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Fuzzy Regression with Applications in Hydrology

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Abstract*— Linear correlation analysis of rainfall station measurements is used in order to depict the relation between a dependent variable (usually the meteorological station with the shortest data-recording time span) and independent variables (neighboring stations with long recording time span). Contrary to classical linear regression where the difference between observation values and estimated values ε_j is considered to be caused by observation errors (random variable), new regression models, based on Fuzzy Logic, have been introduced. In these models, the difference is attributed to the inherent fuzziness of the system and the fuzzy input and output data. In this paper, we attempt to correlate to a trapezoidal membership function model and present two applications: a) the Bisserier case and b) the case of rainfall measurement stations of Aggistro and Upper Vrontou located in Serres, Greece, where input (observational data) is considered to be crisp whereas output (observational data) and model parameters are fuzzy.

***Index Terms*— Fuzzy, membership, rainfall measurement, regression, trapezoidal.**

I. INTRODUCTION

Rainfall measurement models have been extensively used in the design process of water resource projects such as hydrological prediction, spillway design, climatic change studies, rainfall and runoff correlation etc. Rainfall measurements in a specific area are commonly displayed in the form of time series where recorded values can be either continuous or discrete. In many instances, there is a correlation between rainfall time series that belong to different stations and comprise measurements with differing range. E.g. there is an available time series of 15 years for station A and a time series of 30 years for station B. Due to the correlation between them, we can fill in the missing values, in order to extend the shorter time series.

Correlation analysis is used to depict the relation between the dependent variable (usually the meteorological station with the shortest data-recording time span) and the independent variables (neighboring stations with long recording time span.) For this correlation, a multiple linear regression model is used:

$$\mathrm{Y}_i = a_0 + \sum_{j=1}^{n} a_j X_{ij} + \varepsilon_i,$$

$$E(\varepsilon_i) = 0,\ \ \sigma^2(\varepsilon_i) = \sigma^2,\ \ \mathrm{cov}(\varepsilon_i, \varepsilon_j) = 0$$

In classical linear regression, the difference εi between measurement values and estimated values, is a random variable with normal distribution and is considered to be caused by measurement errors. Upper and lower bounds of the estimated value are calculated and the probability that the estimated value will lie between them represents the estimation confidence. According to this, classical regression is considered to be probabilistic and has many uses but can be rendered problematic if the data set is small, if it's hard to prove that error distribution is normal, if there is fuzziness between dependent and independent variables or if linearity acceptance is not proper[13].

Nowadays, new regression models have been introduced, based on fuzzy logic ([10]-[11]-[15]-[16]-[17]-[18] -[19]-[20]). In fuzzy regression the difference between measurement values and estimated values is attributed to the inherent fuzziness of the system as well as to the fuzziness of input and output data. In contrast with classical regression analysis, fuzzy regression analysis uses fuzzy functions for the regression factors. References [12]- [18] usually meet one of the three cases, described below:

- Crisp input values x_{ij} and output values y_j
- Crisp input values x_{ij} and fuzzy output values $\tilde{y}_i$
- Fuzzy input values $\tilde{x}_{ij}$ and fuzzy output values $\tilde{y}_i$

In all of these cases, estimated values $\tilde{\mathrm{Y}}_i$ are fuzzy.
The adjustment of a fuzzy regression model can be achieved through two general methods:

- The **possibilistic** model ([12]-[13]-[15]-[16], etc). Fuzzy regression is possibilistic and the membership function $\mu_{\tilde{F}}$ of a fuzzy number $\tilde{F}$ is considered equal to the possibility distribution function πx(x). The fuzziness of the model is minimized by taking into account the minimum of the spreads around the center of the fuzzy parameters, while considering that the experimental values of every sample are within a specific interval of possible values.
- The **least squares** model ([4]-[9]-[21]). The distance between the estimated output value of the model $\tilde{\mathrm{Y}}_i$ and the observed output value $\tilde{y}_i$ is minimized. This method of [9] is considered to be an extension of the classical linear regression method, based on the notion of model efficiency

optimization depending on data.

In this article, a possibilistic model is described, where membership functions are trapezoidal, measured input values are crisp and measured output values are fuzzy and triangular. The need to use trapezoidal functions is a result of the following reasons [2]:

- We need to optimize the fuzziness of the model.
- We need to restrict experimental data inside the estimated value range.

In order to achieve the restriction, a-cuts are used and we aim to restrict for a level of confidence h=α0 that is high enough. However, that could lead to highly inaccurate parameters. Moreover, solution optimization for level h does not guarantee the same for another level $h' \neq h$. Reference [17] provided the equations for this scenario but they can only be applied when data is crisp, thus, restrictions of high confidence levels interfere with model precision. Moreover, constraint for confidence level h=1 and data with triangular membership functions is impossible with the exception of the special case of collinear data.

Using trapezoidal membership functions for estimated values ([1]-[2]-[3]) allows us to achieve inclusion for output data with triangular membership functions $\tilde{y}_j$ and estimated values with trapezoidal membership functions $\hat{Y}_j$, for confidence level h=1, for which the kernel is not minimized in a point: $[\tilde{y}_j]_{h=1} \subseteq [\hat{Y}_j]_{h=1}$. In addition, for a level of confidence *h=0*, we can achieve inclusion: $[\tilde{y}_j]_{h=0} \subseteq [\hat{Y}_j]_{h=0}$. Due to the linearity of the membership function, inclusion for those levels of confidence allows us to ensure that inclusion is possible for every level of confidence:

Trapezoidal membership function models have been used by: [1]-[2]-[5]-[6]-[7]-[8]. Reference [5] extended Tanaka's method for the case of trapezoidal membership functions with crisp measured input and output values. She used a fuzzy level function $\tilde{f}$, with four crisp level functions f_a, f_b, f_c, f_d. For f_a, f_d she used Tanaka's method [16], whereas for f_b, f_c she used classical linear regression, which led to function $\hat{f}$ and standard deviation σ of the model. Based on this, f_b, f_c, become: $f_b = \hat{f} - \lambda\sigma,\ f_c = \hat{f} + \lambda\sigma,$ with λ being an adjustment factor.

References [1]-[2]-[3] used trapezoidal functions $(S_\ell, K_\ell, K_r, S_r)$ where S_ℓ, S_r are the supports and K_ℓ, K_r are the kernels for a level of confidence *h=0*, with crisp measured input and fuzzy measured output values. Triangular functions were used for the measured output values. Moreover, they introduced into these intervals the median and radius as follows:

$$M_S = (S_\ell + S_r)/2,\ R_S = (S_r - S_\ell)/2,$$

$$M_K = (K_\ell + K_r)/2, \quad R_K = (K_r - K_\ell)/2.$$

They utilized the method of [15], by applying interval inclusion between measured and estimated values and, in order to minimize fuzziness, they used the area of the trapezium. A quite different and rather simple two-phase method was used, in this paper, with one crisp measured input value and one fuzzy measured output value. During phase A, measured input and output values were considered crisp, while parameters were considered fuzzy and supports were estimated using Tanaka's method. In phase B, estimated supports were considered to be the known kernel of the trapezoidal membership functions and in the inclusion they were transferred to the known terms. Supports for the trapezoidal membership functions of the estimation were calculated. Triangular membership functions were used for measured output values.

II. MATHEMATICAL MODEL

A. *Bisserier theory*

In the general case of trapezoidal membership functions, the estimated value is given as follows:

$$\hat{Y}_j = \tilde{A}_0 + \tilde{A}_1 x_{1j} + \tilde{A}_2 x_{2j} + \ldots + \tilde{A}_n x_{nj} = \sum_{i=0}^{n} \tilde{A}_i x_{ij}, \quad x_{0j} = 1 \quad (1)$$

Where:

$$\tilde{A} = ([K_A^-, K_A^+],[S_A^-, S_A^+])$$

$$K_{\tilde{A}} = \mathrm{kernel}(\tilde{A}) = [K_A^-, K_A^+]$$

Based on the above, median and radius are:

Kernel:

$$M_{K_{\tilde{A}}} = (K_A^- + K_A^+)/2,\ R_{K_{\tilde{A}}} = (K_A^+ - K_A^-)/2$$

Supports:

$$M_{S_{\tilde{A}}} = (S_A^- + S_A^+)/2,\ R_{S_{\tilde{A}}} = (S_A^+ - S_A^-)/2$$

In the case of triangular membership function, the following apply:

Kernel: k. In case of symmetry $k = k_A$

Supports:

$$M_{S_{\tilde{A}}} = (S_A^- + S_A^+)/2 = k_A,\ M_{S_{\tilde{A}}} = (S_A^- + S_A^+)/2,$$

$$R_{S_{\tilde{A}}} = (S_A^+ - S_A^-)/2.$$

Based on the above we get the following equations:

$$M_{\tilde{Y}_j} = \sum_{i=0}^{n} M_{A_i} x_{ij},\ \ j = 1,\ldots,M,\ x_{0j} = 1 \quad (2)$$

$$R_{\tilde{Y}_j} = \sum_{i=0}^{N} R_{A_i} |x_{ij}|, \quad j = 1,\ldots,M,\ x_{0j} = 1 \quad (3)$$

The trapezium area is defined as:

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$$\text{area}(\tilde{Y}(x_j)) = (K^+_{\tilde{Y}_j} + S^+_{\tilde{Y}_j})/2 - (K^-_{\tilde{Y}_j} + S^-_{\tilde{Y}_j})/2 = R_{K_{\tilde{Y}_j}} + R_{S_{\tilde{Y}_j}} \quad (4)$$

Therefore, the trapezium area is a function of the radius of median points $R_{K_{\tilde{Y}_j}}$ and the radius of supports $R_{S_{\tilde{Y}_j}}$.

Fuzziness minimization is given by the trapezium area:

$$J_{2Trap}(R_{K_A}, R_{S_A}) = \sum_{j=1}^{M} \text{area}(\tilde{Y}(x_j))$$

$$= M \cdot (R_{S_{A_0}} + R_{K_{A_0}}) + (R_{S_{A_1}} + R_{K_{A_1}}) \cdot \sum_{j=1}^{M} |x_j| \quad (5)$$

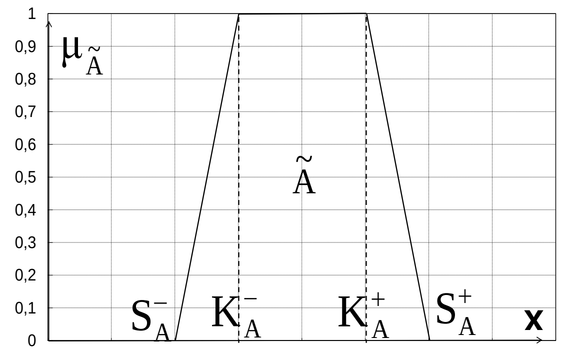

Fig.1. Trapezoidal membership function

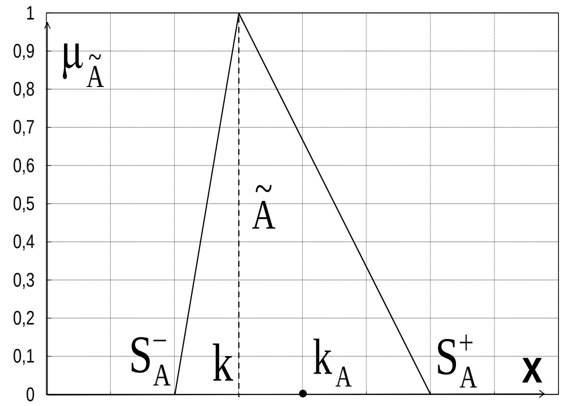

Fig. 2. Triangular membership function

Constraints

Kernel Inclusion: measured values are considered triangular numbers $\tilde{y}_j = (y_\ell, y_m, y_r)$ while estimated values are considered trapezoidal $\hat{Y}_j = (S^-, K^-, K^+, S^+)$.

$$k_{\tilde{y}_j} \in [K^-_{\hat{Y}_j}, K^+_{\hat{Y}_j}] \Leftrightarrow \left| M_{K_{\hat{Y}_j}} - k_{\tilde{y}_j} \right| \le R_{K_{\hat{Y}_j}} \quad (6)$$

Support Inclusion

$$[S^-_{Y_j}, S^+_{Y_j}] \subseteq [S^-_{\hat{Y}_j}, S^+_{\hat{Y}_j}] \Leftrightarrow \left| M_{S_{\hat{Y}_j}} - M_{S_{\tilde{y}_j}} \right| \le R_{S_{\hat{Y}_j}} - R_{S_{\tilde{y}_j}}$$

(7)

Kernel Inclusion inside Supports

$$[K^-_{A_i}, K^+_{A_i}] \subseteq [S^-_{A_i}, S^+_{A_i}] \Leftrightarrow \left| M_{S_{A_i}} - M_{K_{A_i}} \right| \le R_{S_{A_i}} - R_{K_{A_i}} \quad (8)$$

Problem synthesis

$$\text{Minimise}\, J_{2Trap}(R_{K_A}, R_{S_A}) \quad (9)$$

s.t.

Kernel Inclusion

$$\sum_{i=0}^{1} M_{K_{A_i}} x_{ij} - \sum_{i=0}^{1} R_{K_{A_i}} x_{ij} \le k_{Y_j} \le$$
$$\le \sum_{i=0}^{1} M_{K_{A_i}} x_{ij} + \sum_{i=0}^{1} R_{K_{A_i}} x_{ij}, \quad \forall j,\ j = 1,\ldots, M \quad (10)$$

Support Inclusion

$$\sum_{i=0}^{n} M_{S_{A_i}} x_{ij} - \sum_{i=0}^{n} R_{S_{A_i}} x_{ij} \le M_{S_{Y_j}} - R_{S_{Y_j}}, \quad \forall j,\ j = 1,\ldots, M$$
$$\sum_{i=0}^{n} M_{S_{A_i}} x_{ij} + \sum_{i=0}^{n} R_{S_{A_i}} x_{ij} \ge M_{S_{Y_j}} + R_{S_{Y_j}}, \quad \forall j,\ j = 1,\ldots, M \quad (11)$$

Kernel Inclusion inside Supports

$$M_{S_{A_i}} - M_{K_{A_i}} \le R_{S_{A_i}} - R_{K_{A_i}}, \quad i = 0,1$$
$$-M_{S_{A_i}} + M_{K_{A_i}} \le R_{S_{A_i}} + R_{K_{A_i}}, \quad i = 0,1 \quad (12)$$

B. New model

The above problem can be divided into the following two steps:

Step 1 According to the kernel inclusion constraint mentioned above, we have:

$$k_{\tilde{y}_j} \in [K^-_{\tilde{Y}_j}, K^+_{\tilde{Y}_j}] \quad (13)$$

Namely the kernel of experimental values is within the kernel of estimated values. According to [14], for the case of crisp output values (only the kernels of measured values,) if we apply Tanaka's method [16], the range $[\tilde{Y}^\ell, \tilde{Y}^u]^{h=0}$ encircles the kernels of experimental values. In this stage, only triangular functions $\tilde{A}_0(r_0, c_0), \tilde{A}_1(r_1, c_1)$ are applied for the method of Tanaka and the possibilistic model used is: $\hat{Y}_j = \tilde{A}_0 + \tilde{A}_1 x_{1j}$. Thus, the problem of determining the estimated values is:

$$\min(c) = mc_0 + c_1 \sum_{i=1}^{n} x_{1j},\ c_0, c_1 \ge 0 \quad (14)$$

s.t.

$$\text{a)} \quad \tilde{Y}^u = \sum_{i=0}^{1} r_i x_{ij} + \sum_{i=0}^{1} c_i x_{ij} \ge y_j,$$
$$\text{b)} \quad \tilde{Y}^\ell = \sum_{i=0}^{1} r_i x_{ij} - \sum_{i=0}^{1} c_i x_{ij} \le y_j,\ x_{0j} = 1. \quad (15)$$

Through the solution of this system we get the surroundings $[\tilde{Y}^\ell, \tilde{Y}^u]^{h=0}$ as follows:

$$\text{a)} \quad \tilde{Y}^u = (r_0 + c_0) + (r_1 + c_1)x, \quad (16)$$
$$\text{b)} \quad \tilde{Y}^\ell = (r_0 - c_0) + (r_1 - c_1)x.$$

As long as they meet the same constraint, these surroundings coincide with the kernel of trapezoidal functions, resulting in the relations below:

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$$\tilde{Y}^{\ell}=K^{-}{}_{\tilde{Y}_j}=K^{-}{}_{\tilde{A}_0}+K^{-}{}_{\tilde{A}_1}x_{1j},\ j=1,\dots,M,$$
$$\tilde{Y}^{r}=K^{+}{}_{\tilde{Y}_j}=K^{+}{}_{\tilde{A}_0}+K^{+}{}_{\tilde{A}_1}x_{1j},\ j=1,\dots,M. \quad (17)$$

Step 2 We apply support inclusion:

$$[S^{-}_{\tilde{y}_j}, S^{+}_{\tilde{y}_j}]\subseteq[S^{-}_{\tilde{Y}_j}, S^{+}_{\tilde{Y}_j}], \quad (18)$$

where space $[S^{-}_{\tilde{y}_j}, S^{+}_{\tilde{y}_j}]$ is given as follows:

$$[S^{-}_{\tilde{y}_j}=\tilde{y}_j-e_j=\tilde{y}^{\ell}_j,\ S^{+}_{\tilde{y}_j}=\tilde{y}_j+e_j=\tilde{y}^{r}_j] \quad (19)$$

Based on relations (15) and (19), we get:

$$K^{-}{}_{\tilde{Y}_j}-\sum_{i=1}^{2}c_i^{\ell}x_{ij}\le\tilde{y}_j-e_j,\quad x_{1j}=1 \quad (20)$$

$$K^{+}{}_{\tilde{Y}_j}+\sum_{i=1}^{2}c_i^{r}x_{ij}\ge\tilde{y}_j+e_j,\quad x_{1j}=1,\ j=1,2,\dots M \quad (21)$$

Where $K^{+}{}_{\tilde{Y}_j}$ and $K^{-}{}_{\tilde{Y}_j}$ are known since they have been calculated during step 1.
The problem turns now into:

$$\min(c)=m(c_0^{\ell}+c_0^{r})+(c_1^{\ell}+c_1^{r})\sum_{j=1}^{n}x_{1j}, \quad (22)$$
$$c_0^{\ell},c_0^{r},c_1^{\ell},c_{11}^{r}\ge 0,\ .$$

s.t.

$$K^{-}{}_{\tilde{Y}_j}-\sum_{i=1}^{2}c_i^{\ell}x_{ij}\le\tilde{y}_j-e_j,\quad x_{0j}=1$$
$$j=1,2,\dots M \quad (23)$$
$$K^{+}{}_{\tilde{Y}_j}+\sum_{i=1}^{2}c_i^{r}x_{ij}\ge\tilde{y}_j+e_j,\quad x_{0j}=1$$

III. APPLICATIONS

A. Application 1

Given the following experimental data [2] with symmetric fuzzy numbers, we want to assess using the fuzzy regression method, considering trapezoidal membership functions for the assessment.

Table I. Crisp input, fuzzy output data [2]

a/a	x	y	y_l	y_r	M_k	R_s
1	0.1	2.25	1.5	3,00	2.25	0.750
2	0.2	2.88	2,0	3.75	2.88	0.875
3	0.3	2.50	1.5	3.50	2.50	1.000
4	0.4	4.25	2.5	6,00	4.25	1.750
5	0.5	4,00	2.5	5.50	4,00	1.500
6	0.6	5.25	4,0	6.50	5.25	1.250
7	0.7	7.50	5.5	9.50	7.50	2.000
8	0.8	8.50	7,0	10,00	8.50	1.500

By applying the Bisserier theory, we get Fig. 3.

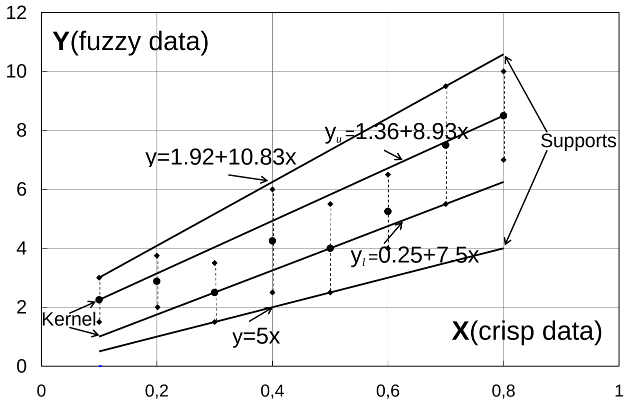

Fig.3. Estimated fuzzy output data [2].

By applying the suggested 2-step method, we get Fig. 4.

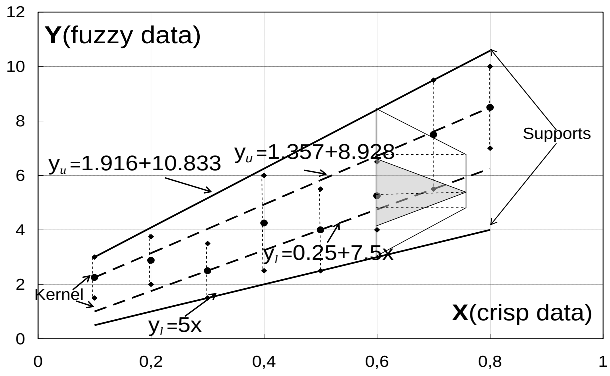

Fig.4. Estimated fuzzy output.

In Fig. 3 and 4, lower lines (Kernel and Support) coincide. For the suggested 2-step method, the upper lines (kernel $y_u=1.357+8.928x$ and straight line of Bisserier $y_u=1.36+8.93x$) differ in the second decimal digit. We apply the distance between measured $\tilde{y}=(y_\ell,y_m,y_r)$ and estimated $\tilde{Y}(S^-,K^-,K^+,S^+)$ values as suggested by [9]:

$$d^2(\tilde{y},\tilde{Y})=(y_m-K^-)^2+(y_m-K^+)^2+ (y_\ell-S^-)^2+(y_r-S^+)^2 \quad (24)$$

This distance is shown on Fig. 5, for both the Bisserier and the suggested method

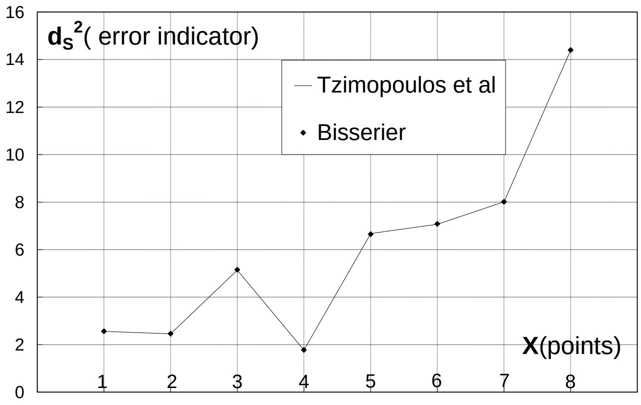

Fig.5. Distance d_s^2 vs x.

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The new method almost coincides with Bisserier method, and the following quantitative error indicator is introduced:

$$Distance=\sum_{j=1}^{M} d^2(\tilde{y},\tilde{Y}) \qquad (25)$$

Quantitative difference between the two methods is shown on Table II.

Table II. Distance between the two methods.

	Tzimopoulos et al	Bisserier
Distance	48,0894	48,080

B. *Application 2*

We consider rainfall measurement stations of Aggistro and Upper Vrontou with the following data:

Table III. Input crisp-output fuzzy data.

T	1929-1930	1930-1931	1931-1932	1932-1933	1933-1934	1934-1935
Aggistro-X	47.6	65.6	40.3	30.8	45.1	54.1
Ano Vrontou-Y	65.8	118.9	57,0	52.5	61.1	73.9
e	13.1	23.7	11.4	10.5	12.2	14.7

T	1935-1936	1936-1937	1937-1938	1938-1939	1939-1940	1940-1941
Aggistro-X	58.7	60.6	55.1	45.9	54.7	47.5
Ano Vrontou-Y	104.4	83.1	78.8	79.0	106.5	77.5
e	20.8	16.6	15.7	15.8	15.9	15.5

Phase 1

We apply Tanaka's model, considering that output data are crisp. For $h=0$, the model is written as follows:

$$\min(12c_0+606c_1)$$

s.t.

$$r_0-c_0+47.6r_1-47.6c_1\le 65.8$$

....

$$r_0-c_0+47.5r_1-47.5c_1\le 77.5$$

$$r_0+c_0+47.6r_1+47.6c_1\ge 65.8$$

....

$$r_0+c_0+47.5r_1+47.5c_1\ge 77.5$$

Solving this problem we get kernel equations:

$$y_\ell=-2.9129+1.419355x,\quad y_u=-2.9129+2.000236x$$

Phase 2

In phase 2, the kernel is considered to be known and the model is written as follows:

$$\min(12c_0^\ell+606c_1^\ell+12_0^u+606c_1^u)$$

s.t.

$$-c_0^\ell-47.6c_1^\ell\le -11.998$$

....

$$-c_0^\ell-47.5c_1^\ell\le -2.496$$

$$c_0^u+47.6c_1^u\ge 0$$

....

$$c_0^u+47.5c_1^u\ge 0.9016$$

Second members of inequalities are now equal as shown below:

$$y_j-e_j-\sum_{i=0}^{1} r_i xij=y_j-e_j-(-2.9129+1.419355x_j),$$

$$y_j+e_j-\sum_{i=0}^{1} r_i xij=y_j-e_j-(-2.9129+2.000235x_j).$$

In Figure 6, calculation results are shown, while support equations are:

$$y_\ell=-2.9129+1.145262x,\quad y_u=-2.9129+2.292283x$$

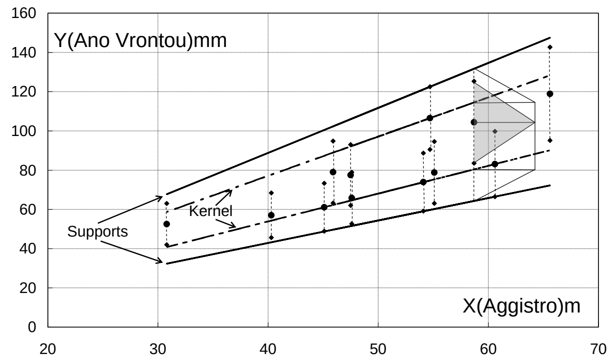

Fig.6. Estimated fuzzy output and data.

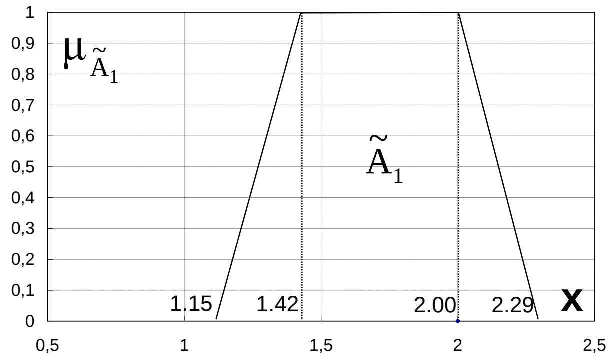

Fig. 7. Trapezoidal function $\tilde{A}_1$.

Trapezoidal function $\tilde{A}_1$ is shown in Fig. 7 with $\tilde{A}_0$ being a crisp number, while in Fig, 8, distance d_s^2 is displayed.

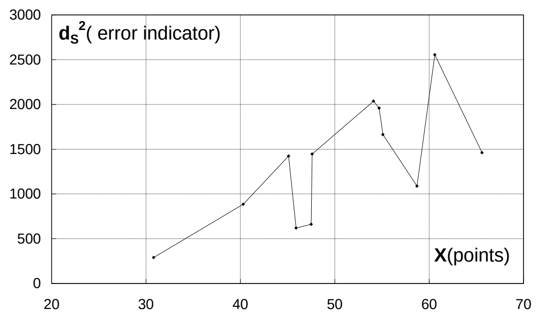

Fig. 8. Distance d_s^2 vs x.

IV. COMMENTS-CONCLUSIONS

The new 2-phase model has the advantage of using only four unknown quantities during each phase, in contrast with Bisserier model that uses eight. Arithmetic results of the Bisserier and the suggested model converge and the difference between quantitative error indicators is close to 0.0002.

The new model can be applied for all confidence levels h and has the following property: data kernels are included into estimated kernels and data supports are also included into estimated supports. In the case of rainfall measurement observations, station association is achieved, even for small samples and we can extend the shorter time series, due to fuzzy correlation of two rainfall stations.

REFERENCES

[1] A. Bisserier, R. Boukezzoula, and S. Galichet, "Linear Fuzzy Regression Using Trapezoidal Fuzzy Intervals," Proceedings of IPMU'08, Torremolinos (Málaga), pp. 181–188, 2008.

[2] A. Bisserier, "Une approche paramétrique de la régression linéaire floue-Formalisation par intervalles," Thèse de Docteur, Université de Savoie, France, p. 172, 2010.

[3] A. Bisserier, R. Boukezzoula, and S. Galichet, "Linear Fuzzy Regression Using Trapezoidal Fuzzy Intervals," Journal of Uncertain Systems, vol.4, no.1, pp. 59-72, 2010.

[4] P.-T. Chang, and E.S. Lee, "A generalized fuzzy least-squares regression," Fuzzy Sets and Systems, vol. 82, pp. 289-298, 1996.

[5] S. Charfeddine, "Optimisation de l'offre d'une compagnie aérienne en environnement incertain," Thèse de docteur, Université de Toulouse, France, p. 188, 2004.

[6] S. Charfeddine, F. Mora-Camino, and M. De Coligny, "Fuzzy linear regression : application to the estimation of air transport demand," International Conference on Fuzzy Sets and Soft Computing in Ecomomics and Finance, Saint-Petersburg, Russia, 2004.

[7] S. Charfeddine, K. Zbidi, F. Mora- Camino, "Fuzzy Regression Analysis using Trapezoidal Fuzzy Numbers," EUSFLAT - LFA 2005, pp. 1213-1218. 2005.

[8] S. Charfeddine, F. Mora-Camino, and M. De Coligny, "Fuzzy linear regression: application to the estimation of air transport demand, HAL Id: hal-01022443, pp. 350-359, 2014.

[9] P. Diamond, "Higher level fuzzy numbers arising from fuzzy regression models," Fuzzy Sets and Systems, vol. 36 , pp. 265-275, 1990.

[10] B. Papadopoulos, and M. Sirpi, "Similarities in Fuzzy Regression Models," Journal of Optimization Theory and Applications, vol. 102, no. 2, pp. 373-383, 1999.

[11] B. Papadopoulos, and M. Sirpi, "Similarities and Distances in Fuzzy Regression Models," Soft Computing, vol. 8, no. 8, pp. 556-561, 2004.

[12] D. T. Redden and W. H. Woodall, "Further examination of fuzzy linear regression," Fuzzy Sets and Systems, vol. 79, pp. 203-211, 1996.

[13] D. A. Savic, and W. Pedrycz, "Evaluation of fuzzy linear regression models," Fuzzy Sets and Systems, vol. 39, pp. 51-63, 1991.

[14] A. F. Shapiro, T. R. Berry-Stölzle, and M.-C. Koissi, "The Fuzziness in Regression Models," ARC2009_Shapiro_06. pdf, p. 6, 2009.

[15] H. Tanaka, S. Uejima, and K. Asai, "Linear regression analysis with fuzzy models," IEEE Trans. Sys. Man and Cyber, vol. 12, no. 6, pp. 903-907, 1982.

[16] H. Tanaka, "Fuzzy data analysis by possibilistic linear models," Fuzzy Sets and Systems, vol. 24, pp. 363-375, 1987.

[17] H. Tanaka, and J. Watada, "Possibilistic linear systems and their application to the linear regression model," Fuzzy Sets and Systems, vol. 27, pp.275-289, 1988.DOI: htp://dx.doi.org/10.1016/0165-0114 (88)90054-1

[18] H. Tanaka, I. Hayashi, and J. Watada, "Possibilistic linear regression analysis for fuzzy data," European Journal of Operational Research, vol. 40, pp. 389-396,1989. DOI: 10.1016/S0898-221(03)90006-X

[19] H. Tanaka, and H. Ishibuchi, "Identification of possibilistic linear systems by quadratic membership functions of fuzzy parameters," Fuzzy Sets and Systems, vol. 41, pp. 145-160, 1991. DOI:10.1016/0165-0114(91)90218-F

[20] C. Tzimopoulos, and B. Papadopoulos, "Fuzzy logic with application in Engineering,"(Greek), Eds. Ziti, p. 627, 2013.

[21] M.-S. Yang, and H.-H. Liu, "Fuzzy least-squares algorithms for interactive fuzzy linear regression models, "Fuzzy Sets and Systems, vol. 135, pp. 305-316, 2003.DOI:10.1016/S0165 -0114(02)00123-9.

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