

Collisional drift waves in a magnetized dusty plasma cylinder

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Abstract- The effect of dust charge fluctuations (DCF) on non-linear coupling between collisional drift waves and a lower hybrid pump wave is studied in a magnetized plasma cylinder. A dispersion relation & growth rate expression for the collisional drift wave is derived incorporating both ponderomotive and ohmic heating effects. It is found that the unstable collisional drift mode frequency increases and the growth rate decreases sharply with the relative density of positively charged dust grains.

Index Terms: Dust grains, Drift waves, growth rate & Lower hybrid waves.

I. INTRODUCTION

Drift waves are low frequency spontaneously excited waves which are produced due to density or temperature gradient in plasma. They are well known source of micro instabilities in plasma devices. The suppression of drift waves by the application of lower hybrid pump wave have been reported by many investigators [1-4]. The parametric excitation and suppression of drift waves by electric field near the lower hybrid frequency was studied by Sundaram and Kaw [1]. Gore *et al* [2] and Liu & Tripathi [3] demonstrated the suppression of drift waves by injection of lower hybrid waves. Recently, there has been a great deal of interest in studying electrostatic waves in dusty plasmas [5-9]. Barkan *et al.* [6] found experimentally that the presence of negatively charged dust grains enhanced the growth rate of the instability of current driven electrostatic ion cyclotron (EIC) wave in a dusty plasma. Sharma and Ajay [9] have studied the effect of dust charge fluctuations on the excitation of upper hybrid wave in a magnetized plasma cylinder. The dust has also been noted to influence a three-wave parametric process in unmagnetised plasmas [10-12] and magnetized plasma [13]. Praburam and Jain [14] have studied the enhancement of collisional drift waves in a plasma cylinder without dust environment. In this paper, we study the effect of dust charge fluctuations on collisional drift waves in a magnetized plasma cylinder.

II. DISPERSION RELATION

We consider a cylindrical dusty plasma column of radius a_1 immersed in a uniform axial magnetic field B_s in the z direction with electron, ion and dust particle densities given as $n_e(r)$, $n_i(r)$ and $n_d(r)$. The charge, mass and temperature of the three species are denoted by $(-e, m_e, T_e)$, (e, m_i, T_i) and $(-Q_{d0}, m_d, T_d)$, respectively. The density of three species varies as

$$n_e(r) = n_{e0} \exp\left(-\frac{r^2}{a_1^2}\right), \quad n_i(r) = n_{i0} \exp\left(-\frac{r^2}{a_1^2}\right) \quad \text{and}$$

$$n_d(r) = n_{d0} \exp\left(-\frac{r^2}{a_1^2}\right)$$

in the interior region of the dusty plasma column and falls off rapidly near the edge. At equilibrium, the electrons acquire a diamagnetic drift velocity

$$v_d = -\frac{\hat{\theta} 2rv_{te}^2}{\omega_{ce} a_1^2}, \quad \text{where } v_{te} = \sqrt{\frac{2T_e}{m_e}}$$

is the electron thermal velocity and $\omega_{ce} = \frac{eB_s}{m_e c}$ is

the electron cyclotron frequency.

This equilibrium is perturbed by a low-frequency electrostatic perturbation (i.e., a drift wave)

$$\phi = \phi(r) \exp[-i(\omega t - l\theta - k_z z)] \quad (1)$$

The high amplitude lower hybrid pump wave $\phi_0 = \phi_0(r) \exp[-i(\omega_0 t - l_0 \theta - k_{z0} z)]$ couples with a drift mode ϕ and two lower hybrid sidebands $\phi_{1,2}$ as

$$\phi_{1,2} = \phi_{1,2}(r) \exp[-i(\omega_{1,2} t - l_{1,2} \theta - k_{z1,2} z)], \quad (2)$$

where $\omega_1 = \omega - \omega_0$, $\omega_2 = \omega + \omega_0$, $k_{z1} = k_z - k_{z0}$, $k_{z2} = k_z + k_{z0}$, $l_1 = l - l_0$, $l_2 = l + l_0$, i.e., phase matching condition. Here we have considered only lowest order coupling. This is a four wave parametric interaction process.

The perturbed densities of electrons, ion and dust are given by

$$n_{e1} = \frac{n_{e0} e}{T_e} (\phi_p + \phi) (1 + i\alpha), \quad (3)$$

where, $\alpha = \frac{v_{ei} (\omega - \omega^*)}{k_z^2 v_{te}^2}$, $\omega^* = k_\theta |v_d| = \frac{2lv_{te}^2}{a_1^2 \omega_{ce}}$ is the

adiabatic drift frequency, v_{ei} is the electron-ion collision frequency and

$$\phi_p = \frac{e}{2im_e \omega_{ce}^2 \omega_0} [(\nabla \phi_0 \times \omega_{ce}) \cdot \nabla \phi_1 - (\nabla \phi_0^* \times \omega_{ce}) \cdot \nabla \phi_2]$$

$$n_{i1} = \frac{n_{i0} e \nabla^2 \phi}{m_i \omega_{ci}^2} + \frac{n_{i0} e \omega^* \phi}{T_e \omega} \quad (4) \quad , \quad \beta = \frac{|I_{e0}|}{e} \left(\frac{n_{d0}}{n_{e0}} \right) = 0.397 \left(1 - \frac{1}{\delta_m} \right) \left(\frac{a}{v_{te}} \right) \omega_{pi}^2 \left(\frac{m_i}{m_e} \right)$$

where $\omega_{ci} \left(= \frac{eB_s}{m_i c} \right)$ is the ion cyclotron frequency.

$$n_{d1} = - \frac{n_{d0} Q_{d0} k^2 \phi}{m_d \omega^2} \quad (5)$$

We obtain dust charge fluctuations by following Jana *et al.* [5] as

$$Q_{d1} = \frac{|I_{e0}|}{i(\omega + i\eta)} \left[\frac{e \nabla^2 \phi}{m_i \omega_{ci}^2} + \frac{e \omega^* \phi}{T_e \omega} - \frac{e}{T_e} (\phi_p + \phi)(1 + i\alpha) \right] \quad (6)$$

where

$$\eta = 0.79 a \left(\frac{\omega_{pi}}{\lambda_{Di}} \right) \left(\frac{1}{\delta_m} \right) \left(\frac{m_i T_i}{m_e T_e} \right)^{\frac{1}{2}}$$

and $\delta_m = n_{i0} / n_{e0}$.

Substituting perturbed densities in the Poisson's equation $\nabla^2 \phi = 4\pi(n_{e1} e - n_{i1} e + n_{d0} Q_{d1} + Q_{d0} n_{d1})$,

we obtain

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \left(p_1^2 - \frac{l^2}{r^2} \right) \phi = \frac{(1 + i\alpha) \phi_p}{L} \left[\frac{\omega_{pe}^2}{v_{te}^2} + \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pi}^2 n_{e0}}{C_s^2 n_{i0}} \right]$$

(7)

where $p_1^2 = p^2 - k_z^2$, $p^2 = \frac{M}{L}$,

$$M = \left[\frac{\omega_{pi}^2 \omega^*}{C_s^2 \omega} - \frac{\omega_{pe}^2}{v_{te}^2} (1 + i\alpha) + \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pi}^2 \omega^* n_{e0}}{C_s^2 \omega n_{i0}} + \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pi}^2 n_{e0}}{C_s^2 n_{i0}} (1 + i\alpha) + \frac{\omega_{pd}^2}{T_{e0}^2} \right]$$

$$L = 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pi}^2 n_{e0}}{\omega_{ci}^2 n_{i0}}$$

$$C_s^2 = \frac{T_e}{m_i}, \omega_{pe} \left(= \sqrt{4\pi n_{e0} e^2 / m_e} \right), \omega_{pi} \left(= \sqrt{4\pi n_{i0} e^2 / m_i} \right)$$

and

$$\omega_{pd} \left(= \sqrt{4\pi n_{d0} Q_{d0}^2 / m_d} \right)$$

are the electron, ion and dust plasma frequencies, respectively and

coupling parameter.

In the absence of non-linear coupling terms, Eq. (7) has a well known solution $\phi = A J_l(p_1 r)$, $p_1 = p_{n1}$. At

$r = a_1$, ϕ must vanish, hence $J_l(p_1 a_1) = 0$, i.e.,

$p_1 a_1 = x_n$ [where x_n are the zeros of the Bessel function $J_l(x)$, $n=1, 2, 3, \dots$]. In the presence of a finite

R.H.S. of Eq. (7), we express ϕ in terms of a complete orthogonal sets of wave function:

$$\phi = \sum_m A_m J_l(p_{m1} r) \quad (8)$$

Substituting the value of ϕ from Eq. (8) in Eq. (7),

multiplying both sides of Eq. (7) by $r J_l(p_{n1} r)$ and

integrating over r from 0 to a_1 (where a_1 is the plasma radius), retaining only the dominant mode ($m = n$), we

obtain

$$\left\{ \frac{\omega^*}{\omega} - (1 + i\alpha) \frac{n_{e0}}{n_{i0}} \frac{\omega_{ci}^2}{C_s^2} - k^2 + \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pi}^2 \omega^* n_{e0}}{C_s^2 \omega n_{i0}} - \frac{i\beta}{(\omega + i\eta)} \frac{\omega_{pi}^2 n_{e0} (1 + i\alpha)}{C_s^2 n_{i0}} \right. \\ \left. - \frac{i\beta}{(\omega + i\eta)} k^2 \frac{n_{e0}}{n_{i0}} + \frac{\omega_{pd}^2 k^2}{\omega^2} \frac{\omega_{ci}^2}{\omega_{pi}^2} \right\} A_n = (1 + i\alpha) \frac{n_{e0} \omega_{ci}^2}{n_{i0} C_s^2} \left(1 + \frac{i\beta}{(\omega + i\eta)} \right) \frac{\int_0^{a_1} J_l(p_{n1} r) \phi_p r dr}{\int_0^{a_1} J_l^2(p_{n1} r) r dr}$$

(9)

where $k^2 = k_z^2 + p_{n1}^2$.

Using $C_s^2 = C_{s0}^2 \left(1 + \frac{\Delta T_e}{T_{e0}} \right)$, where ΔT_e is the increase of

electron temperature under the influence of the pump, T_{e0} is the equilibrium electron temperature and

$C_{s0}^2 = \frac{T_{e0}}{m_i}$. Eq. (9) can be rewritten as

$$\left\{ \frac{\omega_{ci}^2}{C_{s0}^2} \left[\frac{\omega^*}{\omega} \left(1 - \frac{\Delta T_e}{T_{e0}} \right) - \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{e0}} \right) \right] - \left(1 - \frac{\Delta T_e}{T_{e0}} \right) i\alpha \frac{n_{e0}}{n_{i0}} + \frac{i\beta}{(\omega + i\eta)} \frac{n_{e0}}{n_{i0}} \left(\frac{\omega^*}{\omega} - \frac{\omega^* \Delta T_e}{\omega T_{e0}} + \frac{\Delta T_e}{T_{e0}} - 1 \right) + \frac{\alpha\beta}{(\omega + i\eta)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{e0}} \right) \right. \\ \left. + \frac{\omega_{pd}^2 k^2}{\omega^2} \frac{C_{s0}^2}{\omega_{pi}^2} \left(1 - \frac{\Delta T_e}{T_{e0}} \right) \right\} - k^2 - \frac{i\beta}{(\omega + i\eta)} k^2 \frac{n_{e0}}{n_{i0}} \right\} A_n = \frac{n_{e0} \omega_{ci}^2}{n_{i0} C_{s0}^2} \left(1 - \frac{\Delta T_e}{T_{e0}} \right) \left(1 + \frac{i\beta}{(\omega + i\eta)} \right) \frac{\int_0^{a_1} J_l(p_{n1} r) \phi_p r dr}{\int_0^{a_1} J_l^2(p_{n1} r) r dr} \quad (10)$$

Similarly following Praburam *et al.* [4], we obtain

(14)

$$\left(\frac{\omega_{p0}k_z a_1}{(\omega_0 - \omega)} - \lambda_{n_1, l_1}\right) A_{n_1} = -\frac{e\omega^*}{i\omega_1 \omega_{ce}^2 \omega T_e} \int_0^{a_1} r dr \omega_p^2(r) \psi_{n_1, l_1}^{(1)*} (\nabla \phi_0^* \times \omega_{ce}) \cdot \nabla \phi$$

(11)

where A_{n_1} is the constant of wave function ϕ_1 .

$$\left(\frac{\omega_{p0}k_z a_1}{(\omega_0 - \omega)} - \lambda_{n_2, l_2}\right) A_{n_2} = -\frac{e\omega^*}{i\omega_2 \omega_{ce}^2 \omega T_e} \int_0^{a_1} r dr \omega_p^2(r) \psi_{n_2, l_2}^{(1)*} (\nabla \phi_0^* \times \omega_{ce}) \cdot \nabla \phi$$

(12)

where A_{n_2} is the constant of wave function ϕ_2 .

Using the value of ϕ_p , assuming the pump to be azimuthally symmetric ($l_0=0$), and considering the radial mode numbers of the two sidebands to be same, i.e., $n_1 = n_2$, Eqs. (10), (11) and (12) gives a dispersion relation

$$\frac{\omega^*}{\omega} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \left(1 - \frac{\Delta T_e}{T_{eo}}\right) i\alpha \frac{n_{e0}}{n_{i0}} + \frac{i\beta}{(\omega + i\eta)} \frac{n_{e0}}{n_{i0}} \left(\frac{\omega^*}{\omega} - \frac{\omega^* \Delta T_e}{\omega T_{eo}} + \frac{\Delta T_e}{T_{eo}} - 1\right) + \frac{\alpha\beta}{(\omega + i\eta)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) + \frac{\omega_{pd}^2 k^2}{\omega^2} \frac{C_{so}^2}{\omega_{pi}^2} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - k^2 \rho_{so}^2 - \frac{i\beta}{(\omega + i\eta)} k^2 \rho_{so}^2 \frac{n_{e0}}{n_{i0}} = \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) \left(1 + \frac{i\beta}{(\omega + i\eta)}\right) \frac{\mu_1}{\omega}$$

where

$$\mu_1 = 2\mu \frac{\left(\frac{\omega_{p0}k_z a_1}{\omega_0} - \lambda_1\right)}{\left(\frac{\omega_{p0}k_z a_1}{\omega_0} - \lambda_1\right)^2 - \left(\frac{\omega_{p0}k_z a_1}{\omega_0}\right)^2}$$

$\rho_s^2 = C_s^2 / \omega_{ci}^2$, and

$$\mu = \left[\frac{e^2 \omega^{*2}}{2m_e \omega_{ce}^2 \omega_0^2 T_e} \int_0^{a_1} r dr J_l^2(p_{n_l} r) \right] \left[\int_0^{a_1} \omega_p^2(r) r dr \psi_{n_l, l} \frac{\partial \phi_0}{\partial r} J_l(p_{n_l} r) \right]^2$$

Equation (13) can be rewritten as

$$\varepsilon_r(\omega, k) + i\varepsilon_i(\omega, k) = 0, \quad (15)$$

where

$$\varepsilon_r(\omega, k) = \frac{\omega^*}{\omega} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - k^2 \rho_{so}^2 + \frac{\beta\eta}{(\omega^2 + \eta^2)} \frac{\omega^* n_{e0}}{\omega n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{\beta\eta}{(\omega^2 + \eta^2)} k^2 \rho_{so}^2 \frac{n_{e0}}{n_{i0}} - \frac{n_{e0} \mu_1}{n_{i0} \omega} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{\beta\eta}{(\omega^2 + \eta^2)} \frac{\mu_1 n_{e0}}{\omega n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{\beta\eta}{(\omega^2 + \eta^2)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) + \frac{\beta\alpha\omega}{(\omega^2 + \eta^2)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) + \frac{\omega_{pd}^2 k^2 C_{so}^2}{\omega^2 \omega_{pi}^2} \left(1 - \frac{\Delta T_e}{T_{eo}}\right), \quad (16)$$

$$\varepsilon_i(\omega, k) = -\alpha \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) + \frac{\beta\omega}{(\omega^2 + \eta^2)} \frac{\omega^* n_{e0}}{\omega n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{\beta\omega}{(\omega^2 + \eta^2)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{\beta\omega}{(\omega^2 + \eta^2)} \frac{n_{e0}}{n_{i0}} k^2 \rho_{so}^2 - \frac{\beta}{(\omega^2 + \eta^2)} \frac{n_{e0}}{n_{i0}} \mu_1 \left(1 - \frac{\Delta T_e}{T_{eo}}\right) - \frac{\beta\alpha\eta}{(\omega^2 + \eta^2)} \frac{n_{e0}}{n_{i0}} \left(1 - \frac{\Delta T_e}{T_{eo}}\right). \quad (17)$$

Let us write $\omega = \omega_r + i\gamma$ and assume that the wave is either weakly damped or growing (i.e., $|\gamma| \ll \omega_r$).

Then we may set

$$\varepsilon_r(\omega = \omega_r, k) = 0 \quad \text{from} \quad \text{Eq.(15).} \quad (18)$$

Equation (15) yields

Growth rate:

$$\gamma = -\frac{\varepsilon_i(\omega_r, k)}{\partial \varepsilon_r(\omega_r, k) / \partial \omega_r}. \quad (19)$$

Now, we consider two cases of interest

Case I: In the presence of dust charge fluctuations, i.e., dust charging rate η is finite.

Case II: In the absence of dust charge fluctuations, i.e., $Q_{d1} = 0$ when dust charging rate $\eta \rightarrow \infty$.

In the absence of dust grains, i.e., $\delta_m = n_{i0}/n_{e0} = 1, \beta \rightarrow 0$, we recover the expressions for the unstable drift mode frequency and growth rate

of Ref.[14] (cf. page no. 470). Dust grain is negatively charged for $\delta_m > 1$ and positively charged for $\delta_m < 1$.

III. RESULTS AND DISCUSSIONS

To estimate the numerical values of the real frequency and growth rate of the drift wave instability, we use typical dusty plasma parameters: $n_{i0} = 10^9 \text{ cm}^{-3}$, $T_e = 2.0 \text{ eV}$, $T_i = 0.2 \text{ eV}$, $B_s = 0.55 \times 10^3 \text{ G}$, $v_{ei} = 3.0 \times 10^5 \text{ rad./sec}$, $\omega^* = 1.5 \times 10^5 \text{ rad./sec}$, $a_1 = 1.0 \text{ cm}$, length of plasma column $L = 70 \text{ cm}$, $m_i/m_e \approx 7.16 \times 10^4$ (Potassium), average dust grain size $a = 1 \mu\text{m}$, mode number $n = 1$, i.e., the first zero of the Bessel function, $k_{\perp n} = 3.85 \text{ cm}^{-1}$, $k_z = \pi/L$, pump wave amplitude $\phi_0 = 6.6 \times 10^{-4} \text{ esu}$. We vary δ_m from 0.4 to 0.95 for positively charged dust grains.

Using Eq. (18) we have plotted in Fig.1 the normalized real frequency ω_r/ω_{ci} of the unstable collisional drift waves as a function of $\delta_m = n_{i0}/n_{e0}$. In Fig.1 it is seen that the normalized wave frequency ω_r/ω_{ci} increases by a factor ~ 1.44 when δ_m changes from 0.4 to 0.8 if dust charge fluctuations are taken into account under the plasma parameters listed above. Barkan *et al.* [6] have found that the wave frequency was about 10-20% larger than the ion-cyclotron frequency in the presence of negatively charged dust grains. Chow and Rosenberg [7] have shown, in their kinetic analysis on the effect of negatively charged dust grains on the collision less electrostatic ion cyclotron instability, the wave frequency ω_r/ω_{ci} increases about 11% when δ_m is changed from 1 to 4 under similar conditions. Thus the increase is considerably more in case of collisional drift waves in presence of positively charged dust grains in a plasma cylinder.

In Fig.2, we have plotted the normalized growth rate γ/ω_{ci} obtained from Eq. (19) as a function of δ_m for the same parameters as those used in Fig.1. From Fig.2 it can be seen that the normalized growth rate γ/ω_{ci} decreases with δ_m in both cases. However, decrease is more drastic when the dust charge fluctuations are taken into consideration. Thus the dust charge fluctuations can play a significant role in suppression of the collisional drift waves in case of positively charged dust grains.

IV. CONCLUSION

The present work investigates the role of positively charged dust grains in suppression of collisional drift waves. The positively charged dust can play a major role in dusty plasma experiments in the earth ionosphere using space shuttle exhaust. The drift wave instability also plays a crucial role in international ITER and other fusion reactors.

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APPENDIX

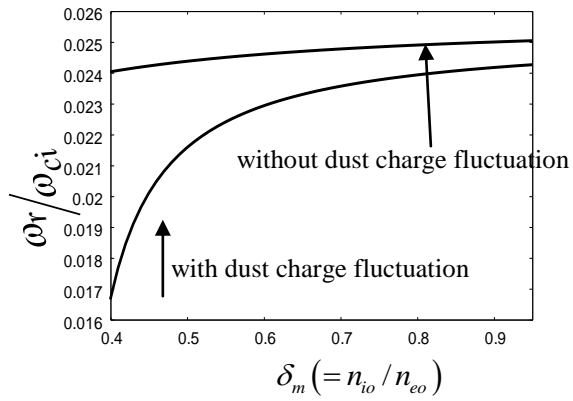


Fig.1: Normalized real frequency ω_r/ω_{ci} of the collisional drift wave as a function $\delta_m (= n_{i0}/n_{e0})$ [with and without dust charge fluctuations].

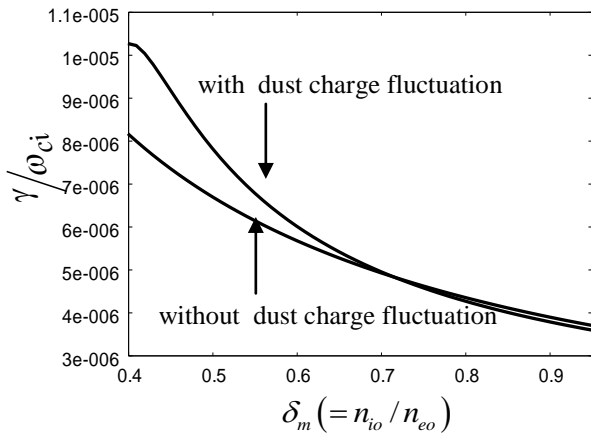


Fig.2: Normalized growth rate γ/ω_{ci} of the collisional drift wave as a function δ_m [with and without dust charge fluctuations].