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OPCL Coupling and Modified Projective Synchronization of Fractional order Differential Systems

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***Abstract*— we design a scheme under mismatched parameters using OPCL control coupling for fractional order differential system to achieve MPS. For the justification of results commensurate fractional order differential system and in commensurate fractional order system is presented.**

***Index Terms*—OPCL, Modified projective synchronization, Numerical Simulation.**

I. INTRODUCTION

The theory of fractional order has been known since the contribution of Leibnitz and L's hospital in 1695[1].In history of fractional calculus [1], the applications of fractional calculus to physics, engineering and now recently area of interest is control processing. Many field as visco elastic systems [2], quantitative finance and complex system evolution and so forth are increasing attention. These fields demonstrate the importance of dynamical systems with fractional- order models. There are many differences between the integer-order system and the fractional-order differential equation systems. There are many properties of integer order which are not simply extended to fractional order system. Chaotic behavior is displayed by several systems such as the fractional-order Rössler system [3], the fractional- market dynamics [4], the Fractional-order Lü system [5], the fractional-order unified System [7], fractional order of new chaotic system [8]. The hyper chaos may be more useful in some fields such as communication, encryption etc. On the other hand the area which attracted much attention is chaos synchronization. Since the seminal work of Pecora and Carroll [6] recently, Fractional-order systems start to attract attention because of Its application in control processing and the advantages in Secure communication. There are many types of Synchronization for the fractional order system such as CS [10] AS [11], PHs [9] and other are studied. Other than above studies, the OPCL control method is a more general and physically realizable that can provide stable CS and Inverse Synchronization [12] in identical and mismatched parameters. Coupled system of Lag synchronization and scaling of chaotic attractor is also studied [13]. On account of the symmetry class of a system OPCL provides synchronization in all the systems without any restriction. The other advantage of OPCL coupling is to obtain synchronization in the presence of mismatched Parameters. OPCL scheme increases a great attention to achieve synchronization in integer order or fractional order Differential system. In MPS the states of master and slave system of non-linear dynamical systems synchronize up to a constant scaling matrix with the CS, AS. In our studies OPCL control coupling is used for achieving MPS of two non-identical systems. The organization of the paper is as follows. In Section II, Dynamical analysis of fractional-order system is studied numerically and analyzed by computer simulations. In Section III, Methodology used for modified projective synchronization by using OPCL coupling is discussed. In Section IV, MPS between two fractional order commensurate systems using OPCL method is presented. In Section V, MPS between two fractional order incommensurate systems using OPCL method is studied. In Section VI, Numerical simulation shows that MPS is achieved between two systems. Finally, section VII, consist the conclusions of this paper.

II. DYNAMICAL ANALYSIS OF FRACTIONAL ORDER SYSTEMS

Consider the fractional order Lorenz system as a master system, with mismatched parameters $\Delta\alpha, \Delta\beta, \Delta\gamma$

$$D_*^{\alpha} x_1 = (\alpha + \Delta\alpha)(x_2 - x_1)$$
$$D_*^{\alpha} x_2 = (\beta + \Delta\beta)x_1 - x_2 - x_1 x_3$$
$$D_*^{\alpha} x_3 = x_1 x_2 - (\gamma + \Delta\gamma)x_3 \qquad (1)$$

And the fractional order response system is

$$D_*^{\alpha} y_1 = a(y_2 - y_1)$$
$$D_*^{\alpha} y_2 = by_1 - cy_2 - y_1 y_3$$
$$D_*^{\alpha} y_3 = y_1^{\,2} - dy_3 \qquad (2)$$

The system (2) exhibit chaotic behavior as shown in the Fig. 1. The initial conditions are (1,-2, 3) and the value of Order is (0.95, 0.95, 0.95), where a=10, b=28, c=8/3, d=1

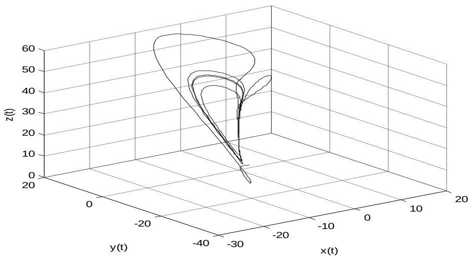

Fig.1 Chaotic attractor

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III. METTHDOLOGY USED FOR MODIFIED PROJECTIVE SYNCHRONIZATION BY USING OPCL COUPLING

In this section we will briefly describe the OPCL coupling method for synchronize two systems. For this we assume

Derive system as $D_*^{\propto} x = f(x) + \Delta f(x)$, where $\Delta f(x)$ contains mismatched parameters. The response system is considered as $D_*^{\propto} y = g(y) + u(t)$,where u (t) is the controller to be designed and $f \neq g$ so these two systems are non -identical system.

The MPS is said to be achieved between the master and slave system if there exist a constant matrix

$k = diag(k_1, k_2, k_3, \ldots . k_n)$, such that $\lim_{t\to\infty} \| e \| = \lim_{t\to\infty} \| y - kx \| = 0$

The controller u(t) is designed in the form of

$u(t) = D_*^{\propto} kx - g(kx) + (H - Jg(kx))(y - kx)$, where $J = \frac{\partial}{\partial(kx)}$ is the jacobian matrix of the dynamical system and H is an constant matrix and $H \in (n * n)$

The form of $g(y)$ is

$g(y) = g(kx) + J(kx)(y - kx) + \cdots$, this expansion is done by using Taylor's series. After putting the value of u (t) and g(y) in the response system we get,

$D_*^{\propto} e = He$

IV. MPS BETWEEN TWO FRACTIONAL ORDER COMMENSURATE SYSTEMS USING OPCL METHOD

The OPCL control method is that which can provide synchronization in identical and non-identical dynamical systems. The fractional order system with different structure, OPCL method is used for getting synchronization.

Now by using the methodology defined in the section II, from the system (2)

The jacobian matrix is obtained

$$Jg(kx) = \begin{bmatrix} -a & a & 0 \\ b - k_3x_3 & -c & -k_1x_1 \\ k_1x_1 & 0 & -d \end{bmatrix} \quad (3)$$

For the response system the constant matrix H is

$$H = = \begin{bmatrix} -a & a & 0 \\ P_1 & -c & P_2 \\ P_3 & 0 & -d \end{bmatrix} \quad (4)$$

For obtaining MPS, we define error vector as

$e = He = (e_1, e_2, e_3)^T$

$= (y_1 - k_1x_1, y_2 - k_2x_2, y_3 - k_3x_3)^T$

System (1) is the derive system and system (2) is the response system which is controlled by OPCL coupling

Therefore

$$D_*^{\propto} y_1 = a(y_2 - y_1) + k_1[(\propto + \Delta\alpha)(x_2 - x_1)] - a(k_2x_2 - k_1x_1)$$
$$D_*^{\propto} y_2 = by_1 - cy_2 - y_1y_3 + k_2[(\beta + \Delta\beta)x_1 - x_2 - x_1x_3] - b(k_1x_1) - c(k_2x_2) - (k_1x_1k_3x_3) + [P_1 + k_3x_3 - b]e_1 + [P_2 + k_1x_1]e_3$$
$$D_*^{\propto} y_3 = y_1^2 - dy_3 + k_3[x_1x_2 - (\gamma + \Delta\gamma)x_3] - [(k_1x_1)^2 - d(k_3x_3)] + (P_3 - k_1x_1)e_1 \quad (5)$$

The error dynamics is

$$D_*^{\propto} e_1 = a(e_2 - e_1) + k_1[(\propto + \Delta\alpha)(x_2 - x_1)] - a(k_2x_2 - k_1x_1)$$
$$D_*^{\propto} e_2 = be_1 - ce_2 - y_1y_3 + k_2[(\beta + \Delta\beta)x_1 - x_2 - x_1x_3] - b(k_1x_1) - c(k_2x_2) - (k_1x_1k_3x_3) + [P_1 + k_3x_3 - b]e_1 + [P_2 + k_1x_1]e_3$$
$$D_*^{\propto} e_3 = e_1^2 - de_3 + k_3[x_1x_2 - (\gamma + \Delta\gamma)x_3] - [(k_1x_1)^2 - d(k_3x_3)] + (P_3 - k_1x_1)e_1 \quad (6)$$

V. MPS BETWEEN TWO FRACTIONAL ORDERS IN COMMENSURATE SYSTEMS USING OPCL METHOD

Consider two fractional system as master and response system

$$D_*^{\propto} x_1 = x_2$$
$$D_*^{\propto 2} x_2 = x_3$$
$$D_*^{\propto 3} x_3 = (\propto + \Delta\alpha)x_1 + (\beta + \Delta\beta)x_2 + (\gamma + \Delta\gamma)x_3 + x_1^3 \quad (7)$$

And the fractional order response system is

$$D_*^{\propto 1} y_1 = a(y_2 - y_1)$$
$$D_*^{\propto 2} y_2 = by_1 - cy_2 - y_1y_3$$
$$D_*^{\propto 3} y_3 = y_1^2 - dy_3 \quad (8)$$

The system (8) also shows chaotic behavior as shown in Fig.2 for different order. The value of order are (0.95 0.92 0.9)

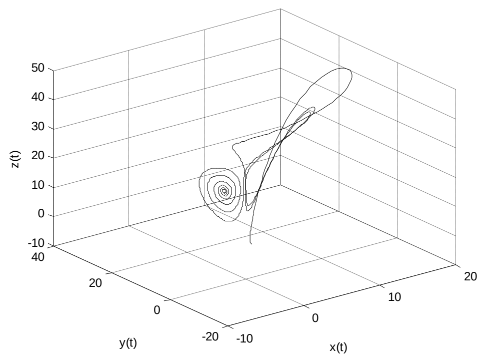

Fig.2. Chaotic Attractor

By using (3) and (4) the fractional order response system controlled by OPCL coupling is

$$D_*^{\propto 1} y_1 = a(y_2 - y_1) + k_1x_2 - a(k_2x_2 - k_1x_1)$$
$$D_*^{\propto 2} y_2 = by_1 - cy_2 - y_1y_3 + k_2[x_3] - b(k_1x_1) - c(k_2x_2) - (k_1x_1k_3x_3) + [P_1 + k_3x_3 - b]e_1 + [P_2 + k_1x_1]e_3$$
$$D_*^{\propto 3} y_3 = y_1^2 - dy_3 + k_3[(\propto + \Delta\alpha)x_1 + (\beta + \Delta\beta)x_2 + (\gamma + \Delta\gamma)x_3 + x_1^3] - [(k_1x_1)^2 - d(k_3x_3)] + (P_3 - k_1x_1)e_1$$

(9)

The error dynamics is

$$D_*^{\propto 1} e_1 = a(e_2 - e_1) + k_1x_2 - a(k_2x_2 - k_1x_1)$$
$$D_*^{\propto 2} e_2 = be_1 - ce_2 - y_1y_3 + k_2[x_3] - b(k_1x_1) - c(k_2x_2) - (k_1x_1k_3x_3) + [P_1 + k_3x_3 - b]e_1 + [P_2 + k_1x_1]e_3$$
$$D_*^{\propto 3} e_3 = e_1^2 - de_3 + k_3[(\propto + \Delta\alpha)x_1 + (\beta + \Delta\beta)x_2 + (\gamma + \Delta\gamma)x_3 + x_1^3] - [(k_1x_1)^2 - d(k_3x_3)] + (P_3 - k_1x_1)e_1$$

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(10)

VI. NUMERICAL SIMULATION

In numerical simulation section, results will be carried out by using *MATLAB*. Two groups are presented one is commensurate and other is incommensurate to justify theoretical results. There is an excellent agreement between the theoretical and computational results. Fig. 3, 4, 5,6,7,8 represents error and state system with the evolution of time for in commensurate system. Fig. 9, 10, 11,12,13,14 represents error and state system with the evolution of time for commensurate system. For both systems, we choose initial conditions for the master system are -1,-1,-1 and for the slave system -1, 1, 2 also the value of k are chosen as 2,-1,-3 and $p_1, p_2, p_3 = -30, -10, 10$ and

$\Delta\alpha, \Delta\beta, \Delta\gamma = 0.1, 0, 0$

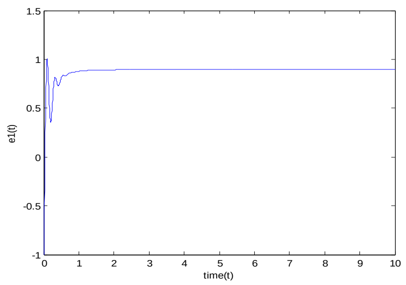

Fig.3.The time evolution of error system

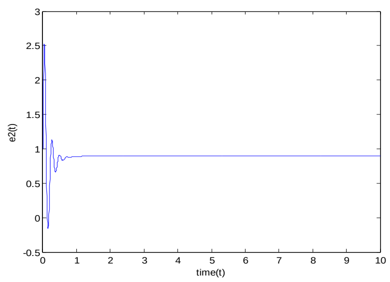

Fig.4.The time evolution of error system

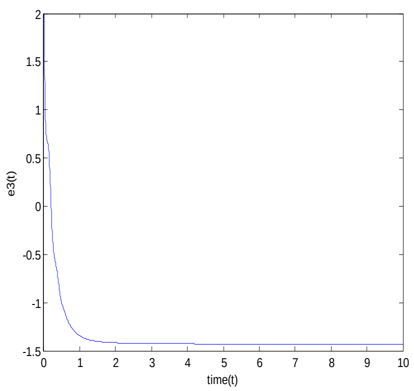

Fig.5.The time evolution of error system

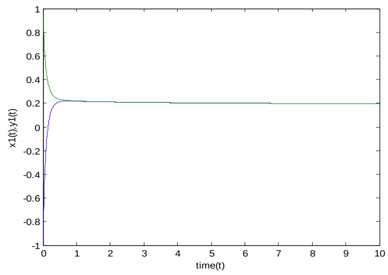

Fig.6.The time evolution of state system

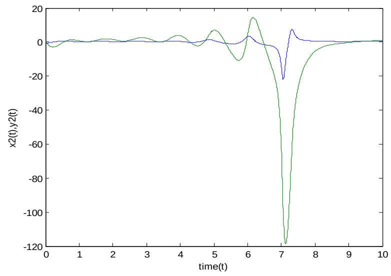

Fig.7.The time evolution of state system

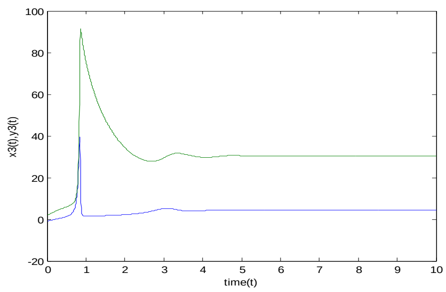

Fig.8.The time evolution of state system

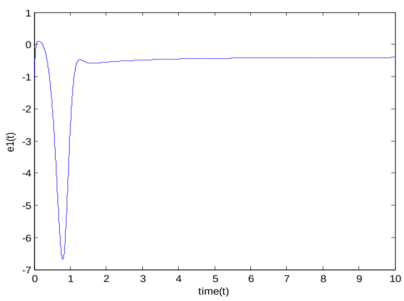

Fig.9.The time evolution of error system

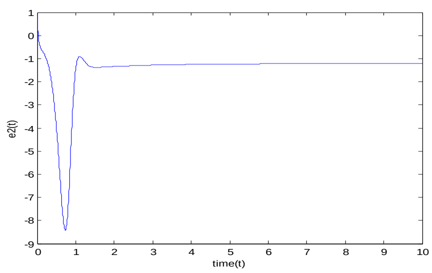

Fig.10.The time evolution of error system

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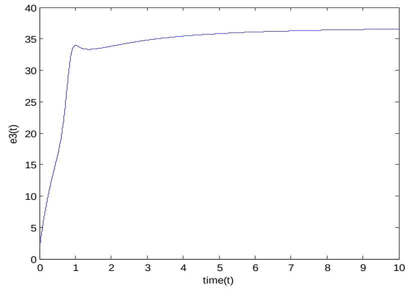

Fig.11.The time evolution of error system

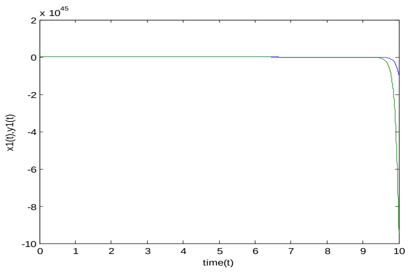

Fig.12.The time evolution of state system

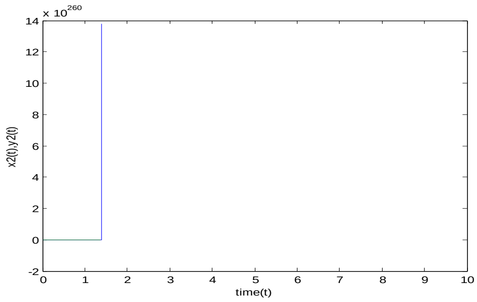

Fig.13. Time evolution of state system

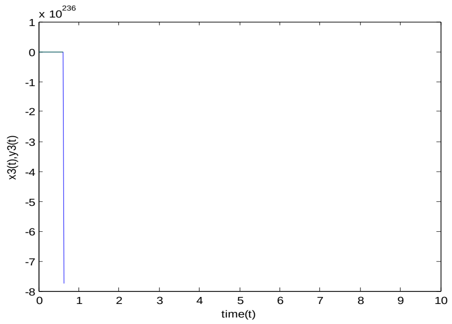

Fig.14. Time evolution of state system

VII. CONCLUSION

For the practical application OPCL coupling is physically realizable method, through OPCL coupling we design controllers for commensurate and incommensurate systems and we can see that between the two systems MPS is achieved. From the numerical simulation it has been established that our analytic and computational results are in excellent agreement.

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