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Product cordial labeling of triple path union on C_3, C4, C5

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***Abstract:** A triple path union P_m (3-G) is obtained by fusing three copies of same graph G at each vertex of path P_m. We take G = C_3, $Fl(C_3)$, and tail(C_3,2p_2) to obtain P_m(3-G) and show that all of them are product cordial graphs.*

Key words: **path union triangle, product cordial, labeling, fusion.**
Subject Classification: 05C78

I. INTRODUCTION

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [9], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[11]. I.Cahit introduced the concept of cordial labeling [7].There are variety of cordial labeling available in labeling of graphs. Sundaram, Ponraj, and Somasundaram [10] introduced the notion of **product cordial labeling**. A product cordial labeling of a graph G with vertex set V is a function f from V to {0,1} such that if each edge uv is assigned the label f(u)f(v), the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a product cordial labeling is called a product cordial graph. We use $v_f(0,1)$ = (a, b) to denote the number of vertices with label 1 are a in number and the number of vertices with label 0 are b in number. Similar notion on edges follows for $e_f(0,1)$ = (x, y).

A lot of work is done in this type of labeling so far. One interested in survey may refer Dynamic survey in Graph labeling by J. Gillian. We mention some part of it. Sundaram, Ponraj, and Somasundaram have shown that trees; unicyclic graphs of odd order; triangular snakes; dragons; helms; Pm∪Pn; Cm∪Pn; Pm∪K1,n; Wm∪Fn (Fn is the fan Pn+K1); K1,m∪K1,n; Wm∪ K1,n; Wm∪Pn; Wm∪Cn; the total graph of Pn (the total graph of Pn has vertex set V (Pn)∪E(Pn) with two vertices adjacent whenever they are neighbors in Pn); Cn if and only if n is odd; $C_n^{(t)}$, the one-point union of t copies of C_n, provided t is even or both t and n are even; K2+mK1 if and only if m is odd; C_m∪P_n if and only if m+n is odd; $K_{m,n}$∪Ps if s >mn; Cn+2∪K1,n; Kn∪Kn,(n−1)/2 when n is odd; Kn∪Kn−1,n/2 when n is even; and P2 n if and only if n is odd. are product cordial graphs.They also prove that $K_{m,n}$ (m,n> 2), P_m ×P_n (m,n> 2) and wheels are not product cordial and if a (p,q)-graph is product cordial graph, then q = 6 (p−1)(p + 1)/4 + 1.In this paper we show that P_m(G) where Graph G is from {C_3, Fl(C_3), and tail(C_3,2p_2) } are product cordial graphs.

II. PRELIMINARIES

Fusion of vertex. Let G be a (p,q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges. If u϶G_1 and v϶G_2, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and q_1 + q_2 edges. Sometimes this is referred as u is identified with the concept is well elaborated in John Clark, Holton [6].

Path union of G ,i.e.P_m(G) is obtained by taking a path p_m and take m copies of graph G . Then fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges. Where G is a (p,q) graph.

III. RESULTS

Theorem 1. G= P_m(3-C_3) is product cordial for all m.
Proof: Take a path P_m=(v_1, e_1, v_2, e_2,..v_m).At i^{th} vertex of Pm fuse three copies of C_3 and are given by $u_{i,1}$=v_i,$u_{i,2}$, $u_{i,3}$,$u_{i,4}$,$u_{i,5}$,$u_{i,6}$, $u_{i,7}$.The vertex$u_{i,1}$ is common to path P_m and all three copies of C_3. |V(G)|= 7m and |E(G)|= 10m-1.Define a function f :V(G)→{0,1} as follows:
Case m = 2x
$f(u_{i,j})$= 0 for all i = 1,2, ..x , j = 1, 2, ..,7
$f(u_{i,j})$ = 1 for all i = x+1, x+2,.. 2x; j = 1, 2, ..,7.
The label number distribution is $v_f(0,1)$ = (7x, 7x) and $e_f(0,1)$ = (10x,10x-1)
Case m = 2x+1
To obtain a labeled copy of $P_{2x+1}(C_3)$ we label P2x(C3) part of it from one end as given above. Further $f(u_{i,j})$ = 1 for j =1, 2, 3, 4 and i = 2x+1;
$f(u_{i,j})$ = 0 for j =5, 6, 7 and i = 2x+1.The label number distribution is $v_f(0,1)$ = (7x+3, 7x+4) and $e_f(0,1)$ = (10x+5,10x+4).

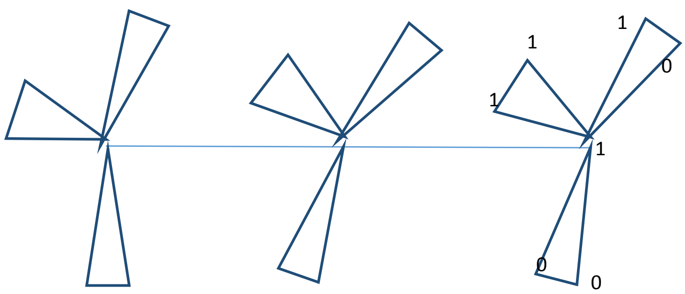

Fig 1 labeled copy

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Thus the graph G is product cordial for all m.

Theorem 2 G= P_m(3-G') is product cordial for all m where G' = FL(C_3).(For all structures)

Proof: Take a path P_m=(v_1, e_1, v_2, e_2,..v_m).At i[th] vertex of P_m fuse three copies of G' and are given by $u_{i,1}$=v_i,$u_{i,2}$, $u_{i,3}$,$u_{i,4}$,$u_{i,5}$,$u_{i,6}$, $u_{i,7}$,$u_{i,8}$,$u_{i,9}$, $u_{i,10}$ where $u_{i,4}$, $u_{i,7}$and $u_{i,10}$ are pendent vertices. The vertex$u_{i,1}$ is common to path P_m and all three copies of C_3. |V(G)|= 10m and |E(G)|= 13m-1.

Define a function f: V(G)→{0,1} as follows:

Structure I: This is obtained when two degree vertex on each of three copies of C_3 are fused with path vertex.

Case m = 2x

f($u_{i,j}$)= 0 for all i = 1, 2, ..x , j = 1, 2, ..,10

f($u_{i,j}$) = 1 for all i = x+1, x+2,.. 2x; j = 1, 2, ..,10.

The label number distribution is v_f(0,1) = (10x, 10x) and e_f(0,1) = (13x,13x-1)

Case m = 2x+1

To obtain a labeled copy of P_{2x+1}(G') we label P_{2x}(G') part of it from one end as given above with only two change given by

f($u_{i,4}$)=0 for i = x+1;

f($u_{i,7}$)=0 for i = x+1

Further f($u_{i,j}$) = 1 for j =1, 2, , …7 and i = 2x+1;

f($u_{i,j}$) = 0 for j =8,9,10 and i = 2x+1.

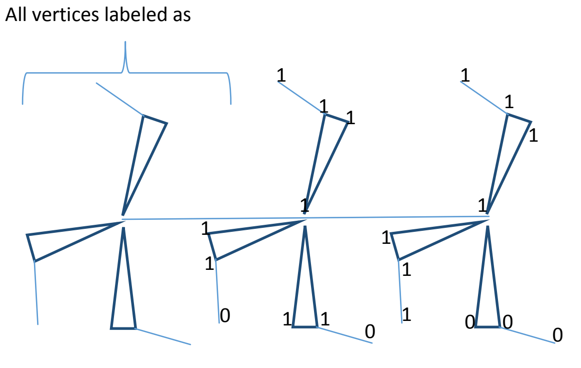

Fig .2 P_3(FL(C_3)): structure 1:v_f(0,1) = (15, 15) and

The label number distribution is v_f(0,1) = (10x+5, 10x+5) and e_f(0,1) = (13x+6,13x+6).

Structure II: This is obtained when three degree vertex on each of three copies of C_3 are fused with path vertex.

Case m = 2x.

f($u_{i,j}$)= 0 for all i = 1, 2, ..x , j = 1, 2, ..,10

f($u_{i,j}$) = 1 for all i = x+1, x+2,.. 2x; j = 1, 2, ..,10.

The label number distribution is v_f(0,1) = (10x, 10x) and e_f(0,1) = (13x,13x-1).

Case m = 2x+1

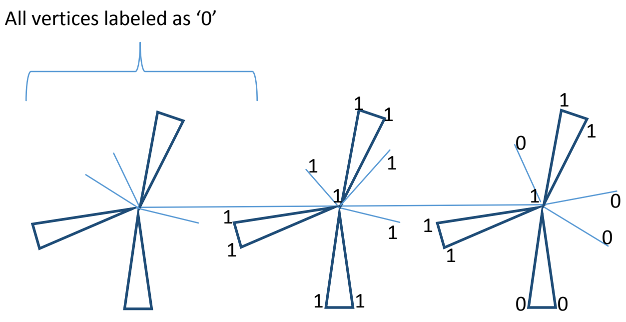

Fig 3 P_3(FL(C_3)): structure 2:v_f(0,1) = (15, 15) and e_f(0,1) =

To obtain a labeled copy of P_{2x+1}(G') we label P_{2x}(G') part of it from one end as given above.

f($u_{i,j}$) = 1 for j =1, 2,3 , 5,6. and i = 2x+1;

f($u_{i,j}$) = 0 for j =4,7,8,9,10 and i = 2x+1.

The label number distribution is v_f(0,1) = (10x+5, 10x+5) and e_f(0,1) = (13x+6,13x+6).

Structure III: This is obtained when any of the pendent vertices on each of three copies of G' are fused with path vertex.

Case m = 2x.

f($u_{i,j}$)= 0 for all i = 1, 2, ..x , j = 1, 2, ..,10

f($u_{i,j}$) = 1 for all i = x+1, x+2,.. 2x; j = 1, 2, ..,10.

The label number distribution is v_f(0,1) = (10x, 10x) and e_f(0,1) = (13x,13x-1).

For m = 2x+1 the product cordial labeling does not exists.

Theorem 3. G= P_m(3-G') is product cordial for all m where G' = tail(C_3,2p_2)

Proof: There are three structures on path union depending on which point on G' is used to obtain a path union by fusing with path vertex. In all structures a path P_m=(v_1, e_1, v_2, e_2,..v_m) remains unchanged. In **Structure I**At i[th] vertex of P_m fuse three copies of G' and are given by $u_{i,1}$=v_i,$u_{i,2}$, $u_{i,3}$,$u_{i,4}$,$u_{i,5}$,$u_{i,6}$, $u_{i,7}$,$u_{i,8}$,$u_{i,9}$, $u_{i,10}$,$u_{i,11}$,$u_{i,12}$, $u_{i,13}$

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where $u_{i,4}$, $u_{i,5}$ and $u_{i,8}, u_{i,9}$, $u_{i,12}, u_{i,13}$ are pendent vertices at $u_{i,3}, u_{i,3}$, $u_{i,7}, u_{i,7}, u_{i,11}, u_{i,11}$ respectively.

Define a function f: V(G)→{0,1} as follows:

Structure 1 is obtained when two degree vertex on each of three copies of G' are fused with path vertex.The vertex $u_{i,1}$ is common to path P_m and all three copies of G'. |V(G)|= 13m and |E(G)|= 16m-1.

Case m = 2x

$f(u_{i,j})$= 0 for all i = 1, 2, ..x , j = 1, 2, ..,13

$f(u_{i,j})$ = 1 for all i = x+1, x+2,.. 2x; j = 1, 2, ..,13.

The label number distribution is $v_f(0,1)$ = (13x, 13x) and $e_f(0,1)$ = (16x,16x-1)

Case m = 2x+1

To obtain a labeled copy of P_{2x+1}(G') we label P_{2x}(G') part of it from one end as given above. Further $f(u_{i,j})$ = 1 for j =1, 2, , .,6,7 and i = 2x+1;

$f(u_{i,j})$ = 0 for j = 8, 9,..,13 and i = 2x+1.

The label number distribution is $v_f(0,1)$ = (13x+6, 13x+7) and $e_f(0,1)$ = (16x+7,16x+8).

Structure II: This is obtained when four degree vertex on each of three copies of G' are fused with path vertex. At the i^{th} vertex of path P_m the design fused has ordinary label given as in the diagram below.

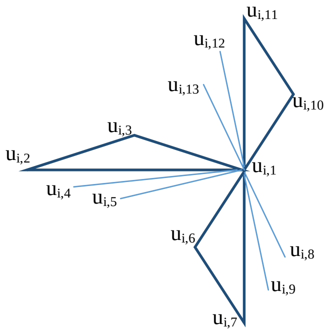

Fig 4 ordinary labeling of structure II fused at i^{th} vertex of P_m

Case m = 2x.

$f(u_{i,j})$= 0 for alli = 1, 2, ..x , j = 5,8,9,11,12,13

$f(u_{i,j})$ = 1 for all i = x+1, x+2,.. 2x; j = 1,2,3,4,6,7,10

The label number distribution is $v_f(0,1)$ = (13x, 13x) and $e_f(0,1)$ = (16x,16x-1).

Case m = 2x+1

To obtain a labeled copy of P_{2x+1}(G') we label P_{2x}(G') part of it from one end as given above.

Further $f(u_{i,j})$ = 0 for j =5,8,9,11,12,13, i = 2x+1;

$f(u_{i,j})$ = 1 for j = 1,2,3,4,6,7,10,11and i = 2x+1.

The label number distribution is $v_f(0,1)$ = (13x+6, 13x+7) and $e_f(0,1)$ = (16x+7,16x+8).

Structure III: This is obtained when any of the pendent vertices on each of three copies of G' are fused

i^{th} path vertex. At the i^{th} vertex of path P_m the design fused has ordinary label given as in the diagram below.

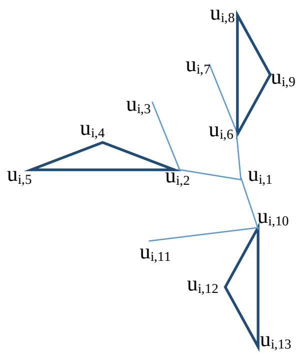

Fig .5 ordinary labeling of structure III fused at i^{th} vertex of P_m

Case m = 2x.

$f(u_{i,j})$= 0 for all i = 1, 2, ..x , j = 1, 2, ..,13

$f(u_{i,j})$ = 1 for all i = x+1, x+2,.. 2x; j = 1, 2, ..,13.

The label number distribution is $v_f(0,1)$ = (13x, 13x) and $e_f(0,1)$ = (16x,16x-1).

Case m = 2x+1

To obtain a labeled copy of P_{2x+1}(G') we label P_{2x}(G') part of it from one end as given above.

Further $f(u_{i,j})$ = 0 for j =7,8,10,11,12,13, i = 2x+1;

$f(u_{i,j})$ = 1 for j = 1,2,3,4,5,6,9 and i = 2x+1.

The label number distribution is $v_f(0,1)$ = (13x+6, 13x+7) and $e_f(0,1)$ = (16x+7,16x+8).

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IV. CONCLUSION

In this paper we have obtained path union by fusing three copies of same graph G at a particular vertex of G with path vertex .This is triple path union and is denoted by Pm(3-G).We discuss resultant structure for product cordial labeling. We have proved that:

1) P_m (3-C_3) is product cordial for all m.

2) P_m(3-G') is product cordial for all m where G' = FL(C_3). (For all structures)

3) = P_m (3-G') is product cordial for all m where G' = tail (C_3, 2p2).

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