

# Multi-Interactive Cellular Neural Network For Associated Memory

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**Abstract**— This paper presents the results of research on associated memory and cellular neural networks, multi-interactive cellular neural network for associated memory based on the connectivity of the network elements (ij), (ij, pq), (ij, pq, mn). This paper presents a corresponding examples for each model of associated memory.

**Index Terms**—Multi-interactive CNN, MiCNN, MiCNN for AM, Associated Memory.

## I. INTRODUCTION

In this paper, we propose a new architecture called multi-interactive cellular neural network (MiCNN) for associated memory, based on the research project proposed by [1] [4] [5] [6]. A multi-interactive cellular neural network is a nonlinear multidimensional connection that is defined by connecting N identical dynamical networks called local cells with a central system, as shown in Figure 1, 2. The internal cells communicate with each other through a central system. Therefore, a multi-interactive CNN with N connections from local cells exist a central system.

<b>H</b> (i-1,j+1)	<b>A</b> (i,j+1)	<b>B</b> (i+1,j+1)
<b>G</b> (i-1,j)	<b>Cen</b> (i,j)	<b>C</b> (i+1,j)
<b>F</b> (i-1,j-1)	<b>E</b> (i,j-1)	<b>D</b> (i+1,j-1)

Fig 1. Cellular Neural Network 3x3

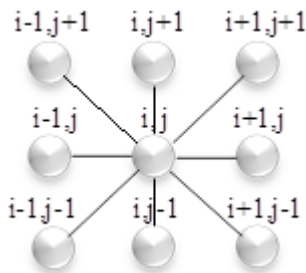


Fig 2. Connections of each neuron in the CNN

### A. The structure of the neural elements of the standard cells

In the cellular neural network [1][6] each cell  $C_{ij}$  with  $i$  is the number of rows,  $j$  is the number of columns links with neighboring cells  $C_{kl}$  in neighboring  $Nr(i, j)$  of radius  $r$  ( $r$  is a

positive integer). Each cell is an input processor  $v_{uij}$ , state  $v_{xij}(t)$ , the output  $v_{yij}(t)$ . Dynamical equations describe the structure of a cell neurons as follows:

$$C \frac{dv_{xij}(t)}{dt} = -\frac{1}{R_x} v_{xij}(t) + \sum_{C(k,l) \in Nr(i,j)} A(i, j; k, l) v_{ykl}(t) + \sum_{C(k,l) \in Nr(i,j)} B(i, j; k, l) v_{ukl} + I \quad (1)$$

$$1 \leq i \leq M; 1 \leq j \leq N$$

Output function:

$$v_{yij}(t) = \frac{1}{2} \left( |v_{xij}(t) + 1| - |v_{xij}(t) - 1| \right) \quad (2)$$

$$\text{Input signal: } v_{uij} = E_{ij} \quad (3)$$

$$\text{The conditions } |v_{xij}(0)| \leq 1; |v_{uij}| \leq 1 \quad (4)$$

Other theories  $A(i, j; k, l) = A(k, l; i, j)$  and  $C > 0; R_x > 0$

$$\text{With } 1 \leq i, k \leq M; 1 \leq j, l \leq N \quad (5)$$

Where  $M, N$  is the size of the network [1]

### B. Stability of cellular neural network

Leon O. Chua and Yang have proposed Lyapunov function:

$$E(t) = -\frac{1}{2} \sum_{(i,j)} \sum_{(k,l)} A(i, j; k, l) v_{yij}(t) v_{ykl}(t) + \frac{1}{2R_x} \sum_{(i,j)} v_{yij}(t)^2 - \sum_{(i,j)} \sum_{(k,l)} B(i, j; k, l) v_{yij}(t) v_{ukl} - \sum_{(i,j)} I v_{yij}(t) \quad (6)$$

Conditions and demonstration of the network stability have been made in [1]

## II. MULTI-INTERACTIVE CELLULAR NEURAL NETWORK

### A. The structure of multi-interactive cellular neural network

Multi-interactive cellular neural network is associated with the sum of the controlling links and basic feedback to the sum of the input controlling signal accumulations and the output sum of the feedback signal accumulations at any point  $(k, l)$  and  $(m, n)$  in the vicinity of the point  $(i, j)$ . Dynamical equation for multi-interactive cellular neural network as follows:

$$C \frac{dv_{xij}(t)}{dt} = -\frac{1}{R_x} v_{xij}(t) + \sum_{C(k,l) \in Nr(i,j)} A(i, j; k, l) v_{ykl}(t) + \sum_{C(k,l) \in Nr(i,j)} B(i, j; k, l) v_{ukl} + I$$

$$+ \sum_{C(k,l), C(m,n) \in Nr(i,j)} A(i, j; k, l, m, n) v_{ykl(t)} v_{ymn(t)} + \sum_{C(k,l), C(m,n) \in Nr(i,j)} B(i, j; k, l; m, n) v_{ukl} v_{umn} \quad (7)$$

$$v_{yij}(t) = \frac{1}{2} \left( |v_{xij}(t) + 1| - |v_{xij}(t) - 1| \right) \quad (8)$$

Where:

A (i, j, k, l, m, n) and B (i, j, k, l, m, n) is the ratio of the accumulation of the two feedback signals from the output and controls at point respectively (k, l) and (m, n) at point (i, j). The input signals, assumptions and similar binding conditions (3), (4), (5).

**B. Stability of output state**

The problem in multi-interactive cellular neural network is that the network state must be stable to be able to put in the application?

Consider the largest state:

$$v_{\max} = \max_{(i,j)} \left\{ 1 + R_x |l| + R_x \sum_{C(k,l) \in Nr(i,j)} (|A(i, j; k, l)| + R_x \sum_{C(m,n) \in Nr(i,j)} (|A(i, j; k, l; m, n)| + |B(i, j; k, l; m, n)|) \right\} \quad (9)$$

It is necessary to prove limited state (stability of output state). Conditions and demonstration of the stability of the network have been made in [2].

**III. ASSOCIATED MEMORY USING MULTI INTERACTIVE CELLULAR NEURAL NETWORK**

**A. Associated memory**

An information storage device is called an associated memory if it permits the recall of information on the basis of a partial knowledge of its content, but without knowing its storage location. The associated memories are described as follows: Consider M binary (-1/1) patterns p1. In associated memory Makoto and L.O.Chua defined as follows (\*):

$$S_{ij} = \frac{1}{N} \sum_{m=1}^M p_i^m p_j^m \quad (10)$$

Assign the initial state  $v_1(n+1) = \text{sgn} \left( \sum_{j=1}^N s_{ij} v_j(n) \right)$

$$p^1 = \begin{bmatrix} 1 \\ p_1 \\ 1 \\ p_2 \\ 1 \\ p_3 \\ \vdots \\ 1 \\ p_N \end{bmatrix} \cdot p^2 = \begin{bmatrix} p_1^2 \\ p_2^2 \\ p_3^2 \\ \vdots \\ p_N^2 \end{bmatrix} \dots p^M = \begin{bmatrix} p_1^M \\ p_2^M \\ p_3^M \\ \vdots \\ p_N^M \end{bmatrix} \quad (*)$$

Where  $\text{sgn}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

**B. Associated memory using cellular neural network**

For binary matrix k with m x n size P<sup>1</sup>, P<sup>2</sup>, P<sup>3</sup>, ..., P<sup>k</sup> and k is the sample which needs storing, w<sub>ij, pq</sub> are associated weights between neurons (i, j) and neurons (p, q), w<sub>ij, pq, mn</sub> is associated weights between neurons (i, j), neural (p, q) and neural (m, n). In standard cellular neural network, associated memory is determined by the weights [7]

$$w_{ij, pq} = \begin{cases} 0 & |i-p| > 1 \vee |j-q| > 1 \\ \sum_{l=1}^k P_{ij}^l P_{pq}^l |i-p| \leq 1 \wedge |j-q| \leq 1 \end{cases} \quad (11)$$

Example: There are three patterns as input P<sup>1</sup>, P<sup>2</sup>, P<sup>3</sup> as follows:

P<sup>1</sup> = [-1 1 -1 1 1 -1 -1 1 -1 1 -1 1 1 1 1]  
 P<sup>2</sup> = [-1 -1 -1 1 1 -1 1 1 -1 1 1 -1 1 1 -1]  
 P<sup>3</sup> = [-1 1 1 1 -1 1 -1 1 -1 1 -1 -1 1 1 1]

According to (10) with N = 16 and M = 3

$$S_{ij} = \frac{1}{16} \sum_{m=1}^3 p_i^m p_j^m$$

We have

$$S_{12} = \frac{1}{16} \sum_{m=1}^3 p_1^m p_2^m = \frac{1}{16} (p_1^1 p_2^1 + p_1^2 p_2^2 + p_1^3 p_2^3) = \frac{1}{16} [(-1) \times (1) + (-1) \times (-1) + (-1) \times (1)] = \frac{-1}{16}$$

$$S_{13} = \frac{1}{16} \sum_{m=1}^3 p_1^m p_3^m = \frac{1}{16} (p_1^1 p_3^1 + p_1^2 p_3^2 + p_1^3 p_3^3) = \frac{1}{16} [(-1) \times (-1) + (-1) \times (-1) + (-1) \times (1)] = \frac{1}{16}$$

$$S_{23} = \frac{1}{16} \sum_{m=1}^3 p_2^m p_3^m = \frac{1}{16} (p_2^1 p_3^1 + p_2^2 p_3^2 + p_2^3 p_3^3) = \frac{1}{16} [(1) \times (-1) + (-1) \times (-1) + (1) \times (1)] = \frac{1}{16}$$

Calculating respectively, we have value matrix 16[s<sub>ij</sub>] =

$$\begin{bmatrix} 3 & -1 & 1 & -3 & -1 & 1 & 1 & -3 & 3 & -3 & 1 & 3 & -1 & -3 & -1 & -1 \\ -1 & 3 & 1 & 1 & -1 & 1 & -3 & 1 & -1 & 1 & -3 & -1 & -1 & 1 & 3 & -1 \\ 1 & 1 & 3 & -1 & -3 & 3 & -1 & -1 & 1 & -1 & -1 & 1 & -3 & -1 & 1 & -3 \\ -3 & 1 & -1 & 3 & 1 & -1 & -1 & 3 & -3 & 3 & -1 & -3 & 1 & 3 & 1 & 1 \\ -1 & -1 & -3 & 1 & 3 & -3 & 1 & 1 & -1 & 1 & 1 & -1 & 3 & 1 & -1 & 3 \\ 1 & 1 & 3 & -1 & -3 & 3 & -1 & -1 & 1 & -1 & 1 & -3 & -1 & 1 & -3 & 1 \\ 1 & -3 & -1 & -1 & 1 & -1 & 3 & -1 & 1 & -1 & 3 & 1 & 1 & -1 & -3 & 1 \\ -3 & 1 & -1 & 3 & 1 & -1 & -1 & 3 & -3 & 3 & -1 & -3 & 1 & 3 & 1 & 1 \\ 3 & -1 & 1 & -3 & -1 & 1 & 1 & -3 & 3 & -3 & 1 & 3 & -1 & -3 & -1 & -1 \\ -3 & 1 & -1 & 3 & 1 & -1 & -1 & 3 & -3 & 3 & -1 & -3 & 1 & 3 & 1 & 1 \\ 1 & -3 & -1 & -1 & 1 & -1 & 3 & -1 & 1 & -1 & 3 & 1 & 1 & -1 & -3 & 1 \\ 3 & -1 & 1 & -3 & -1 & 1 & 1 & -3 & 3 & -3 & 1 & 3 & -1 & -3 & -1 & -1 \\ -1 & -1 & -3 & -1 & 3 & -3 & 1 & 1 & -1 & 1 & 1 & -1 & 3 & 1 & -1 & 3 \\ -3 & 1 & -1 & 3 & 1 & -1 & -1 & 3 & -3 & 3 & -1 & -3 & 1 & 3 & 1 & 1 \\ -1 & 3 & 1 & 1 & -1 & 1 & -3 & 1 & -1 & 1 & -3 & -1 & -1 & 1 & 3 & -1 \\ -1 & -1 & -3 & 1 & 3 & -3 & 1 & 1 & -1 & 1 & 1 & -1 & 3 & 1 & -1 & 3 \end{bmatrix}$$

According to (10), we have stored chain, P<sup>1</sup>, P<sup>2</sup>, P<sup>3</sup>, ..., P<sup>k</sup> corresponding to the range S<sup>1</sup>, S<sup>2</sup>, S<sup>3</sup>, ..., S<sup>k</sup> that can be determined as follows:

$$S^l = N_{ij}(P^l) \quad l = 1, 2, 3, \dots, k$$

Weights connected to a second neuron cells ij is determined by

$$w_{ij,pq} = \sum_{l=1}^k S_{pq}^l S_{ij}^l \quad (12)$$

P identified between i-1 and i + 1 and q determined between j-1 and j + 1

Similar to the calculation of the above, it is applied to (11) and (12)

We have weight matrix: 16[ w<sub>ij,pq</sub> ]=

$$\begin{bmatrix} 3 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & 1 & 0 & -1 & 1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & -1 & 0 & 3 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 0 & 0 & -1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 3 & -3 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 3 & 0 & -3 & 3 & -1 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & -1 & -1 & 0 & -1 & 3 & -1 & 0 & -1 & 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 0 & 0 & -1 & 3 & 0 & 0 & -1 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 3 & -3 & 0 & 0 & -1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 & -1 & 0 & -3 & 3 & -1 & 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 & -1 & 0 & -1 & 3 & 1 & 0 & -1 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 & 0 & 1 & 3 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 3 & -1 & 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 3 \end{bmatrix}$$

Next 16th matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -3 & 0 & 0 & 1 & 1 \end{bmatrix}$$

IV. CONCLUSION

This paper presents an overview of one direction associated memory with the values ij and cellular neural network applications for associated memory with the connection between the central cell to bidirectional neighboring cells (ij, pq) and multi-association with the connection between the central cell to multidimensional neighboring cells (ij, pq, ml). With each type of memory, there is a given corresponding result.

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C. Associated memory using multi-interactive cellular neural network

With multi-interactive cellular neural network, associated memory recommended is determined by weights as follows:

$$w_{ij,pq,mm} = \begin{cases} 0 & |i-p| > 1 \vee |j-q| > 1 \vee |i-m| > 1 \\ & \vee |j-n| > 1 \\ \sum_{l=1}^k P_{ij}^l P_{pq}^l P_{mn}^l & |i-p| \leq 1 \wedge |j-q| \leq 1 \\ & \wedge |i-m| \leq 1 \wedge |j-n| \leq 1 \end{cases} \quad (13)$$

With P<sub>ij</sub><sup>l</sup> P<sub>pq</sub><sup>l</sup> P<sub>mn</sub><sup>l</sup> are the inputs (i, j; pq; mn) of sample P<sup>l</sup>

According to (13), similar to the above, we have 16 weight matrices as follows:

First matrix

$$\begin{bmatrix} -3 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -3 & 0 & 0 & -1 & -1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & -3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 3 & -3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$