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Thermoelastic Problem of a Thin Rectangular Plate with Moving Heat Source

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ABSTRACT- This paper is concern with the thermoelastic solution of a thin rectangular plate in which an attempt has been made to determine the temperature distribution and thermal stresses with the help of integral transform technique. The results are obtained in term of Bessel's function in the form of infinite series.

KEY WORDS: Moving heat source, Marchi-Fasulo transform, Fourier Cosine Transform.

I. INTRODUCTION

Several researchers have considered the heat conduction equation with internal heat generation and find the solution by various methods. N.W.Khobragade and P.C.Wankhede [6] studied an inverse unsteady-state thermoelastic problem of a thin rectangular plate. Recently D.T.Solanke and M.H.Durge [18] has been considered the heat conduction equation with internal moving heat source for a Neumann's thin rectangular plate and find the thermal stresses by using Green's theorem. Also D.T.Solanke and M.H.Durge [19] has been determined the temperature distribution and thermal stresses in thin rectangular plate with moving line heat source taking second type boundary condition by using integral transform technique and Green's theorem. Kidawa-Kukla J [9] has studied the temperature distribution in a Rectangular Plate heated by a Moving Heat Source.

In present paper author considered thermoelastic problem with second and third kind boundary condition in a thin rectangular plate occupying the region $D: -a \le x \le a, -b \le y \le b, 0 \le z \le h\}$.The solution of the problem is obtained by using finite Marchi-Fasulo transform and finite Fourier cosine transform techniques in the form of infinite series.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the region $D: -a \le x \le a, -b \le y \le b, 0 \le z \le h\}$. The plate is subjected to the motion of moving point heat source at the point $(0, y', z')$ which move its place along x, y, z axes with constant velocity vector

$$\bar{v} = v_1 i + v_2 j + v_3 k$$

where v_1, v_2, v_3 are component of velocity vector along x, y, z axes respectively. The temperature distribution of the rectangular plate is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x,y,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

Where k is the thermal conductivity and α is thermal diffusivity of the material of the plate. Consider an instantaneous moving point heat source at point $(0, y', z')$ and releasing its heat spontaneously at time t' Such volumetric moving heat source in rectangular coordinates is given by (2.4.1)

$$g(x,y,z,t) = g_0 \delta(x)\delta(y-y')\delta(z-z')\delta(t-t')$$

Hence equation (1) becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{k} g_0 \delta(x)\delta(y-y')\delta(z-z')\delta(t-t') = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

Where $y' = v_2 t$ and $z' = v_3 t$, with initial condition

$$T(x,y,z,0) = T_0 \tag{3}$$

And the boundary conditions are given by

$$\left[T(x,y,z,t) + k_1 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = G_1(y,z,t) \tag{4}$$

$$\left[T(x,y,z,t) + k_2 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = G_2(y,z,t)$$

(5)

$$\left[T(x,y,z,t) + k_3 \frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=b} = G_3(x,z,t) \tag{6}$$

$$\left[T(x,y,z,t) + k_4 \frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=-b} = G_4(x,z,t) \tag{7}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=0} = f_1(x,y,t) \tag{8}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=h} = f_2(x,y,t) \tag{9}$$

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Let us introduce a thermal stress function χ related to Component of stress in the rectangular coordinates system as in [6] is

$$\chi = \chi_c + \chi_p \tag{10}$$

where χ_c is the complementary solution and χ_p is particular solution.

χ_c and χ_p are governed by equations:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_c = 0 \tag{11}$$

and

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_p = -\alpha E \Gamma . \tag{12}$$

Since plate is thin z is negligible and where $\Gamma = T - T_0$, T_0 is initial temperature.

Also component of stress functions are given by

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2} \tag{13}$$

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2} \tag{14}$$

$$\sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} \tag{15}$$

The boundary conditions are

$\sigma_{yy} = 0$, $\sigma_{xy} = 0$ at $y = b$.

Equations (1) to (15) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform two times and then finite Fourier Cosine transform to equation (2), also using given boundary conditions we get

$$-a_l^2 \overline{\overline{T}}^* - b_m^2 \overline{\overline{T}}^* - \frac{n^2\pi^2}{h^2}\overline{\overline{T}}^* + G + \frac{n\pi}{h}[(-1)^{n+1} f_2 + f_1] + \frac{1}{k} g_0 4 a_l a$$

$$\cos(a_l a)\cos(b_m b)\delta(t-t')\cos(d_n y')\cos\left(\frac{n\pi z'}{h}\right) = \frac{1}{\alpha}\frac{d\overline{\overline{T}}^*}{dt} \tag{16}$$

Solving above equation and using initial condition we get,

$$\overline{\overline{T}}^* = e^{-\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)t}$$

$$\times \left[\int \exp[\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)t \left\langle G + \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] \right.\right.$$

$$\left. \left. \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')\delta(t-t') \cos\left(\frac{n\pi z'}{h}\right) dt + \Omega \right] \right. \tag{17}$$

Where

$$\Omega = -\left[\int \exp[\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)t \left\langle G + \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] + \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')\delta(t-t') \cos\left(\frac{n\pi z'}{h}\right) \right\rangle dt \right]_{t=0}$$

And G = G (G_1, G_2, G_3, G_4)

Taking inverse finite Fourier cosine transform, and Marchi-Fasulo transform two times we get

$$T = \frac{2}{h}\sum_{l,m,n,=1}^{\infty} \frac{p_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m} \times \left\{ \left[\left[\int \exp[\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)(t-t') \left\langle G + \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] \right. \right.\right.\right.$$

$$\left.\left.\left. + \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')\delta(t-t') \times \cos\left(\frac{n\pi z'}{h}\right) dt \right\rangle \cos\left(\frac{n\pi z}{h}\right) \right\} \tag{18}$$

And

$$\Gamma = \frac{2}{h}\sum_{l,m,n,=1}^{\infty} \frac{p_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m} \times \left\{ \left[\left[\int e^{\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)(t-t')} \left\langle G + \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] \right. \right.\right.\right.$$

$$\left.\left.\left. + \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')\delta(t-t') \times \cos\left(\frac{n\pi z'}{h}\right) dt \right\rangle \cos\left(\frac{n\pi z}{h}\right) \right\} - T_0$$

IV. DETERMINATION OF STRESS FUNCTION

Let the suitable form of χ_c satisfying (11) is given by

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$$\chi_c = \sum_{l,m,n=1}^{\infty}\left\{ y^2\left[c_1 e^{\frac{n\pi x}{a}} + c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + y^2\left[c_3 e^{\frac{n\pi x}{a}} + c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\right\} \quad (19)$$

Let the suitable form of χ_p satisfying (12) is given by

$$\chi_p = \frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times\Big\{ \Big[\int \exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+ \frac{a}{k}g_0 4a_l m_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t') \sin\frac{n\pi z'}{h}\Big\rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\Big\} - T_0 \quad (20)$$

Substituting equation (19) and (20) in (10), one obtains

$$\chi = \sum_{l,m,n=1}^{\infty}\left\{ y^2\left[c_1 e^{\frac{n\pi x}{a}} + c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + y^2\left[c_3 e^{\frac{n\pi x}{a}} + c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\right\} + \frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times\Big\{ \Big[\int \exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+ \frac{a}{k}g_0 4a_l m_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t') \sin\frac{n\pi z'}{h}\Big\rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\Big\} - T_0 \quad (21)$$

Using (21) in (13), (14), (15) we get

$$\sigma_{xx} = \sum_{l,m,n=1}^{\infty}\left\{ 2\left[c_1 e^{\frac{n\pi x}{a}} + c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + 2\left[c_3 e^{\frac{n\pi x}{a}} + c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\right\} + \frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\left\{\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times \Big[\int \exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+ \frac{a}{k}g_0 4a_l m_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t') \sin\frac{n\pi z'}{h}\Big\rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\right\}_{yy} \quad (22)$$

$$\sigma_{yy} = \sum_{l,m,n=1}^{\infty}\left\{ y^2\frac{n^2\pi^2}{a^2}\left[c_1 e^{\frac{n\pi x}{a}} + c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + y^2\frac{n^2\pi^2}{a^2}\left[c_3 e^{\frac{n\pi x}{a}} + c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\right\} + \frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\left\{\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times \Big[\int \exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+ \frac{a}{k}g_0 4a_l b_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t') \sin\frac{n\pi z'}{h}\Big\rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\right\}_{xx}$$

(23)

$$\sigma_{xy} = \sum_{l,m,n=1}^{\infty}\left\{ 2y\frac{n\pi}{a}\left[c_1 e^{\frac{n\pi x}{a}} - c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + 2y\frac{n\pi}{a}\left[c_3 e^{\frac{n\pi x}{a}} - c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\right\} + \frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\left\{\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times \Big[\int \exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+ \frac{a}{k}g_0 4a_l b_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t')\right.$$

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$$\sin\frac{n\pi z'}{h}\ \rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}\Bigg]_{xy} \tag{24}$$

Using the boundary conditions $\sigma_{xy}=0, \sigma_{yy}=0$, at $y=b$ and equation (22) and (23) we get

$$C_1=-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\right]$$

$$C_2=-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\right]e^{\frac{2n\pi x}{a}}+\frac{a\gamma_2}{2bn\pi}e^{\frac{n\pi x}{a}}$$

And $C_3=C_4=0$

Where

$$\gamma_1=\frac{2\alpha Ea^2b^2}{h\left(a^2+b^2\right)}\Bigg\{\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times$$

$$\Bigg[\int\exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+$$

$$\frac{a}{k}g_0 4a_l b_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t')$$

$$\sin\frac{n\pi z'}{h}\ \rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}\Bigg]_{xx}\Bigg\}_{y=b} \tag{25}$$

$$\gamma_2=\frac{2\alpha Ea^2b^2}{h\left(a^2+b^2\right)}\Bigg\{\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times$$

$$\Bigg[\int\exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+$$

$$\frac{a}{k}g_0 4a_l b_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t')$$

$$\sin\frac{n\pi z'}{h}\ \rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}\Bigg]_{xy}\Bigg\}_{y=b} \tag{26}$$

Substituting the above values in (22) to (24), it gets

$$\sigma_{xx}=\sum_{l,m,n=1}^{\infty}\left\{2\left[\begin{array}{l}\left[-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\right]\right]e^{\frac{n\pi x}{a}}\\ +\left[-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\right]e^{\frac{2n\pi x}{a}}+\frac{a\gamma_2}{2bn\pi}\right]\end{array}\right]\right.$$

$$\sin\left(\frac{n\pi z}{h}\right)$$

$$+\frac{2\alpha Ea^2b^2}{h\left(a^2+b^2\right)}\Bigg\{\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times$$

$$\Bigg[\int\exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+$$

$$\frac{a}{k}g_0 4a_l m_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t')$$

$$\sin\frac{n\pi z'}{h}\ \rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}\Bigg]_{yy} \tag{27}$$

$$\sigma_{yy}=\sum_{l,m,n=1}^{\infty}\left\{y^2\frac{n^2\pi^2}{a^2}\left[\begin{array}{l}\left(-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\right]\right)e^{\frac{n\pi x}{a}}\\ +\left(-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\right]e^{\frac{2n\pi x}{a}}+\frac{a\gamma_2}{2bn\pi}\right)\end{array}\right]\right.$$

$$\sin\left(\frac{n\pi z}{h}\right)$$

$$+\frac{2\alpha Ea^2b^2}{h\left(a^2+b^2\right)}\Bigg\{\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times$$

$$\Bigg[\int\exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+$$

$$\frac{a}{k}g_0 4a_l b_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t')$$

$$\sin\frac{n\pi z'}{h}\ \rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}\Bigg]_{xx} \tag{28}$$

$$\sigma_{xy}=\sum_{l,m,n=1}^{\infty}\left\{2y\frac{n\pi}{a}\left[\begin{array}{l}\left(-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\right]\right)e^{\frac{n\pi x}{a}}\\ -\left(-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\right]e^{\frac{2n\pi x}{a}}+\frac{a\gamma_2}{2bn\pi}\right)\end{array}\right]\right.$$

$$\sin\left(\frac{n\pi z}{h}\right)$$

$$+\frac{2\alpha Ea^2b^2}{h\left(a^2+b^2\right)}\Bigg\{\sum_{l,m,n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m}\times$$

$$\Bigg[\int\exp[\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')\Big\langle G+\frac{n\pi}{h}\left[(-1)^{n+1}f_2+f_1\right]+$$

$$\frac{a}{k}g_0 4a_l b_m\cos(a_l a)\cos(b_m b)\cos(a_n y')\delta(t-t')$$

$$\sin\frac{n\pi z'}{h}\ \rangle dt\ \Big]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}\Bigg]_{xy} \tag{29}$$

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V. SPECIAL CASE AND NUMERICAL RESULTS

Set

$$f_1(x,y,t) = (x-a)^2 (x+a)^2 (y-b)^2 (y+b)^2 \left(T_0 e^{-t}\right)$$

$$f_2(x,y,t) = (x-a)^2 (x+a)^2 (y-b)^2 (y+b)^2 e^h \left(T_0 e^{-t}\right)$$

$G_1 = G_2 = G_3 = G_4 = 0.$

a = 2, b = 4, h = 2, t = 1sec in equation (18), we get

$$T = \sum_{l,m,n,=1}^{\infty} \frac{p_1(x)}{\lambda_1} \frac{R_m(y)}{\mu_m}$$

$$\times \Bigg\{ \Bigg[\int \exp[\alpha \left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{4} \right) (1-t') \Big\langle G + \frac{n\pi}{2} \left[(-1)^{n+1} f_2 + f_1 \right]$$

$$+ \frac{a}{k} g_0 4 a_l b_m \cos(2a_l) \cos(4b_m) \cos(d_n y') \delta(t-t')$$

$$\cos\left(\frac{n\pi z'}{2}\right) dt \Big\rangle \cos\left(\frac{n\pi z}{2}\right) \Bigg\} \tag{30}$$

VI. CONCLUSION

In this paper, the temperature distribution and thermal stresses have been found by using the finite Marchi-Fasulo transform and the finite Fourier cosine transform with moving heat point source. The results are obtained in the form of infinite series.

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