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A different approach on a Pythagorean triangle which satisfies

$$a(Hypotonuse)-4a\frac{(Area)}{(Perimeter)}=\alpha^2$$

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***Abstract:* We obtain non-trivial integral values for the sides f the Pythagorean triangle such that its $a(Hypotonuse)-4a\frac{(Area)}{(Perimeter)}=\alpha^2$. A few interesting relations between the sides of the Pythagorean triangle are presented.**

***Key words:* Integral solutions, Pythagorean triangles, MSC classification number 11D09.**

I. INTRODUCTION

One well known set of solutions of the Pythagorean equation $x^2+y^2=z^2$ are $x=2ab,\ \ y=a^2-b^2\ and\ z=a^2+b^2$. Many mathematicians has been used this set of solutions to obtain the non-zero integral values for x,y and z [1-3]. As a new approach, in this paper we introduce another set of solutions

$$x=2A+1,\ \ y=2A^2+2A\ and\ z=2A^2+2A+1$$

for the equation $x^2+y^2=z^2$. By using this solution we obtain three non-zero integers x,y and z under certain relations satisfying the equation $x^2+y^2=z^2$ [4-6]. In this communication, we present yet another interesting Pythagorean triangle where in each of which the ratio $a(Hypotonuse)-4a\frac{(Area)}{(Perimeter)}$ may be expressed as a perfect square.

II. METHOD OF ANALYSIS

Taking A>0 to be the generators of the Pythagorean triangle (x,y,z), the assumption that $a(Hypotonuse)-4a\frac{(Area)}{(Perimeter)}=\alpha^2$ leads to the Pellian equation $Y^2=DX^2+a$ where D = 2a, not a perfect square and A = X.

For the clear understanding we consider the following two cases:

i) a = 3 (odd number) so that D = 6

ii) a = 4 (even number) so that D = 8

Case (i):

When a = 3 the equation

$$Y^2=DX^2+a \qquad (1)$$

becomes

$$Y^2=6X^2+3 \qquad (2)$$

Let $(x_0,y_0)=(1,3)$ be the initial solution of (2). Consider the Pellian

$$Y^2=6X^2+1 \qquad (3)$$

Let $(\tilde{x}_0,\tilde{y}_0)=(2,5)$ be a solution of equation (3).

n	A_{n+1}	Y_{n+1}
-1	1	3
0	11	27
1	109	267
2	1079	2643
3	10681	26163

Using Brahmagupta lemma the general solution $(\tilde{x}_n,\tilde{y}_n)$ of equation (3) is given by

$$\tilde{y}_n+\sqrt{6}\tilde{x}_n=\left[5+2\sqrt{6}\right]$$

Where n = 0,1,2,3….. (4)

Since irrational roots occur in pairs

$$\tilde{y}_n-\sqrt{6}\tilde{x}_n=\left[5-2\sqrt{6}\right]$$

Where n = 0,1,2,3… (5)

is also a solution.

From equations (4) and (5), we obtain

$$\tilde{y}_n=\frac{1}{2}\left[\left(5+2\sqrt{6}\right)^{n+1}+\left(5-2\sqrt{6}\right)^{n+1}\right] \qquad (6)$$

and

$$\tilde{x}_n=\frac{1}{2\sqrt{6}}\left[\left(5+2\sqrt{6}\right)^{n+1}-\left(5-2\sqrt{6}\right)^{n+1}\right]$$

(7)

Using the equations (6) and (7), the solutions of equation (2) is given by

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$$A_{n+1} = X_{n+1} = \frac{1}{2\sqrt{6}}\left[\left(3+\sqrt{6}\right)\left(5+2\sqrt{6}\right)^{n+1} - \left(3-\sqrt{6}\right)\left(5-2\sqrt{6}\right)^{n+1}\right]$$

n = -1,0,1,2,3…

$$Y_{n+1} = \frac{1}{2}\left[\left(3+\sqrt{6}\right)\left(5+2\sqrt{6}\right)^{n+1} - \left(3-\sqrt{6}\right)\left(5-2\sqrt{6}\right)^{n+1}\right]$$

n = -1,0,1,2,3…

III. NUMERICAL EXAMPLES

Observations:

1. The Recurrence relations for X and Y are $X_{n+3} - 10X_{n+2} + X_{n+1} = 0$ and $Y_{n+3} - 10Y_{n+2} + Y_{n+1} = 0$.
2. For all values of n, both X and Y are odd.
3. For all values of n, Y_{n+1} is divisible by 3.
4. $24X_{n+1}Y_{n+1}$ is difference of two squares.
5. $X_{n+3} + X_{n+1} \equiv 0 \pmod{10}$ and $Y_{n+3} + Y_{n+1} \equiv 0 \pmod{10}$

Case (ii):

When a = 4, the equation (1) leads to

$$Y^2 = 8X^2 + 4 \quad (8)$$

Let $(x_0, y_0) = (2,6)$ be initial solution of equation (8).

To obtain the general solution of (8) consider the Pellian

$$Y^2 = 8X^2 + 1 \quad (9)$$

Let $(\tilde{x}_0, \tilde{y}_0) = (1,3)$ be the initial solution of equation (9)

Then the general solution of equation (9) is given by

$$\tilde{y}_n = \frac{1}{2}\left[\left(3+\sqrt{8}\right)^{n+1} + \left(3-\sqrt{8}\right)^{n+1}\right]$$

and

$$\tilde{x}_n = \frac{1}{2\sqrt{8}}\left[\left(3+\sqrt{8}\right)^{n+1} - \left(3-\sqrt{8}\right)^{n+1}\right]$$

n = 0,1,2,3,…

Therefore, the general solution of equation (8) is

$$A_{n+1} = X_{n+1} = \frac{1}{\sqrt{8}}\left[\left(3+\sqrt{8}\right)^{n+1} - \left(3-\sqrt{8}\right)^{n+1}\right]$$

$$Y_{n+1} = \left[\left(3+\sqrt{8}\right)^{n+1} + \left(3-\sqrt{8}\right)^{n+1}\right]$$

n = 0,1,2,3…

Numerical Examples

n	A_{n+1}	Y_{n+1}
0	2	6
1	12	34
2	70	198
3	408	1154
4	2378	6726

Observations:

1. The recurrence relations for X and Y are $X_{n+3} - 6X_{n+2} + X_{n+1} = 0$ and $Y_{n+3} - 6Y_{n+2} + Y_{n+1} = 0$
2. For all values of n, both X_{n+1} and Y_{n+1} are even.
3. For all values of n, Y_{n+1} is divisible by 3.
4. $X_{n+3} + X_{n+1} \equiv 0 \pmod{6}$ and $Y_{n+3} + Y_{n+1} \equiv 0 \pmod{6}$

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