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Thermal Deflection of an Annular Disc due to Heat Generation

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Abstract-In this paper, an attempt has been made to study thermoelastic response of an annular disc occupying the space $D: a \le r \le b,\ -h \le z \le h,$ ***due to heat generation with radiation type boundary conditions. Here we apply integral transform techniques to find the thermoelastic solution.***

Keywords: **Thermo elastic problem, annular disc, Thermal deflection, integral transform.**

I. INTRODUCTION

Khobragade et al. [3-11] have investigated temperature distribution, displacement function, and stresses of a thin as well as thick Circular Plate.

This paper deals with the determination of thermal deflection due to internal heat generation within it. The disc is considered having arbitrary initial temperature and subjected to radiation type boundary conditions which are fixed at $(r = a)$ and $(r = b)$. The non-homogeneous type boundary conditions are maintained on plane surfaces of the disc. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in series form in terms of Bessel's functions. The results for thermal deflection have been computed numerically and are illustrated graphically.

II. STATEMENT OF THE PROBLEM

Consider the disc of thickness $2h$ occupying the space D defined by $a \le r \le b;\ -h \le z \le h$. The disc is considered having arbitrary initial temperature and subjected to radiation type boundary conditions which are fixed at $(r = a)$ and $(r = b)$. The non homogeneous type boundary conditions are maintained at plane surfaces of the disc. For time $t > 0$, heat is generated within the disc at the rate $g(r,z,t)$. Under these conditions the thermal deflections in the disc due to heat generation are required to be determined.

The differential equation satisfying the deflection function $\omega(r,t)$ is given by **[2]**

$$\nabla^4 \omega = \frac{\nabla^2 M_T}{D(1-\nu)} \tag{1}$$

where M_T is the thermal moment of the plate defined as

$$M_T = a_t E \int_{-h}^{h} T(r,z,t)\, z\, dz \tag{2}$$

D is the flexural rigidity of the disc denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{3}$$

a_t, E and ν are the coefficients of the linear thermal expansion, Young's modulus and Poisson's ratio of the disc material respectively and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{4}$$

Since the edge of an annular disc is fixed and clamped,

$$\omega = \frac{\partial \omega}{\partial r} = 0, \text{ at } r = a, b \tag{5}$$

The temperature distribution $T(r,z,t)$ of the plate at time t satisfies the differential equation as [**2**]

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{6}$$

with initial and boundary conditions

$$T(r,z,0) = F(r,z) \tag{7}$$

$$T + k_1 \frac{\partial T}{\partial r} = G_1(z,t) \text{ at } r = a,\ t > 0 \tag{8}$$

$$T + k_2 \frac{\partial T}{\partial r} = G_2(z,t) \text{ at } r = b,\ t > 0 \tag{9}$$

$$T + k_3 \frac{\partial T}{\partial z} = f_1(r,t) \text{ at } z = -h,\ t > 0 \tag{10}$$

$$T + k_4 \frac{\partial T}{\partial z} = f_2(r,t) \text{ at } z = h,\ t > 0 \tag{11}$$

where k_1, k_2, k_3 and k_4 are radiation constants on curved surfaces
and plane surfaces of the disc respectively and α is thermal diffusivity of the material of the disc.

The differential equation governing the displacement function *U(r,z)* as [**1**] is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu) a_t T \quad (2.2.1) \tag{12}$$

with *U = 0* at *r = a* and *r = b* (13)

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ (2.2.2) as [**1**] are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \tag{14}$$

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$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \tag{15}$$

where μ is the Lame's constant, while each of the stress function σ_{rz}, σ_{zz} and $\sigma_{\theta z}$ are zero within the disc in the plane state of stress.

Equations (1) – (15) constitute mathematical formulation of the problem.

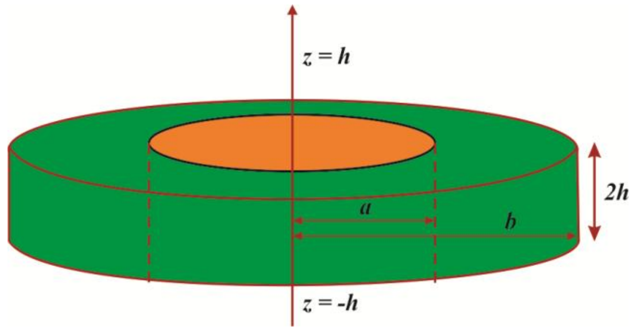

Fig shows the geometry of the problem

III. SOLUTION OF THE PROBLEM

Applying Marchi-Zgrablich transform defined [7] to the equation (6), one obtains

$$\frac{d^2\overline{T}}{dz^2} + \frac{g}{k} - \mu_m^2 \overline{T} + \psi = \frac{1}{\alpha}\frac{d\overline{T}}{dt} \tag{16}$$

where $\overline{T}$ is the Marchi-Zgrablich transform of T and m is the Marchi-Zgrablich transform parameter.

where

$$\psi(z,t) = \frac{b}{k_2} S_0(\alpha, \beta, \mu_m b)\, G_2(z,t) - \frac{a}{k_1} S_0(\alpha, \beta, \mu_m a) G_1(z,t) \tag{17}$$

Applying Marchi-Fasulo transform defined [7] to equation (16), one obtains

$$\frac{d\overline{T}^*}{dt} + \alpha p^2 \overline{T}^* = \Phi \tag{18}$$

where $p^2 = \mu_m^2 + \lambda_n^2$. (19)

$$\Phi = \alpha\left[\frac{P_n(h)}{k_4}\overline{f}_2(r,t) - \frac{P_n(-h)}{k_3}\overline{f}_1(r,t) + \frac{\overline{g}^*}{k} + \psi^*\right] \tag{20}$$

Solution of the differential equation (18) is given by

$$\overline{T}^* = e^{-\alpha p^2 t}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right) \tag{21}$$

Applying inverse Marchi-Fasulo transform and Marchi Zgrablich transform to the equation (21), one obtains

$$T(r, z, t) = \sum_{m,n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_m r)}{\mu_m^2} \frac{P_n(z)}{\lambda_n^2}$$

$$\times e^{-\alpha p^2 t}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right) \tag{22}$$

(2.2.15)

IV. DETERMINATION OF THERMAL DEFLECTION $\omega(r,t)$

Using expression for temperature distribution from equation (22) in equation (2) one obtains

$$M_T = a_T E \sum_{m,n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_m r)}{\lambda_n^2 \ \mu_m^2}$$

$$e^{-\alpha p^2 t}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right)\int_{-h}^{h} z P_n(z) dz \tag{23}$$

We assume the solution of equation (23) satisfying condition (5) as

$$\omega(r,t) = \sum_{m=1}^{\infty} C_m(t)[S_0(k_1, k_2, \mu_m r) - S_0(k_1, k_2, \mu_m b)] \tag{24}$$

where μ_m are the positive roots of the transcendental equation

$$S_0(k_1, k_2, \mu_m a) - S_0(k_1, k_2, \mu_m b) = 0 \tag{25}$$

It can be easily seen that (2.3.1)

$$\omega = \frac{\partial \omega}{\partial r} = 0 \text{ at } r = a, b \tag{26}$$

Hence solution (23) satisfies condition (5).

Now

$$\nabla^4 \omega = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)^2 \sum_{m=1}^{\infty} C_m[S_0(k_1, k_2, \mu_m r) - S_0(k_1, k_2, \mu_m b)] = 0 \tag{27}$$

(2.3.2)

We use the well known result

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) S_0(k_1, k_2, \mu_m r) = -\mu_m^2\ S_0(k_1, k_2, \mu_m r)]$$

(2.3.3)

in equation (27) to obtain

$$\nabla^4 \omega = \sum_{m=1}^{\infty} C_m \mu_m^4\ S_0(k_1, k_2, \mu_m r)$$

and

$$\nabla^2 M_T = -a_T E \sum_{m,n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_m r)}{\lambda_n^2}$$

$$e^{-\alpha p^2 t}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right)\int_{-h}^{h} z P_n(z) dz \tag{28}$$

(2.3.4)

Using equation (27) and equation (28) in the equation (1), one obtains

$$\sum_{m=1}^{\infty} C_m \mu_m^4 S_0(k_1, k_2, \mu_m r) = \frac{-a_T E}{D(1-\nu)} \sum_{m,n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_m r)}{\lambda_n^2}$$

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$$\times e^{-\alpha p^2 t}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right)\int_{-h}^{h} zP_n(z)dz \quad (29)$$

On solving equation (29), one obtains

$$C_m(t) = \frac{-a_T E}{D(1-\nu)}$$

$$\times \sum_{m,n=1}^{\infty} \frac{e^{-\alpha p^2 t}}{\mu_m^{\ 4}\ \lambda_n^{\ 2}}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right)\int_{-h}^{h} zP_n(z)dz \quad (30)$$

Using equation (30) in equation (24), we get

$$\omega(r,t) = \frac{-a_t E}{D(1-\nu)}$$

$$\times \sum_{m,n=1}^{\infty} \frac{e^{-\alpha p^2 t}}{\mu_m^{\ 4}\ \lambda_n^{\ 2}}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right)\int_{-h}^{h} zP_n(z)dz$$

$$\times[S_0(k_1,k_2,\mu_m r) - S_0(k_1,k_2,\mu_m b)] \quad (31)$$

The displacement function is given by

$$U(r,z,t) = -(1+\nu)a_t \sum_{m,n=1}^{\infty} \frac{S_0(k_1,k_2,\mu_m r)}{\mu_m^{\ 4}}\frac{P_n(z)}{\lambda_n^{\ 4}}$$

$$\times e^{-\alpha p^2 t}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right) \quad (32)$$

The radial and tangential stresses are given by

$$\sigma_{rr} = \frac{2\mu(1+\nu)a_t}{r}\sum_{m,n=1}^{\infty} \frac{S_0'(k_1,k_2,\mu_m r)}{\mu_m^{\ 3}}\frac{P_n(z)}{\lambda_n^{\ 4}}$$

$$\times e^{-\alpha p^2 t}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right) \quad (33)$$

$$\sigma_{\theta\theta} = 2\mu(1+\nu)a_t \sum_{m,n=1}^{\infty} \frac{S_0''(k_1,k_2,\mu_m r)}{\mu_m^{\ 2}}\frac{P_n(z)}{\lambda_n^{\ 4}}$$

$$\times e^{-\alpha p^2 t}\left(\overline{F}^* + \int_0^t \Phi e^{\alpha p^2 t'} dt'\right) \quad (34)$$

V. SPECIAL CASE AND NUMERICAL RESULTS

Setting

$$f_1(r,t) = \delta(r-r_0)\times(1-e^{-t}) \ ,$$

$$f_2(r,t) = \delta(r-r_0)\times(1-e^{-t}) \ ,$$

$$F = 0 \Rightarrow \overline{F}^* = 0 \quad (35)$$

a =2, b =3 , h =1, k_1 = 0.25, k_2 =0.25, k =0. 86, r_0 =0.75,

t = 1 sec., in equation (22), one obtains the expression for temperature distribution as

$$T(r,z,t) = \sum_{m,n=1}^{\infty} \frac{S_0(0.25, 0.25, \mu_m r)}{\mu_m^{\ 2}}\frac{P_n(z)}{\lambda_n^{\ 2}}$$

$$\times e^{-\alpha p^2 t}\left(\int_0^1 \Phi e^{\alpha p^2 t'} dt'\right) \quad (36)$$

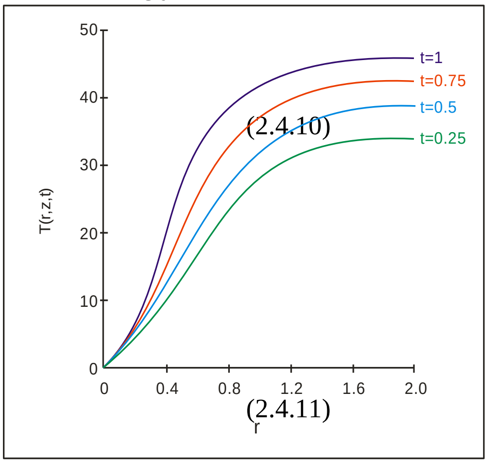
VI. CONCLUSION

In this paper, the temperature distribution, thermal deflection, displacement function and thermal stresses of a thin annular disc are investigated with the help of Marchi-Fasulo transform and Marchi-Zgrablich transform techniques.

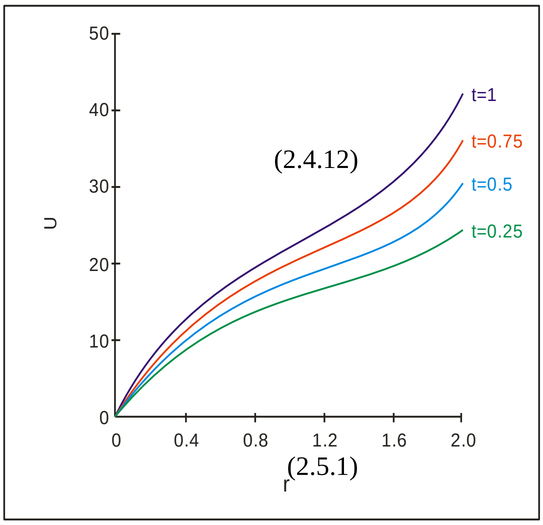
Using the formulas derived, calculations have been made using MathCAD, and graphs have been plotted for the temperature distribution, thermal deflection, displacement function and thermal stresses versus different values of time and radius.

The results of a thin annular disc made of aluminium have been determined in series form in terms of Bessel's functions.

The author has plotted the graphs taking the material properties of aluminium, and the numerical computation has been inferred accordingly.



Graph 1: Temperature distribution vs radius



Graph 2: Displacement component vs radius

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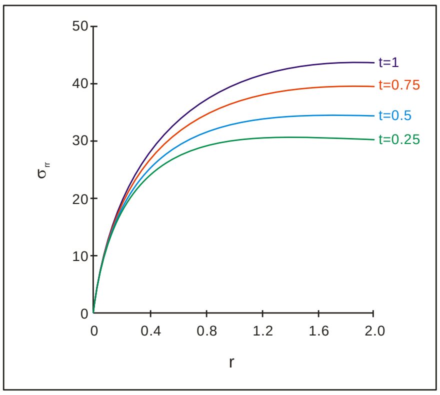
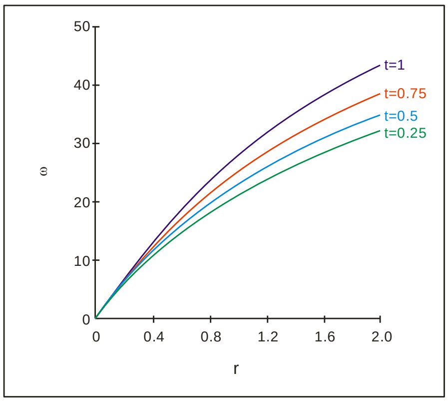


Graph 3: Radius stresses vs radius



Graph 4: Thermal deflection vs radius

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Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems (Thermoelasticity in particular) and Operations Research. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and six students submitted their thesis in University for award of Ph.D Degree under their guidance.