filter length.

Index Terms — **Mmax NLMS Algorithm, Variable Step-Size, Performance, Convergence Characteristics, Filter Coefficients, Adaptive Algorithm.**

I. INTRODUCTION

Finite impulse response (FIR) with Adaptive filtering techniques finds extensive application in signal processing. The normalized least-mean-square (NLMS) algorithm [*1*][*2*] is treated as one of the most popular adaptive algorithms in many applications. Since the NLMS algorithm requires *O*(2*L*) multiply accumulate (MAC) operations per sampling period, it is very desirable to reduce the computational workload of the processor. Partial update adaptive algorithms differ in the criteria used for selecting filter coefficients to update at each of the iteration. It is found that as the number of filter coefficients updated per iteration in a partial update adaptive filter is reduced, the computational complexity is also reduced but at the expense of some loss in performance. The aim of this paper is to propose improving the convergence characteristics of adaptive algorithm. It has been shown in [*6*] that the convergence performance of MMax-NLMS is dependent on the step-size. Analysis of the mean-square deviation of MMax-NLMS is first presented and then a variable step-size in order to increase its rate of convergence is derived. The simulation results verify that the proposed variable step-size MMax-NLMS (MMax- NLMSvss) algorithm achieves *higher* rate of convergence with lower computational complexity compared to NLMS for white Gaussian noise (WGN).

II. THE MMAX-NLMS ALGORITHM

The output at the *n*th iteration, $v(n) = \mathbf{u}^T(n)\mathbf{h}(n)$ where $\mathbf{u}(n) = [u(n), u(n-1), \ldots, u(n-L+1)]^T$ is the tap-input vector while the unknown impulse response $h(n) = [h_o(n), \ldots, h_{L-1}(n)]^T$ is of length *L*. An adaptive filter $\hat{h}(n) = [\hat{h}_o(n), \ldots, \hat{h}_{L-1}(n)]^T$ which assumed [*3*] to be of equal length to the unknown system $\mathbf{h}(n)$, is used to estimate $\mathbf{h}(n)$ by adaptively minimizing *a priori* error signal $e(n)$ using $\hat{v}(n)$ defined by

$$e(n) = u^T(n)h(n) - \hat{v}(n) + g(n) \quad \text{Eq. (1)}$$

$$\hat{v}(n) = u^T(n)\hat{h}(n-1) \quad \text{Eq. (2)}$$

with g(*n*) being the measurement noise.
In the MMax-NLMS algorithm [*4*], only those taps corresponding to the *M* largest magnitude tap-inputs are selected for updating at each iteration with $1 \le M \le L$. Defining the sub-selected tap-input vector ,

$$\hat{u}(n) = Q(n)u(n) \quad \text{Eq. (3)}$$

where $\mathbf{Q}(n) = \mathrm{diag}\{q(n)\}$ is an *L* x *L* tap selection matrix and $\mathbf{Q}(n) = [q_0(n), \ldots, q_{L-1}(n)]^T$ element $q_j(n)$ for $j = 0, 1, \ldots, L-1$ is given by,

$$q_j(n) = \begin{cases} 1 & |u(n-j)| \in \{M \text{ Maxima of } |\mathbf{u}(n)|\} \\ 0 & \textit{otherwise} \end{cases} \quad \text{Eq. (4)}$$

Where

$$|\mathbf{u}(n)| = \left[|u(n)|, \ldots\ldots, |u(n-L+1)|\right]^T$$

Defining $\|.\|^2$ as the squared l_2-norm, the MMax-NLMS tap-update equation is then

$$\hat{\boldsymbol{h}}(n) = \hat{\boldsymbol{h}}(n-1) + \frac{\mu \mathbf{Q}(n)\mathbf{u}(n)e(n)}{\|\mathbf{u}(n)\|^2 + C} \quad \text{Eq. (5)}$$

where *C* is the regularization parameter. Defining $\mathbf{I}_{L\times L}$ as the L x L identity matrix, it is noted that if $\mathbf{Q}(n) = \mathbf{I}_{L\times L}$, i.e., with $M = L$, the update equation in (Eq. 5) is equivalent to the NLMS algorithm. Similar to the NLMS algorithm, the step-size μ in (Eq. 5) controls the ability of MMax-NLMS to track the unknown system which is reflected by its rate of convergence. To select the *M* maxima of $|\mathbf{u}(n)|$ in (Eq. 4), MMax-NLMS employs the SORTLINE algorithm [*7*] which requires $2\log_2 L$ sorting operations per iteration. The computational complexity in terms of multiplications for MMax-NLMS is $O(L+M)$ compared to $O(2L)$ for NLMS. The performance of MMax-NLMS normally reduces with the number of filter coefficients updated per iteration. This tradeoff between complexity and convergence can be shown by first defining $\xi(n)$, the normalized misalignment as

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$$\xi(n) = \frac{\left\| \boldsymbol{h}(n) - \hat{\boldsymbol{h}}(n) \right\|^2}{\left\| \boldsymbol{h}(n) \right\|^2} \quad \text{Eq. (6)}$$

Fig.1 and Fig.2 shows the variation in convergence performance of MMax-NLMS with M for the case of L = 128 and μ = 0.1 using a white Gaussian noise (WGN) as input. For this illustrative example, WGN g(n) is added to achieve a signal-to-noise ratio (SNR) of 15dB. It can be seen that the rate of convergence reduces with reducing M as expected.

III. MEAN SQUARE DEVIATION OF MMAX-NLMS

It has been shown in [5] that the convergence performance of MMax- NLMS is dependent on the step-size μ when identifying a system. Since the aim of this paper is to reduce the degradation of convergence performance due to partial updating of the filter coefficients, from Fig.2 it is clear that the convergence performance decreases as M=L/4. Fig.3 shows the Normalized misalignment verses Time.

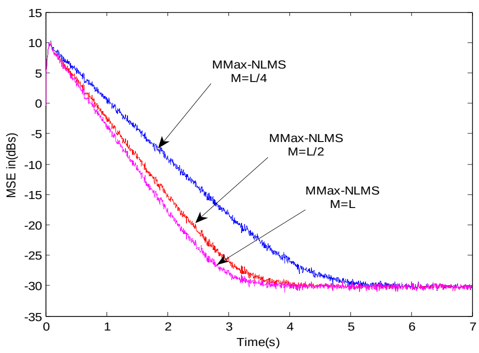

Fig1: Convergence curves of MMax-NLMS for different M.

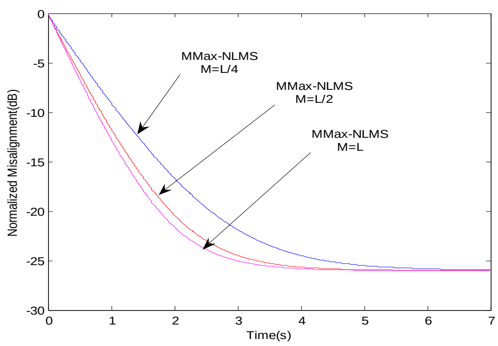

Fig 2: Normalized Misalignment curves for different M.

The MSD of MMax-NLMS can be obtained by first defining the system deviation as

$$\grave{o}(n) = \boldsymbol{h}(n) - \hat{\boldsymbol{h}}(n) \quad \text{Eq. (7)}$$

$$\grave{o}(n-1) = \boldsymbol{h}(n) - \hat{\boldsymbol{h}}(n-1) \quad \text{Eq. (8)}$$

Subtracting (8) from (7) and using (5), we obtain

$$\grave{o}(n) = \grave{o}(n-1) + \frac{\mu \mathbf{Q}(n)\mathbf{u}(\mathrm{n})\mathrm{e}(\mathrm{n})}{\mathbf{u}^T(n)\mathbf{u}(\mathbf{n}) + C} \quad \text{Eq. (9)}$$

Defining $\varphi\{.\}$ as the expectation operator and taking the mean square of (9), the MSD of MMax-NLMS can be expressed iteratively as

$$\varphi\left\{\left\|\grave{o}(\mathrm{n})\right\|^2\right\} = \varphi\left\{\grave{o}^T(\mathrm{n})\grave{o}(\mathrm{n})\right\} = \varphi\left\{\left\|\grave{o}(n-1)\right\|^2\right\} - \varphi\{\Phi(\mu)\} \quad \text{Eq. (10)}$$

Where

$$\varphi\{\Phi(\mu)\} = \varphi\left\{ \frac{2\mu\tilde{\mathbf{u}}^T(n)\grave{o}(n-1)\mathrm{e}(n)}{\left\|\mathbf{u}(\mathrm{n})\right\|^2} - \frac{\mu^2 \left\|\tilde{\mathbf{u}}(n)\right\|^2 \mathrm{e}^2(n)}{\left[\left\|\mathbf{u}(n)\right\|^2\right]} \right\}$$

Eq. (11)

Assume that the effect of the regularization term Con the MSD is small. As can be seen from (Eq. 10), in order to increase the rate of convergence for the MMax-NLMS algorithm, step-size μ is chosen such that $\varphi\{\Phi(\mu)\}$ is maximized.

IV. THE PROPOSED MMAX-NLMS$_{VSS}$ ALGORITHM

Following the approach of [6], differentiating (Eq. 11) with respect to μ and setting the result to zero,

$$\varphi\left\{ \frac{\mu(n)\left\|\tilde{\mathbf{u}}(n)\right\|^2 \mathrm{e}^2(n)}{\left[\left\|\mathbf{u}(\mathrm{n})\right\|^2\right]} \right\} = \varphi\left\{ \grave{o}^T(n-1)\tilde{\boldsymbol{u}}(n)\left[\left\|\boldsymbol{u}(n)\right\|^2\right]^{-1} e(n) \right\}$$

giving the variable step-size

$$\mu(n) = \mu_{max} \times \frac{\grave{o}^T(\mathrm{n}-1)\tilde{\mathbf{u}}(\mathrm{n})\left[\left\|\mathbf{u}(\mathrm{n})\right\|^2\right]^{-1} \mathbf{u}^T(n)\grave{o}(n-1)\left\|\mathbf{u}(\mathrm{n})\right\|^2}{\left\|\tilde{\mathbf{u}}(n)\right\|^2 \grave{o}^T(\mathrm{n}-1)\mathbf{u}(\mathrm{n})\left[\left\|\mathbf{u}(\mathrm{n})\right\|^2\right]^{-1} \mathbf{u}^T(n)\grave{o}(n-1) + \sigma_g^2 \mathrm{M}(n)}$$

where $0 < \mu_{\max} \leq 1$ limits the maximum of $\mu(n)$ and from **[6]**

$$\mathrm{M}(\mathrm{n}) = \frac{\left\|\tilde{\mathrm{u}}(\mathrm{n})\right\|^2}{\left\|\mathrm{u}(\mathrm{n})\right\|^2} \quad \text{Eq. (12)}$$

is the ratio between energies of the sub-selected tap-input vector $\tilde{\mathbf{u}}(n)$ and the complete tap-input vector $\mathbf{u}(n)$, while $\sigma_g^2 = \varphi\{g^2(n)\}$. To simplify the numerator of $\mu(n)$ further, considering $\tilde{\mathbf{u}}(n)\mathbf{u}^T(n) = \tilde{\mathbf{u}}(n)\tilde{\mathbf{u}}^T(n)$

$$\mu(n) = \mu_{max} \times \frac{\grave{o}^T(\mathrm{n}-1)\tilde{\mathbf{u}}(\mathrm{n})\left[\left\|\mathbf{u}(\mathrm{n})\right\|^2\right]^{-1} \tilde{\mathbf{u}}^T(n)\grave{o}(n-1)\left\|\mathbf{u}(\mathrm{n})\right\|^2}{\left\|\tilde{\mathbf{u}}(n)\right\|^2 \grave{o}^T(\mathrm{n}-1)\mathbf{u}(\mathrm{n})\left[\left\|\mathbf{u}(\mathrm{n})\right\|^2\right]^{-1} \mathbf{u}^T(n)\grave{o}(n-1) + \sigma_g^2 M(n)}$$

$\mu(n)$ can be further simplified by letting

$$\tilde{\mathbf{P}}(n) = \tilde{\mathbf{u}}(n)\left[\mathbf{u}^T(n)\mathbf{u}(\mathrm{n})\right]^{-1} \tilde{\mathbf{u}}^T(n)\grave{o}(n-1) \quad \text{Eq. (13)}$$

$$\mathbf{P}(n) = \mathbf{u}(n)\left[\mathbf{u}^T(n)\mathbf{u}(\mathrm{n})\right]^{-1} \mathbf{u}^T(n)\grave{o}(n-1) \quad \text{Eq. (14)}$$

from which it is then shown that [9]

$$\left\|\tilde{\mathbf{P}}(n)\right\|^2 = \mathrm{M}(n)\grave{o}^T(\mathrm{n}-1)\tilde{\mathbf{u}}(n)\left[\left\|\mathbf{u}(\mathrm{n})\right\|^2\right]^{-1} \tilde{\mathbf{u}}^T(n)\grave{o}(n-1)$$

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$$\left\|\mathbf{P}(n)\right\|^2 = \grave{o}^T(n-1)\mathbf{u}(n)\left[\left\|\mathbf{u}(n)\right\|^2\right]^{-1}\mathbf{u}^T(n)\grave{o}(n-1)$$

Following the approach in [7], and defining 0 << α <1 as the smoothing parameter, $\tilde{\mathbf{P}}(n)$ and $\mathbf{P}(n)$ are estimated iteratively by

$$\tilde{\mathbf{P}}(n) = \alpha\tilde{\mathbf{P}}(n-1) + (1-\alpha)\tilde{\mathbf{u}}(n)\left[\mathbf{u}^T(n)\mathbf{u}(n)\right]^{-1}e_a(n) \quad \text{Eq. (15)}$$

$$\mathbf{P}(n) = \alpha\mathbf{P}(n-1) + (1-\alpha)\mathbf{u}(n)\left[\mathbf{u}^T(n)\mathbf{u}(n)\right]^{-1}e(n) \quad \text{Eq. (16)}$$

where $e(n) = \mathbf{u}^T(n)\grave{o}(n-1)$ in (Eq. 16), the error $e_a(n)$ due to active filter coefficients $\tilde{\mathbf{u}}(n)$ in (Eq. 15) is given as

$$e_a(n) = \tilde{\mathbf{u}}^T(n)\grave{o}(n-1) = \tilde{\mathbf{u}}^T(n)\left[\boldsymbol{h}(n) - \hat{\boldsymbol{h}}(n-1)\right] \quad \text{Eq. (17).}$$

It is important to note that since $\tilde{\mathbf{u}}^T(n)\mathbf{h}(n)\mathbf{h}(n)$ is unknown, $e_a(n)$ is to be approximated. Defining $\bar{\mathbf{Q}}(n) = \mathbf{I}_{L\times L} - \mathbf{Q}(n)$ [9] as the tap-selection matrix which selects the inactive taps, we can express $e_i(n) = \left[\bar{\mathbf{Q}}(n)\mathbf{u}(n)\right]^T\grave{o}(n-1)$ as the error contribution due to the inactive filter coefficients such that the total error $e(n) = e_a(n) + e_i(n)$. As explained in [6], for $0.5L \le M < L$, the degradation in M(n) due to tap-selection is negligible. This is because, for M large enough, elements in $\bar{\mathbf{Q}}(n)\mathbf{u}(n)$ are small and hence the errors $e_i(n)$ are small, as is the general motivation for MMax tap-selection [8]. Approximating $e_a(n) \approx e(n)$ in (Eq. 15) gives

$$\tilde{\mathbf{P}}(n) \approx \alpha\tilde{\mathbf{P}}(n-1) + (1-\alpha)\tilde{\mathbf{u}}(n)\left[\mathbf{u}^T(n)\mathbf{u}(n)\right]^{-1}e(n) \quad \text{Eq. (18)}$$

Using (Eq. 16) and (Eq. 18), the variable step-size is then given as

$$\mu(n) = \mu_{max}\frac{\left\|\tilde{\mathrm{P}}(n)\right\|^2}{M^2(n)\left\|\mathbf{P}(n)\right\|^2 + C} \quad \text{Eq. (19)}$$

where $C = M^2(n)\sigma_g^2$. Since σ_g^2 is unknown, it is shown that approximating C by a small constant, typically 0.0001 [8]. The computation of (Eq. 16) and (Eq. 18) each requires M additions. In order to reduce computation even further, and since for M large enough the elements in $\bar{\mathbf{Q}}(n)\mathbf{u}(n)$ are small, approximating, $\left\|\mathbf{P}(n)\right\|^2 = \left\|\tilde{\mathrm{P}}(n)\right\|^2$ gives

$$\mu(n) = \mu_{max}\frac{\left\|\tilde{\mathrm{P}}(n)\right\|^2}{M^2(n)\left\|\tilde{\mathrm{P}}(n)\right\|^2 + C} \quad \text{Eq. (20)}$$

When $\mathbf{Q}(n) = \mathbf{I}_{L\times L}$, i.e., $M = L$, MMax-NLMS is equivalent to the NLMS algorithm and from (Eq. 12), $\mathbf{M(n)}$ = 1 and $\left\|\tilde{\mathrm{P}}(n)\right\|^2 = \left\|\mathbf{P}(n)\right\|^2$. As a consequence, the variable step-size $\mu(n)$ in (Eq. 20) is consistent with that presented in [8] for $M = L$.

V. SIMULATION RESULTS

The performance of MMax-NLMS$_{vss}$ in terms of the normalized misalignment is determined and defined in (Eq. 6) using WGN input. With a sampling rate of 8 kHz and a reverberation time of 256 ms, the length of the impulse response is L = 1024. Similar to [9], C = 0.0001, α = 0.15 are taken, WGN $g(n)$ is added to v(n) to achieve an SNR of 15dB. The value of μ_{max} = 1 is taken for MMax-NLMS$_{vss}$ while step-size μ for the NLMS algorithm is adjusted so as to achieve the same steady-state performance for all simulations. Fig.4 shows the improvement in convergence performance of MMax-NLMS$_{vss}$ over MMax-NLMS for the cases of M = L/4.

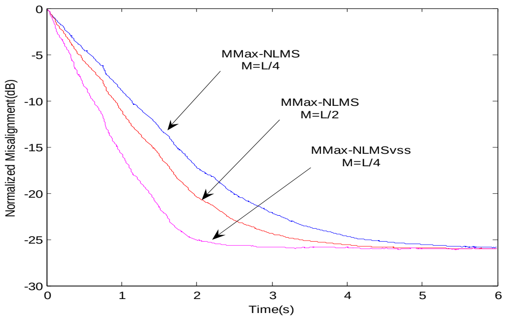

Fig 3: Improvement in convergence performance of MMax-NLMSvss over MMax-NLMS for different M.

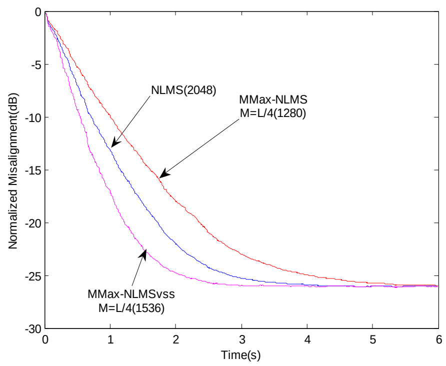

Fig 4: Comparison curves of Convergence performance of MMax-NLMSvss with NLMS and MMax-NLMS.

The step-size of NLMS has been adjusted in order to achieve the same steady-state normalized misalignment. This corresponds to μ = 0.1. More importantly, the proposed MMax-NLMS$_{vss}$ algorithm outperforms NLMS even with lower complexity when M =256. This improvement in normalized misalignment of 7 dB (together with a reduction of 25% in terms of multiplications) over NLMS is due to variable step-size for MMax-NLMS$_{vss}$. The MMax-NLMS$_{vss}$ achieves the same convergence performance as the NLMSvss [8] when M = L. In order to illustrate the benefits of the proposed algorithm, M = 256 taken for both MMax-NLMS and MMax-NLMS$_{vss}$. This gives a 25% savings in multiplications per iteration for MMax-NLMS$_{vss}$ over

NLMS. As can be seen, even with this computational savings, the proposed MMax-NLMS$_{vss}$ algorithm achieves an improvement of 1.5 dB in terms of normalized misalignment over NLMS.

VI. CONCLUSION

By analyzing the mean-square deviation of MMax-NLMS we can derive a partial update MMax-NLMS algorithm with a variable step-size during adaptation for improvement of convergence characteristics. Computation of (Eq. 18), computations of $\|\tilde{\mathrm{P}}(\mathrm{n})\|^2$ for (Eq. 20) require M multiplications each. The computation of $\|\mathrm{u}(\mathrm{n})\|^2$ and $\|\tilde{\mathrm{u}}(\mathrm{n})\|^2$ for $M(n)$ in (Eq. 12) requires 2 multiplications and a division using recursive means. Since the term $\tilde{\mathbf{u}}(n)\left[\mathbf{u}^T(n)\mathbf{u}(\mathrm{n})\right]^{-1} e(n)$ is already computed in (Eq. 18), no multiplications are now required for the update equation in (Eq. 5). Hence including the computation of $u^T(n)\hat{h}(n-1)$ for $e(n)$, MMax-NLMSvss requires $O(L + 2M)$ multiplications per sample period compared to $O(2L)$ for NLMS. The number of multiplications required for MMax-NLMSvss is thus less than NLMS when $M < L/2$. Although MMax-NLMSvss requires an additional 2 log2 L sorting operations per iteration using the SORTLINE algorithm [7], its complexity is still lower than NLMS. As with MMax-NLMS, we would expect the convergence performance for MMax-NLMSvss to degrade with reducing M. However, simulation results show that any such degradation is offset by the improvement in convergence rate due to $\mu(n)$. In terms of convergence performance, the proposed MMax-NLMS$_{vss}$ algorithm achieves approximately 3 dB improvement in normalized misalignment over NLMS for WGN input. More importantly, the proposed algorithm can achieve higher rate of convergence with lower computational complexity compared to NLMS.

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AUTHOR BIOGRAPHY

USN Rao obtained Master's Degree in Engineering from National Institute of Technology Bhopal from the year 2005, research interests are in Adaptive Signal Processing, Renewable Energy Systems, and VLSI Signal Processing. Currently working as Associate Professor & Head of the Department of ECE at Sri Venkateswara Engg College, Haryana, India.

Manish Garg doing Master's in Engineering from Hindu College of Engineering and working as Asst. Professor in the Dept. of ECE at Sri Venkateswara Engg College, Haryana. His research Interests are Digital Signal Processing & Wireless Communications.

Prateek doing Master's in Engineering from Samalka Group of Institutions and working as Asst. Professor in the Dept. of ECE at Sri Venkateswra Engg College, Haryana. His research Interests are in DSP, Digital system Design.