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A Comparison on Efficiency of X-bar and S Control Charts for Skewed Distribution

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Abstract: ***The objective of this research is to compare the efficiency of the widely used X-bar and S control charts for monitoring mean and standard deviation of skewed processes. Three methods used to construct the control charts are the weighted variance (WV) method, the parametric bootstrap (PB) method and the classical Shewhart method. The control charts obtained by the three methods are compared in terms of type I error rates when the distributions of the underlying process are Weibull, lognormal and gamma with different parameters and sample sizes. The results from simulation studies show that the control charts form the PB method outperform those from the WV and Shewhart methods for the processes with skewed distributions.***

Keywords: **Control chart, skewed distribution, type I error rate.**

I. BACKGROUND/ OBJECTIVES AND GOALS

In numerous industries, such as electronics, telecommunications and mechanical engineering, X-bar and S control charts are commonly used in monitoring a production process. They are widely used to decide whether the process variation is in control or out of control. Theoretically, these control charts assume that the monitored variables are normally distributed. In practice, however, there are several situations in which the process distributions are not normal but skewed. For example, the lifetime of a product, time to failure for electrical and mechanical components and systems and duration of a telephone call are often skewed. To deal with skewed distributions of process variables, three approaches commonly used are (i) transforming the original data into a normal distribution, (ii) increasing the sample size until the sample mean is approximately normally distributed, and (iii) employing heuristic methods to design control charts for skewed underlying process distributions. Nevertheless, the first approach is not easy to apply, whereas the second approach is expensive and practically impossible in some cases. Therefore, this paper considers the third approach to establish control charts for mean and standard deviation. The heuristic methods employed in this study are the weighted variance (WV) method proposed by Choobineh and Ballard (1987), the parametric bootstrap (PB) method recently proposed by Lukin and Yaschenko (2015) and the classical Shewhart method (Nichols and Padgett, 2005). Accordingly, the main objective of this research is to compare the efficiency of X-bar and S control charts constructed by the three methods for monitoring mean and standard deviation of skewed process.

II. METHODS

In this paper, the performances of the heuristic methods for establishing X-bar and S control charts are compared by evaluating the efficiency of control charts constructed using three considering methods. A comparison on the efficiency of the X-bar and S control charts is made in terms of the probability of signaling a false out-of-control or type I error rate. When the process is in-control, it is desirable for the type I error rate, specifically the probability of a subgroup X-bar or S falling outside the ±3 sigma control limits is 0.0027 or 0.27%. Simulation studies are carried out to establish X-bar and S control charts using the PB, the WV and the Shewhart methods for skewed distributions with respect to sample sizes of n = 10, 15 and 20. These sample sizes are suggested by Khoo et al. (2009) as the WV

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method performs well when the n is moderate or large ($n \geq 10$). The skewed distributions considered are Weibull, lognormal and gamma distributions. These distributions are chosen because their characteristics represent a variety of shapes from nearly symmetric to highly skewed. By using Monte Carlo Simulation written in R programming language, type I error rates of X-bar and S control charts based on the PB method, the WV and the Shewhart methods are evaluated and then compared. The process measurements (the samples of X) used in the simulation study are generated in accordance with Weibull, lognormal and gamma distributions. The parameters of each distribution considered in this study are Weibull (λ, β) ; $(\lambda = 2, \beta = 0.5)$ and $(\lambda = 10, \beta = 2)$, lognormal (μ, σ^2) ; $(\mu = -1, \sigma^2 = 1)$ and $(\mu = 0, \sigma^2 = 1)$ and gamma (α, λ); $(\alpha = 5, \lambda = 15)$ and $(\alpha = 10, \lambda = 35)$. The upper and lower control limits (UCL and LCL) for X-bar and S control charts by Shewhart and the WV methods are constructed based on three-sigma limits, while the control limits of the PB method are constructed using the quantiles 99.865% UCL and 0.135% LCL. The control limits by the three methods are obtained from the following formulas.

A. The Parametric Bootstrap method (PB method)

According to Lukin and Yaschenko (2015), X-bar and S control charts are constructed as the following steps:

Step 1: Draw a sample of size n for k subgroups.

Step 2: Compute the sample average ($\bar{X}_i$), the sample standard deviation (S_i) and the sample variance (S_i^2) for each subgroup; $i = 1,2,\ldots, k$.

Step 3: Compute the grand average ($\bar{\bar{X}}$), the average standard deviation ($\bar{S}$) and the pooled variance ($\bar{S}^2$).

Step 4: Compute the estimations of parameters of lognormal, Weibull and gamma distributions.

Step 5: Generate a sample of size N distributed in accordance with the chosen distributions.

Step 6: Arrange a sample obtained in step 5 randomly into $B = N/n$ subgroups (of size n). For each subgroup compute the sample average ($\bar{X}_i^*$) and the sample standard deviation (S_i^*).

Step 7: Rank B values of $\bar{X}_i^*$ in ascending order, then the values with the sequence numbers $(\alpha/2)\cdot B$ and $[1-(\alpha/2)]\cdot B$ are respectively the lower and upper control limit (LCL and UCL) of X-bar chart. Note that α = 0.0027 is used in the simulation study.

Step 8: Rank B values of S_i^* in ascending order, then the values with the sequence numbers $(\alpha/2)\cdot B$ and $[1-(\alpha/2)]\cdot B$ are the lower and upper control limit of S chart.

Therefore, the control limits of X-bar and S charts by the PB method are

$$UCL_{PB} = (\alpha/2)\cdot B$$

$$LCL_{PB} = [1-(\alpha/2)]\cdot B$$

B. The Weighted Variance method (WV method)

According to Khoo et al. (2009), the control limits of X-bar chart by the WV method are

$$UCL_{WV} = \bar{\bar{X}} + A_U\bar{S}$$

$$LCL_{WV} = \bar{\bar{X}} - A_L\bar{S},$$

Where $\bar{\bar{X}}$ is the grand average, $\bar{S}$ is the average of the sample standard deviations, A_U and A_L are constants based on the values of P_X and sample size n. Here, P_X is the probability that the quality variable X will be less than or equal to its mean μ_X i.e., $P_X = P(X \leq \mu_X)$. If the parameter μ_X is unknown, the estimator $\hat{\mu}_X = \bar{\bar{X}}$ is used. The control limits of S chart are

$$UCL_{WV} = B_U\bar{S}$$

$$LCL_{WV} = B_L\bar{S},$$

Where B_U and B_L are constants based on the values of P_X and sample size n. The values of the constants A_U,

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A_L, B_U and B_L for selected combinations of P_X and n are given in Khoo et al. (2009).

C. The Shewhart method

According to Alwan (2000) and Montgomery (2009), the control limits of X-bar chart are given by

$$UCL_{SH} = \bar{\bar{X}} + \frac{3\bar{S}}{C_4\sqrt{n}} \tag{7}$$

$$LCL_{SH} = \bar{\bar{X}} - \frac{3\bar{S}}{C_4\sqrt{n}}, \tag{8}$$

Where $\bar{\bar{X}}$, $\bar{S}$, c_4 and n represent the grand average, the average of the sample standard deviations, a constant that depends on n and the sample size respectively. The control limits of S control chart are given by

$$UCL_{SH} = \bar{S} + 3\frac{\bar{S}}{C_4}\sqrt{1-C_4^2} \tag{9}$$

$$LCL_{SH} = \bar{S} - 3\frac{\bar{S}}{C_4}\sqrt{1-C_4^2}, \tag{10}$$

Where $\bar{S}$ and c_4 are the average of the sample standard deviations and a constant that depends on the sample size n respectively. To compare the efficiency of X-bar and S charts, type I error rates are estimated by the percentage of X-bar and S outside the control limits. In this study type I rates are obtained based on 100,000 simulation trials and sample sizes, n = {10, 15, 20}. Based on the skewed distributions which are Weibull, Lognormal and Gamma, given that the process is in control, a sample of size n for k subgroups are generated and the control limits of the charts using the PB method, the WV and the Shewhart methods are estimated. Then to compute the type I error rate, another 10,000 in control samples, each of size n are generated and the proportion of points falling outside the control limits are computed. The average type I error rates are finally recorded.

III. RESULTS

By using Monte Carlo simulation, the Shewhart, the WV and the PB methods are applied to construct X-bar and S control charts for a process with skewed distribution. Then type I error rates of both control chart are obtained. Table 1 contains the numerical values of type I error rates of X-bar and S charts from the three methods for different skewed distributions and sample sizes. The following conclusions can be drawn from the comparison of type I error rates.

(1) Based upon the results in Table 1, it is clearly seen that type I error rates of both X-bar and S control charts using the PB method are closer to the nominal value 0.27% than those of control charts using the WV and Shewhart methods in almost all cases. Thus X-bar and S charts by the PB method are found to be superior to those by the Shewhart and WV methods.

(2) The X-bar chart by the WV method performs better than those by the PB and Shewhart methods when the distribution of a process is Weibull with parameter $(\lambda = 2, \beta = 0.5)$ and n = 10 and 15.

(3) The X-bar and S charts by Shewhart method poorly performs in almost all cases.

(4) Finally, the results will help practitioners in making decisions on which control charts is preferable to meet their needs when monitoring the mean and the standard deviation of a skewed process.

Table 1: Type I error rates of control charts based on three methods when the process distributions are Lognormal, Weibull and Gamma with different parameters and sample sizes.

Distributions	Methods	n = 10		n = 15		n = 20	
		X-bar chart	S chart	X-bar chart	S chart	X-bar chart	S chart
Lognormal $(\mu = -1, \sigma$	Shewhart	0%	14.53%	0%	20.678%	0%	25.036%
	WV	0.831%	0.927%	0.696%	0.854%	0.6%	0.778%
	PB	0.27	0.26	0.27	0.30	0.29	0.27

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		8%	%	2%	4%	4%	6%
Lognormal ($\mu = 0, \sigma^2$	She whar t	0%	14.3 32%	0%	20.6 48%	0%	25.1 95%
	WV	1.6 %	4.22 8%	1.39 9%	4.45 %	1.25 7%	8.74 %
	PB	0.25 4%	0.27 8%	0.26 2%	0.35 6%	0.26 %	0.35 2%
Weibull ($\lambda = 2, \beta =$	She whar t	0%	0.60 7%	0%	1.12 4%	0%	0.48 5%
	WV	0.24 %	0.03 7%	0.27 1%	0.01 9%	0.3 %	0.01 8%
	PB	0.19 7%	0.37 7%	0.25 2%	0.23 %	0.24 %	0.26 6%
Weibull ($\lambda = 10, \beta$	She whar t	0%	0.84 3%	0%	0.88 4%	0%	0.29 5%
	WV	0.38 7%	0.99 %	0.1 %	0.37 %	0.31 1%	1.02 1%
	PB	0.28 2%	0.30 7%	0.28 4%	0.29 4%	0.28 6%	0.26 1%
Gamma ($\alpha = 5, \lambda =$	She whar t	0%	1.65 9%	0%	1.71 1%	0%	1.70 4%
	WV	0.23 %	0.11 7%	0.33 4%	0.07 8%	0.29 8%	0.06 5%
	PB	0.29 8%	0.26 6%	0.28 5%	0.29 1%	0.26 5%	0.28 7%
Gamma ($\alpha = 10, \lambda$	She whar t	0%	1.00 2%	0%	0.93 5%	0%	0.90 9%
	WV	0.34 6%	0.91 6%	0.37 6%	0.90 5%	0.32 3%	0.92 %
	PB	0.26 6%	0.26 8%	0.26 3%	0.3%	0.27 9%	0.25 8%

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