The influence of core radius on propagation constant, quadratic and cubic dispersion in optical fibers

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Abstract—In this study, the influence of core radius on propagation constant and its high order \( \beta_0, \beta_1, \beta_2, \beta_3 \) (\( \beta \), \( d^2 \beta / d \), \( d^3 \beta / d \)) have been analyzed and simulated. The variation of \( \beta_0, \beta_1, \beta_2, \beta_3 \) verses wavelength (\( \lambda \)) is shown carefully, as we have moved to high order of \( \beta \), the effects of influence core radius is seem clearly. The quadratic dispersion and cubic dispersion is derived from \( \beta_2, \beta_3 \). Their variations with wavelength is shown, a comparison between quadratic dispersion and cubic dispersion for three core radius models are considered as the zero dispersion shifted to the shorter wavelength. The results are analyzed and simulated by matlab software.

Index Terms—Optical fiber, Influence of core radius, quadratic dispersion, Constant Propagation.

I. INTRODUCTION

Fiber optics system having single-wavelength data rates of 100 Gbps, multiple wavelength system have achieved capacities of a few Tb/sec. Fiber systems find use anywhere from small isolated networks within a building or facility to gigantic intercontinental systems that include transoceanic links. In addition to the demand posed by the internet for increased bandwidth drives the development of even faster fiber networks. The principal material in the fiber material is silica, which is very cheap and abundant, enables low-loss propagation of infra red and visible wavelengths, this feature not available in metallic waveguides. The use of optical carrier frequencies (around 10\(^{14}\)Hz). Actually the low loss useful optical frequencies of best fiber is on order of (10\(^{13}\) Hz) [1].

The capacity of the fiber link of a specified length is determined by how many time slots (bits)can occur by second. This number is limited by the maximum switching speed of the modulator at the input, the speed of detector at the receiver beside the loss and dispersive effects in the fiber.

The dispersion in single mode fibers, which has the effect of broadening the pulses as they propagate. The broadened pulses are reduced in amplitude and spread into adjacent time slots, thus increasing the uncertainty that the receiver will correctly interpret the energy within a given slot [2]. A major effort in the development of optical fibers has been in design of fibers that simultaneously exhibit low loss and controlled amount of dispersion.

II. PROPAGATION OF WAVE IN THE MEDIUM

The medium exhibit chromatic dispersion if the propagation constant (for the field of the single mode) varies non-linearly with frequency. Signal distortion arising from group velocity dispersion occurs as the frequency components of the signal propagate with different velocities.

In optical fiber waveguide, this effect arises from two mechanisms:

1. The refractive index varies with wavelength.
2. Waveguide dispersion.

The interplay with these two effects in optical fiber has led to number of successful ways of minimizing dispersion over specific wavelength range by using special refractive index profiles and specific core diameter. A curve fit to experimentally measured index data can be accomplished using Sellmeier formula for refractive index[3][4].

\[
 n^2 - 1 = \sum_{p=1}^{3} \frac{A_p \lambda^2}{\lambda^2 - \lambda_0^2} \tag{1}
\]

The first three terms in Taylor expansion for \( \beta(\omega) \) are used as following equation [3][5].:

\[
 \beta(\omega) \approx \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2\beta_2 + \frac{1}{6}(\omega - \omega_0)^3\beta_3 \ldots \tag{2}
\]

Where:

\[
 \beta_0 \approx \frac{N}{2}\omega_0, \beta_1 = \frac{dN}{d\omega}\omega_0, \beta_2 = \frac{d^2N}{d\omega^2}\omega_0, \beta_3 = \frac{d^3N}{d\omega^3}\omega_0
\]

And the group velocity function \( v_g(\omega) \) [6][7].:

\[
 v_g = \frac{d\omega}{d\beta} \tag{3}
\]

The higher order term (higher than \( \beta_3 \)) is needed for fully characterized dispersive material of pulse order of femto seconds. For optical fiber applications, these higher order terms are not needed, and the inclusion of \( \beta_3 \) is sufficient [8][9].

Dispersion parameter:

\[
 D(\lambda) = -\frac{1}{c^2} \frac{d^2N}{d\lambda^2} \tag{4}
\]

\[
 D(\lambda) = \frac{dD}{d\lambda} = \frac{d\omega}{d\lambda} \frac{d^2 \beta}{d\lambda^2} = \frac{2\pi c}{\lambda^2} \frac{d^2 \beta}{d\omega^2} \tag{5}
\]

\[
 \beta_2 = \frac{d^2 \beta}{d\lambda^2} = \frac{2\pi c}{\lambda^2} D(\lambda) \tag{6}
\]

\[
 \beta_3 = \frac{d^3 \beta}{d\lambda^3} = \frac{2\pi c}{\lambda^2} \frac{dD}{d\lambda} + \frac{d^2D}{d\lambda^2} \tag{7}
\]

\[
 \frac{dD}{d\lambda} = -\frac{1}{c} \left( \frac{d^2N}{d\lambda^2} + \lambda^2 \frac{d^2N}{d\lambda^4} \right) \tag{8}
\]
III. SIMULATION OF RESULTS

The wavelength dependence of refractive index is approximated by Sellmeier’s formula, where the coefficients for composition of pure silica SiO$_2$ for cladding and 13.5%GaO$_2$.865%SiO$_2$ for core [9]. Eq.(2) is plotted in Figure (1) for core 13.5%GaO$_2$.865%SiO$_2$ and cladding refractive index pure SilicaSiO$_2$ for range from 1.2µm to (1.6µm). This figure shows that the difference between them is not constant for assuming range and the slightly increasing in the magnitude of the refractive index of the core to the cladding refractive index which is essential assumption for weakly approximation wave guide.

\[
\beta_2 = \frac{d^2E}{d\omega^2} = \frac{\lambda^2}{2\pi c^2} \frac{d^2N_{eff}}{d\lambda^2} + \lambda \frac{d^2N_{eff}}{d\lambda^2} \quad \text{(9)}
\]

\[
\beta_3 = \frac{d^2E}{d\omega^2} = -\frac{\lambda^2}{4\pi^2 c^3} \left( 3 \frac{d^2N_{eff}}{d\lambda^2} + \lambda \frac{d^2N_{eff}}{d\lambda^2} \right) \quad \text{(10)}
\]

In figure (3) the varies of the group velocity (\(v_g\)) with wavelength is shown, there is increased in group velocity until 1.33µm and then decreased at longer wavelength within our range. Another important thing it's magnitude is decrease with increasing radius of the core.

**Fig.(3) variation of group velocity with wavelength for different core radius.**

In Figure (4) the high order as in the term (\(\beta_2\)), which relates to the important design parameter as the dispersion will decreased and becomes (zero)ps2/km, at 1.33µm and about (-20) ps2/km at 1.55µm. Also as the core radius increased led to (\(\beta_2\)) decreased slightly.

**Fig.(4): Relation between \(\beta_2\) and wavelength for different core radius.**

Figure (5) shows the relation between (\(\beta_3\)) and wavelength, which reffered to the slope curve of the smallest core radius is less than the other two core radius.

**Fig.(5): The reactive index of the core 13.5% GaO$_2$.865% and cladding SilicaSiO$_2$ for range from 1.2µm to (1.6µm)**

**Fig.(2) Relation between the propagation constant with the wavelength for different core radius.**
Fig. (5): Relation between $\beta_3$ and wavelength for different core radius.

Figure (6) describing the behave of the dispersion against wavelength, according to eq. (6), as the core radius increased small shift to the lower wavelength occurred.

Fig. (6): Effect of three core radius on dispersion .

Figure (7) shows the plot of dispersion with wavelength for (2.75\(\mu\)m) core.

Fig. (7): Variation of dispersion derived from Equ.(4) and from Equ.(5).

Radius, the first curve using Eq. (4) and the second using Eq. (5) and accepted coincidence is founded. The dispersion slope Eq.(7) with wavelength is shown in figure (8), for three core radius, from this curves it can be estimate the dispersion slope for particular wavelength.

Fig.(8): relation between dispersion slope and wavelength for different core radius.

Fig.(9): Variation of dispersion slope derived from Equ.(7) and Equ.(8).

From coincidence of the Fig.8. and Fig.9, it can be consider the dependence on the result of propagation constant and it’s high orders are acceptable.

V. CONCLUSIONS

The results show the influence of core radius for optical fibers was carried out using MATLAB software program. It can be noted the effect of radius on $\beta_2$ is inconsiderable, while group velocity increased proportionally with radius of core. The maximum value of $(\beta_2)$ start at wavelength value (1.2 \(\mu\)m) then decreases in convergence way for different radius core to reach its minimum value (-20 ps/\(\mu\)m) at (\(\lambda=1.6\) \(\mu\)m) as shown in fig.(4), which leads the zero dispersion to be shifted to the shorter wavelength values. The most important factor the dispersion which increased at the edges of limited range with the core radius and become inconsiderable around 1.3 \(\mu\)m. Finally the flatten of dispersion slope decreases as the core radius increased.

The future work is the study of the effects of $\Delta$ (refractive index difference) between the core and the cladding of the fiber on the three types of dispersions and the total dispersion.

REFERENCES


