ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 1, Issue 4, April 2012

Cost Based Performance Evaluation of H_2/D/1 and E_2/D/1 Traffic Model

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Abstract*—Wireless Cost based performance of a network is very much essential for a network planner prior implementation of a network. In this paper we propose a cost based analysis of H_2/D/1 and E_2/D/1 traffic models based on the concept of a previous analysis of M/G/1 model. Here, the arrival rate of packets follows hyper-exponential and Erlangian distribution and the service time of both traffic are deterministic. It has been found that distinct minima of the cost profile of H_2/D/1 but that of E_2/D/1 is almost hyperbolic which gives the opportunity to a network planner to select exact traffic parameters to attain the service of minimum cost.

***Index Terms*— LST, Mean Waiting Time, SCV, Probability of Immediate Service, Delay, Functional Equation.**

I. INTRODUCTION

Non–Markovian traffic model of deterministic service time is essential for analysis of packet traffic of fixed sized cell especially for ATM networks. Three basic parameters: probability density function (pdf) of call/packet arrival rate, pdf of length of service time and size of buffer of a router are expressed in generalized notation A/B/C/D as explained in [1]-[3]. For example: Recently, M/G/1/K traffic model is widely accepted to evaluate performance of packet traffic. In standard M/G/1/K system the arrival process follows Poisson rule with λ (packets per unit time), the service time is identically distributed random variable which follows general distribution. In this model only one server is available and the length of the queue or the size of buffer is k-1, i.e. the maximum k-1 users can be placed in the waiting position. Statistical analysis of M/G/1 system is shown in [4] and [5]. A similar analysis is also shown in context of network traffic in [6], where the authors presented the analytical model of M/D/1/m traffic to evaluate the performance of connection oriented and connectionless ATM traffic taking deterministic service time. In [7], the authors relate the total cost of a call with the number of high usage circuit to get optimum number of high usage links in a network of alternate routing provision. The paper only considers the rent of high usage link and tandem exchange link. Like [8], in this paper, four parameters: cost of losing a customer, customer delay, size of buffer and processor speed are related with the probability states to get the optimum cost in routing a call. In [9] and [10], authors evaluate non-Markovian traffic parameters of M/D/1(m) system suitable for ATM network. In [11], the author deals with non-Markovian arrival process to determine the stationary state distribution, where two traffic models Hn/G/∞ and En/G/∞ are considered. The paper shows that the performance of En/G/∞ is better than that of Hn/G/∞. In [12], processing time of a manufacturing system is taken as a random variable to determine the mean waiting time in a GI/D/1 queue. The paper also gives the complete analytical method of determination of mean waiting time of M/D/1 system using DTMC where the authors consider that a queue experiences failure when a job arrival process is Poissonian. In [13], a cost model of internet traffic is proposed where the cost of a network elements are calculated and distributed over different services/users taking the quality of service (QoS) as a parameter. The cost model is named as Total Element Long Run Incremental Cost (TELRIC) and authors calculate the total cost as the sum of fixed cost of network element and cost per BW×BW provided by the network. The paper compares unit cost against BW for M/M/1 and G/G/1 traffic models. In this paper our aim is to determine the traffic parameters (functional equation, mean waiting time, probability of delay and immediate service) analytically then bring them under cost based approach of [3] and [14] to achieve optimum cost condition of the network. The rest of the paper is organized as follows: Section II provides the spectral solution of the GI/G/1 traffic in generalized form, Sec. III deals with the derivation of the traffic parameters of H_2/D/1 and E_2/D/1 systems along with the cost analysis of the network while Sec. IV provides the results according to the proposal and analysis of the previous part and finally Sec. V concludes the entire analysis.

II. SPECTRAL SOLUTION OF GI/G/1

The timing diagram of GI/G/1 traffic model is shown in Fig. 1. The arrival instants of n and (n+1)-th cells are C_n and C_{n+1} where the corresponding interarrival time is t_{n+1}. The waiting time and service time of the n-th cell are W_n and X_n respectively. The traffic parameters of GI/G/1 model are the followings: $A(t)$ is the probability density function (pdf) for interarrival times between customers, $B(x)$ is the pdf of service time for the customers (independent), Service discipline: first come first service (FCFS), C_n is the n-th arriving customer, $t_n = T_n - T_{n-1}$ is the interarrival time between C_n and C_{n-1}; here T_n and T_{n-1} are the arrival instants of the n-th and (n-1)-th calls, x_n is the service time of C_n, w_n is the waiting time (in queue) for C_n. The interarrival time $\{t_n\}$ and the service time $\{x_n\}$ are two random variables; where they are described by the pdfs $A(t)$ and $B(x)$ independent of index n. The waiting times for (n+1)-th call is the difference between the arrival instant of C_{n+1} and the instant of start of its service is expressed as [15]:

ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
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$$W_{n+1} = \begin{cases} W_n + X_n - t_{n+1} & if\ W_n + X_n - t_{n+1} \ge 0, \\ 0 & if\ W_n + X_n - t_{n+1} < 0. \end{cases}$$

(1)

Let us define a random variable:

$$u_n = x_n - t_{n+1}. \quad (2)$$

According to [15], $C(y-w) = 0$ as $w \to \infty$, similarly $w(0^-) = 0$. Therefore, we have

$$W(y) = \begin{cases} -\int_{0^-}^{\infty} W(w)dC(y-w); & y \ge 0 \\ 0 & y < 0 \end{cases} \quad (9)$$

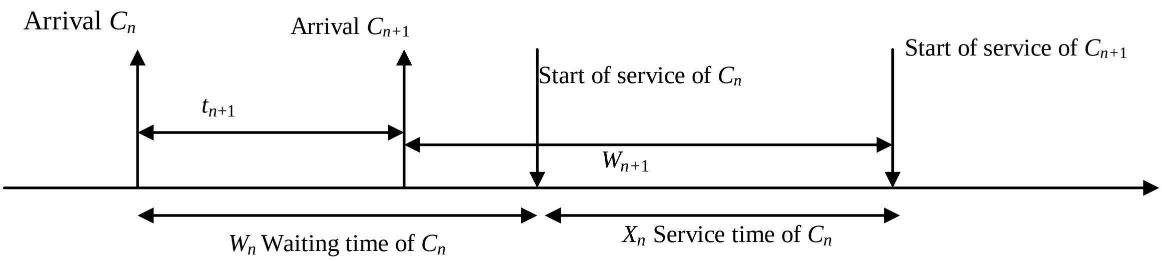

Fig. 1 Interarrival and Service Time of GI/G/1 Traffic

From Eqs. (1) and (2), we have

$$W_{n+1} = \begin{cases} W_n + U_n & if\ W_n + U_n \ge 0, \\ 0 & if\ W_n + U_n < 0. \end{cases} \quad (3)$$

The probability $C_n(u)$ is defined as

$$C_n(u) = P[x_n - t_{n+1} \le u]$$
$$= \int_{t=0}^{\infty} P[x_n \le u + t \mid t_{n+1} = t]\, dA(t) \quad (4)$$
$$= \int_{t=0}^{\infty} B(u + tdB(t)) = C(u).$$

We have

$$W_{n+1}(y) = P[w_n + u_n \le y]$$
$$= \int_{w=0^-}^{\infty} P[w_n \le y - w \mid w_n = w]\, dW_n(w).$$

(5)

Here u_n is independent of w_n, therefore, Eq. (5) becomes

$$W_{n+1}(y) = \int_{w=0^-}^{\infty} C_n(y-w)\, dW_n(w); \quad y \ge 0. \quad (6)$$

Taking $n \to \infty$, Eq. (6) becomes

$$W(y) = \int_{w=0^-}^{\infty} C(y-w)\, dW(w); \quad y \ge 0, \quad (7)$$

Where we have dropped the subscripts. Equation (7) above is known as Lindley's integral equation [16], where $W(y) = 0$ for $y < 0$. Let us perform integration by parts in Eq. (7), we obtain

$$W(y) = C(y-w)W(w)\Big|_{w=0^-}^{\infty} - \int_{0^-}^{\infty} W(w)dC(y-w)$$
$$= Lt_{w \to \infty} C(y-w)W(w) \quad (8)$$
$$- C(y)W(0^-) - \int_{0^-}^{\infty} W(w)dC(y-w).$$

Changing the variable $u = y - w$, we get another form of Eq. (8):

$$W(y) = \begin{cases} \int_{u=-\infty}^{\infty} W(w)dC(y-w); & y \ge 0, \\ 0 & y < 0. \end{cases} \quad (10)$$

The 'complementary' waiting time of $W(y)$ is

$$W\_(y) = \begin{cases} 0 & y \ge 0, \\ \int_{u=-\infty}^{\infty} W(y-u)dC(u); & y < 0. \end{cases}$$

(11)

Adding Eqs. (10) and (11), we obtain

$$W(y) + W\_(y) = \int_{-\infty}^{y} W(y-u)\, c(u)\, du,$$

(12)

for all real y. The Laplace transform (LT) of $W\_(y)$ is

$$\phi\_(s) = \int_{-\infty}^{\infty} W\_(y)e^{-sy}dy = \int_{-\infty}^{0} W\_(y)e^{-sy}dy.$$

(13)

Similarly, the LT of waiting time PDF $W(y)$ is

$$\phi_+(s) = \int_{-\infty}^{\infty} W\_(y)e^{-sy}dy = \int_{0^-}^{\infty} W\_(y)e^{-sy}dy.$$

(14)

According to the definitions of $\phi_+(s)$ and $W^*(s)$, we can relate them as

$$s\phi_+(s) = W^*(s).$$

(15)

Defining the pdf of u as $c(u) = dC(u)/du = a(-u) * b(u)$, we have

$$C^*(s) = A^*(-s)B^*(s). \quad (16)$$

ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
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Taking the LT of Eq. (12), we have

$$\phi_+(s)+\phi_-(s)=\phi_+(s)C^*(s)=\phi_+(s)A^*(-s)B^*(s)\,,$$

(17)

which can be written as

$$\phi_-(s)=\phi_+(s)\left\{A^*(-s)B^*(s)-1\right\}.$$

(18)

Let us consider *A*(s)* and *B*(s)* to be rational functions of *s*, where

$$A^*(-s)B^*(s)-1=\frac{\psi_+(s)}{\psi_-(s)}\,.$$

(19)

From Eqs. (18) and (19), we have

$$\phi_-(s)=\phi_+(s)\frac{\psi_+(s)}{\psi_-(s)}\,.$$

(20)

From the Liouville's theorem, we know that if *f (r)* is analytic and bounded function for all finite values of *r*, then *f (r)* is a constant. Therefore, we can write

$$\phi_-(s)\psi_-(s)=\phi_+(s)\psi_+(s)=K\,,$$

where *K* is a constant, we can then write

$$\phi_+(s)=\frac{K}{\psi_+(s)}\,.$$

(21)

To determine *K*, we can use

$$s\phi_+(s)=W^*(s)=\int_{y=0^-}^{\infty}e^{-sy}dW(y)\,,$$

(22)

which implies

$$Lt_{s\to 0}s\phi_+(s)=Lt_{s\to 0}\int_{0^-}^{\infty}e^{-sy}dW(y)=1=Lt_{s\to 0}\frac{sK}{\psi_+(s)}\,,$$

and from this, we finally obtain

$$K=Lt_{s\to 0}\frac{s\psi_+(s)}{s}\,.$$

(23)

Example-1

Let us consider the M/M/1 traffic model where, the cumulative distribution function (cdf) of the interarrival time is $A(t)=1-e^{-\lambda t}$ and the pdf of interarrival time is $a(t)=\lambda e^{-\lambda t}$. The cdf of the service time is, $B(t)=1-e^{-\mu t}$ and the corresponding pdf is $b(t)=\mu e^{-\mu t}$. Taking LT, we get $A^*(s)=\lambda/(s+\lambda)$ and $B^*(s)=\mu/(s+\mu)$. Now,

$$A^*(-s)B^*(s)-1=\frac{\psi_+(s)}{\psi_-(s)}=\left(\frac{\lambda}{\lambda-s}\right)\left(\frac{\mu}{s+\mu}\right)-1$$

$$=\frac{s^2+s(\mu-\lambda)}{(\lambda-s)(s+\mu)}.$$

The pole-zero diagram of $\frac{\psi_+(s)}{\psi_-(s)}$ is given in Fig. 2.

Here $\psi_+(s)$ must be analytic and zero-free for Re(s) > 0, therefore, collecting the two zeros and one pole on the left half plane for $\psi_+(s)$, we have

$$\psi_+(s)=\frac{s(s+\mu-\lambda)}{s+u}$$

and

$$\psi_-(s)=\lambda-s\,.$$

Therefore,

$$K=Lt_{s\to 0}\frac{\psi_+(s)}{s}=Lt_{s\to 0}\frac{(s+\mu-\lambda)}{s+u}=1-\lambda/\mu=1-\rho\,,$$

and

$$\phi_+(s)=\frac{(1-\rho)(s+\mu)}{s(s+\mu-\lambda)}\,.$$

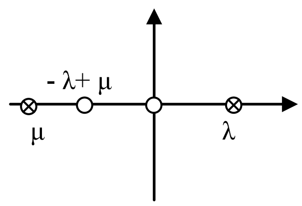

Fig. 2 Pole-zero diagram of $\psi_+(s)/\psi_-(s)$.

Again, $W^*(s)=s\phi_+(s)$, therefore, inverting $W^*(s)$, we obtain

$$W(y)=1-\rho\, e^{\mu(1-\rho)y};\ \ y\geq 0.$$

Example-2

Let us consider the D/M/1 traffic model. The cdf, pdf and LT of the interarrival time are

$$A(t)=\begin{cases}0 & t<T\\ 1 & t\geq Y\end{cases},$$

$a(t)=\delta(t-T)$ and $A^*(s)=e^{-sT}$ respectively.

The cdf, pdf and LT of the service time are $B(t)=1-e^{-\mu t}$, $b(t)=\mu e^{-\mu t}$ and $B^*(s)=\mu/(s+\mu)$ respectively. Therefore,

$$A^*(-s)B^*(s)-1=\frac{\psi_+(s)}{\psi_-(s)}=\left(e^{sT}\mu/s+\mu\right)-1.$$

Now,

$$C(u)=\int_{t=0}^{\infty}B(u+t)dA(t)$$

$$=\int_{t=0}^{\infty}B(u+t)a(t)dt$$

$$=\int_{t=0}^{\infty}B(u+t)\delta(t-T)dt=B(u+T),$$

ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
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Where $u \geq -T$. Therefore,

$$c(u) = b(u+T);\ u \geq -T.$$

Again,

$$D(y) = \int_{\infty}^{y} D(y-u)dC(u) = \int_{\infty}^{y} D(y-u)c(u)du,$$

Which implies

$$D(y-T) = \int_{-\infty}^{y-T} D((y-T)-u)c(u)\,du$$
$$= \int_{-\infty}^{y-T} D(y-T-u)b(u+T)\,du$$
$$= \int_{0}^{y} D(y-u)b(u)\,du.$$

Considering, $D(y) = 1 + l_1 e^{l_2 y}$, a possible solution of the above equation implies

$$1 + l_1 e^{l_2(y-T)} = \int_0^y \{1 + l_1 e^{l_2(y-u)}\}\mu e^{-\mu u}\,du$$
$$= \int_0^y \mu e^{-\mu u}\,du + l_1 \mu e^{l_2 y}\int_0^y e^{-u(l_2+\mu)}du.$$

Therefore,

$$1 + l_1 e^{l_2(y-T)} = 1 - e^{-\mu y} - \frac{l_1 \mu e^{-\mu y}}{l_2+\mu} + \frac{l_1 \mu e^{l_2 y}}{l_2+\mu}.$$

Taking the co-efficient of $e^{l_2 y}$, we have

$$l_1 e^{-l_2 T} = \frac{l_1 \mu_1}{l_2+\mu} \Rightarrow \frac{\mu_1}{l_2+\mu} = e^{-l_2 T}.$$

Similarly, taking the co-efficient of $e^{-\mu y}$, we have

$$0 = -1 + \frac{l_1 \mu}{l_2+\mu} = 0 \Rightarrow \frac{1}{\mu} + \frac{l_1}{l_2+\mu} = 0.$$

Since $D(0) = 1 + l_1$ and using the above relations, we get $1-D(0)=e^{-\mu D(0)T}$; which has a zero and a nonzero root. Let ρ be the nonzero root of the above equation. Then the pdf of the delay is

$$D(y) = 1 - (1-\rho)e^{\mu\rho y};\ y \geq 0.$$

Example-3

Let us consider the case of a voice traffic (variable bit rate voice source) where the interarrival time follows the hyper-exponential pdf and the service time follows the Erlangian distribution. The Kendal's notation of the traffic becomes H2/E2/1.

The cdf, pdf and LT of the interarrival time are:

$$A(t) = 1 - p_1 e^{-\lambda_1 t} - p_2 e^{-\lambda_2 t},$$
$$a(t) = \lambda_1 p_1 e^{-\lambda_1 t} + \lambda_2 p_2 e^{-\lambda_2 t},$$

and

$$A^*(s) = \frac{p_1\lambda_1}{s+\lambda_1} + \frac{p_2\lambda_2}{s+\lambda_2}$$

Respectively.

Similarly, the cdf, pdf and LT of the service time are:

$$B(t) = 1 - (1+2\mu t)e^{-2\mu t},$$
$$b(t) = 2\mu(2\mu t)e^{-2\mu t},$$

and

$$B^*(s) = \frac{(2\mu)^2}{(s+2\mu)^2}$$

Respectively. Thus,

$$A^*(-s)B^*(s) - 1 = \frac{\psi_+(s)}{\psi_-(s)} = \left(\frac{p_1\lambda_1}{\lambda_1 - s} + \frac{p_2\lambda_2}{\lambda_2 - s}\right)\frac{(2\mu)^2}{(2\mu+s)^2} - 1$$

,

From which, we obtain

$$\frac{\psi_+(s)}{\psi_-(s)} = \frac{sP(s)}{(\lambda_1 - s)(\lambda_2 - s)(2\mu+s)^2},$$

Where

$$P(s) = s^3 + \{4\mu - (\lambda_1+\lambda_2)\}s^2 + \{\lambda_1\lambda_2 - 4\mu(\lambda_1+\lambda_2) + 4\mu^2\}s + 4\mu\{\lambda_1\lambda_2 - \mu(p_2\lambda_1 + p_1\lambda_2)\}.$$

We have found three roots (one positive and two negative) of $P(s)$, and these are: $\lambda_2 < \rho_1 < \lambda_1$, $-2\mu < -\rho_2 < 0$ and $-\rho_3$. Finally,

$$\psi_+(s) = \frac{s(s+\rho_2)(s+\rho_3)}{(2\mu+s)^2},$$
$$\psi_-(s) = \frac{-(\lambda_1 - s)(\lambda_2 - s)}{(s-\rho_1)},$$
$$\phi_+(s) = \left(\frac{\rho_2\rho_3}{4\mu^2}\right)\frac{(2\mu+s)^2}{s(s+\rho_2)(s+\rho_3)},$$

and the pdf of the delay is

$$D(y) = 1 + \left(\frac{\rho_3}{4\mu^2}\right)\frac{(2\mu-\rho_2)^2}{(\rho_2-\rho_3)}e^{-\rho_2 y} - \left(\frac{\rho_2}{4\mu^2}\right)\frac{(2\mu-\rho_3)^2}{(\rho_2-\rho_3)}e^{-\rho_3 y};\ y \geq 0.$$

III. The Traffic Model

The most convenient way of deriving the parameters of H_2/D/1 or E_2/D/1 traffic is the spectral solution technique of general independent (GI) arrival traffic model of the previous section. Let us consider the *n*-th call of GI/G/1 system where the service and waiting times are X_n and W_n respectively. If the interarrival time between the *n*-th and (*n*+1)-th calls is Y_{n+1} then, $W_{n+1} = \max(W_n + X_n - Y_{n+1}, 0)$. At steady state condition, i.e as $n \to \infty$, $W_n \to W$, $(X_n - Y_{n+1}) \to U$, then $W = \max(0, W+U)$.

Let $u(t)$ and $w(t)$ be the density functions of U and W respectively, then we have [17]:

$$W(t) + W_-(t) = \int_{-t}^{t} W(t-v)u(v)dv\ ;\ -\infty < t < \infty. \qquad (24)$$

ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
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Since $U = X - Y$, the Laplace-Steieltjes transform (LST) of U is: $u(\theta) = a^*(\theta)b^*(-\theta)$; when $a^*(\theta)$ and $b^*(\theta)$ are the LST of X and Y respectively. Taking LT of Eq. (24), we have

$$W^*(\theta) + W_-^*(\theta) = u^*(\theta)W^*(\theta),$$

From which we obtain

$$W^*(\theta)\{a^*(\theta)b^*(-\theta) - 1\} = W_-^*(\theta). \qquad (25)$$

It is to be mentioned here that $a^*(\theta)b^*(-\theta) - 1$ has a special factorization of the form [18]:

$$a^*(\theta)b^*(-\theta) - 1 = \frac{\psi_+^*(\theta)}{\psi_-(\theta)}, \qquad (26)$$

Where the conditions of $\psi(\theta)$ are given in [14]. Using the above relations and LST of $W(t)$, we get the probability of immediate service as

$$W(0^+) = \underset{\theta \to 0}{Lt} \frac{\psi_+(\theta)}{\theta},$$

and the mean waiting time

$$W = Lt_{\theta \to 0} \frac{d\{\theta W^*(\theta)\}}{d\theta}.$$

In $H_2/D/1$ traffic model, the interarrival time follows the pdf of the Hyper-exponential distribution, whose LST is:

$$a^*(\theta) = \frac{k\lambda_1}{\theta + \lambda_1} + \frac{(1-k)\lambda_2}{\theta + \lambda_2}. \qquad (27)$$

The service time is deterministic whose LST is:

$$b^*(\theta) = e^{-h\theta}. \qquad (28)$$

To determine the mean waiting time, we need the roots of the following functional equation [8]:

$$y(\theta)b^*(\theta) - (\lambda_1 - \theta)(\lambda_2 - \theta) = 0, \qquad (29)$$

where

$$y(\theta) = k\lambda_1(\lambda_2 - \theta) + (1-k)\lambda_2(\lambda_1 - \theta). \qquad (30)$$

The characteristics equation is derived by taking $\psi_+(0)$ and $\psi_-(0)$ of Eq. (26) for $H_2/G/1$ traffic.

The mean waiting time of $H_2/G/1$ is expressed as

$$W = W_M + \frac{[k\lambda_1 + (1-k)\lambda_2]\rho - \lambda}{\lambda_1\lambda_2(1-\rho)} + \frac{1}{\theta_0}, \qquad (31)$$

where W_M is the mean waiting time of the equivalent M/G/1 model (with the same arrival rate) and is given by

$$W_M = \frac{\rho(1 + C_s{}^2)h}{2(1-\rho)}, \qquad (32)$$

Where C_s^2 is the squared coefficient of variation (SCV) and is expressed as $C_s^2 = \sigma^2 / m^2$.

The probability of immediate service P_I is given by the following expression:

$$P_I = (1-\rho)\frac{\lambda_1\lambda_2}{\lambda\theta_0}. \qquad (33)$$

The probability of delay is then

$$P_D = 1 - P_I. \qquad (34)$$

In particular, if only λ and C_a^2 (called the squared coefficient of variance (SCV)), are given, and setting the symmetric condition, we have

$$\frac{k}{\lambda_1} = \frac{1-k}{\lambda_2},$$

where

$$k = \frac{1}{2}\left(1 + \sqrt{1 - \frac{2}{1 + C_a^2}}\right), \; \lambda_1 = 2k\lambda, \; \lambda_2 = 2(1-k)\lambda. \qquad (35)$$

In $E_2/D/1$ traffic model, the interarrival time follows the pdf of the Erlangian distribution, whose LST is

$$a^*(\theta) = \left(\frac{k\lambda}{\theta + k\lambda}\right)^k. \qquad (36)$$

To determine the mean waiting time, we need the roots of the following functional equation:

$$(k\lambda)^k b^*(\theta) - (k\lambda - \theta)^k = 0. \qquad (37)$$

The mean waiting time of $E_2/G/1$ is expressed as:

$$W = W_M - \frac{1 - C_a{}^2}{2\rho(1-\rho)}h + \sum_{i=1}^{k-1} \frac{1}{\theta_i}. \qquad (38)$$

The probability of immediate service P_I can be written as

$$P_I = \frac{(k\lambda)^k(1-\rho)}{\lambda} \sum_{i=1}^{k-1} \frac{1}{\theta_i}. \qquad (39)$$

Probability of delay is then given by Eq. (34).

The cost parameters used in this paper are the following [1, 8]:

B = Waiting cost for one customer,
W = Mean waiting time,
P_I = Probability of immediate service,
P_D = Probability of delay,
$C/\bar{x}^c$ = Cost of buffer of sufficient size to hold one Customer,
$D/\bar{x}^c$ = Cost of a processor which can process Customers at rate $1/\bar{x}$,
$\bar{x}$ = Mean service time.

Now, we find the total cost of operating the system per unit time. Since the proposed traffic model processes infinite queue hence cost due to lost of customer of [8] is excluded here in evaluating the total cost of operating system per unit time. The expression of cost Ξ is

$$\Xi(C, D, \bar{x}) = BW + P_D \frac{C}{\bar{x}^c} + P_I \frac{D}{\bar{x}^d}. \qquad (40)$$

Let us assume that speed of the buffer memory matches with that of the processor, then selecting for c = 1. Let us select d = 2 like [8], therefore, the expression of the cost takes the following form:

$$\Xi(C, D, \bar{x}) = BW + P_D \frac{C}{\bar{x}} + P_I \frac{D}{\bar{x}^2}. \qquad (41)$$

ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 1, Issue 4, April 2012

IV. RESULTS

The objective of this research work is to evaluate the relationship between the size of buffer and the mean service time ($\bar{x}$) that minimizes the cost for the four different service time distribution cases. The parameters that we used here are: buffer size varies from 1 to 60; call arrival rate, λ = 0.93 cells/ms; call termination rate, μ = 1cells/ms; cell length, L_c = 53 bytes for ATM transmission; transmission speed, c = 150 Mbps; cost of losing one customer, A=4Mu; waiting cost for one customer, B = 0.03MU; cost of buffer of sufficient size to hold one customer, C = 0.02MU; cost of a processor which can process customers at a rate $1/\bar{x}$, D = 0.07MU; call termination rate-1, μ_1 = 1 per unit time; and call termination rate-2, μ_2 = 1.2 per unit time. Let us first determine the roots of the functional equation of H_2/D/1 graphically. The roots are found varying h from zero crossing points of Fig. 3 as shown in Table 1.

After getting the roots θ_0 of both traffic, the mean waiting time W in seconds is evaluated for all h and plotted in Fig. 5. The mean delay in H_2/D/1 is found larger than that of E_2/D/1. The variation of W is wider at higher value of h, especially for $h \geq 0.7$ TU. Let us now observe the profile of the probability of immediate service and delay against variation of packet length.

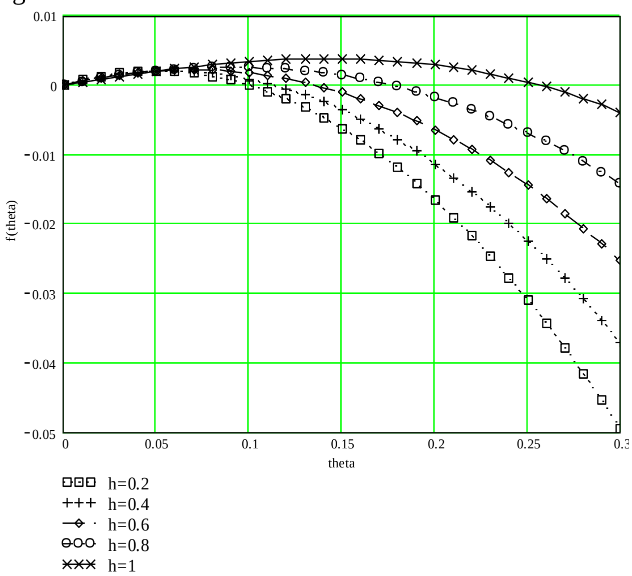

Fig. 3 Profile of Functional Equation of H_2/D/1.

The variations of P_I and P_D are shown in Fig. 6. It is observed that P_I decreases with the increase in h but the situation is reverse for P_D. Probability of immediate service of E_2/G/1 is found much larger than that of H_2/D/1 case. The rate of decrement of P_I of H_2/D/1 is more prominent than that of E_2/D/1 traffic for $h \leq 0.6$ TU and beyond that point both curves become almost parallel. Again probability of delay of H_2/D/1 is greater than that of E_2/D/1 case. The slope of P_D of H_2/D/1 case is found greater than that of E_2/D/1 case for $h \leq$ 0.6 TU, but the curves become parallel for h>0.6TU. Finally, the costs of two traffic models are compared in Fig. 7. The cost of E_2/D/1 is found much greater than that of H_2/D/1, where $h \leq 0.5$ TU. Beyond that point costs are very close to each other but for $h \geq 1.4$ cost of H_2/D/1 is much higher than that of E_2/D/1. The two curves intersect at h = 1.1 TU. The cost profile of H_2/D/1 has distinct minima whereas E_2/D/1 shows the profile of hyperbola as is visualized from Fig. 7.

Table 1: Roots of Functional Equation

θ_0	0.1	0.105	0.112	0.12	0.134	0.15	0.173	0.22	0.26
H	0.2	0.300	0.400	0.50	0.600	0.70	0.800	0.90	1.00

Table 2: Roots of Functional Equation

θ_0	1.87	1.82	1.74	1.68	1.58	1.55	1.52	1.51	1.49
H	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

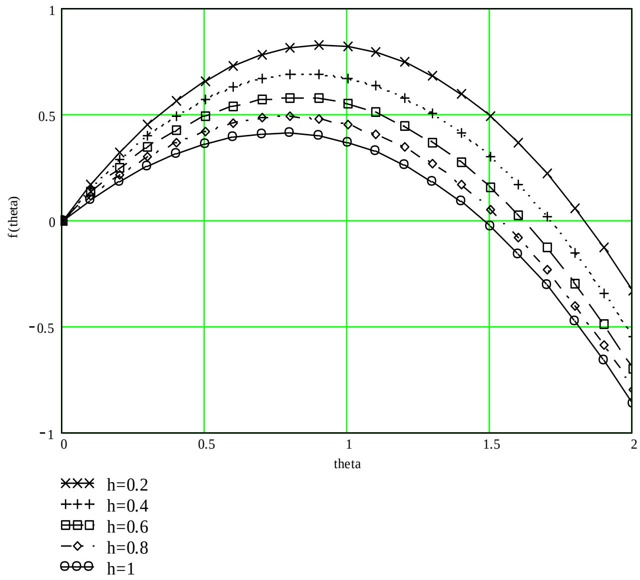

Fig. 4 Profile of Functional Equation of E_2/D/1.

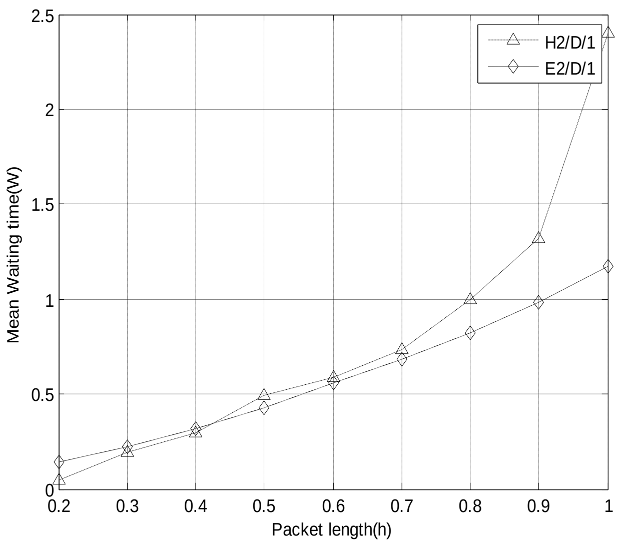

Fig. 5 Comparison of Mean Delay between H_2/D/1 and E_2/D/1.

ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 1, Issue 4, April 2012

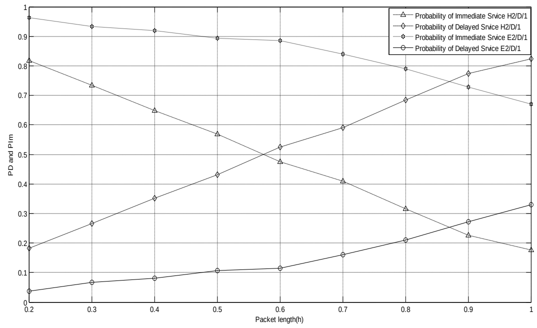

Fig. 6 Comparison of P_I and P_D of $H_2/D/1$ and $E_2/D/1$.

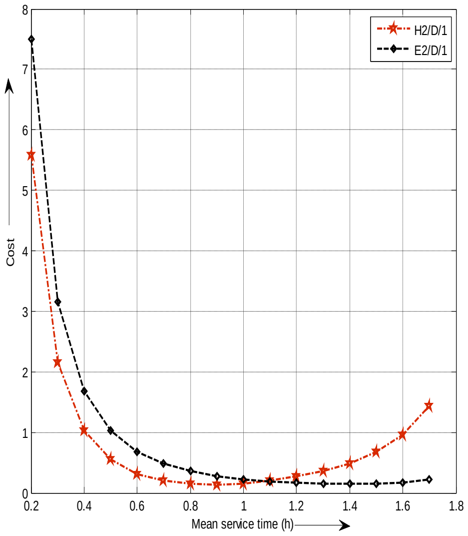

Fig. 7 Comparison of Cost of $H_2/D/1$ and $E_2/D/1$.

V. CONCLUSION

In this paper we have derived traffic cost of $H_2/D/1$ and $E_2/D/1$ traffic models which match with voice traffic of variable bit rate with fixed length of packet. Such traffic model is applicable in voice communication through ATM link. Our finding is that $H_2/D/1$ traffic has an optimum length of packet to achieve minimum traffic cost but $E_2/D/1$ does not have any optimum point. However, our analysis can suggest the minimum length of the packet beyond which traffic will maintain its minimum cost. The entire work can be extended for $E_2/G/1$ and $H_2/G/1$ of variable length packet (using the spectral solution technique of Sec. II) to observe the performance of voice over IP (VoIP) traffic.

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ISSN: 2277-3754
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 1, Issue 4, April 2012

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