Job shop Scheduling under Answer Set Programming Environment

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Abstract—Answer set programming (ASP) is a new programming language paradigm combining the declarative aspect with non-monotonic reasoning. In this paper, we will show an application of job shop scheduling in Answer Set Programming. The problem is a highly combinatorial and is generally solved by specific programs written in procedural languages. We present an approach to solve the job shop scheduling problem in Answer Set Programming and compare them with the standard ones. It turns out that, although Answer Set Programming greatly simplifies the problem statement. It is comparable in efficiency to specialized programs.

Index Terms— answer set programming, answer set semantic, job shop scheduling, logic programming.

I. INTRODUCTION

Many real-life problems belong to the class of NP-complete problems [1]. Logic Programming under answer set semantics provides a powerful language for a logical formulation of these problems. Its nondeterministic computation liberates the user from the tree-search programming. Answer set programming (ASP) is a form of declarative programming that is emerged from logic programming with negation and reasoning formalism that is based on the answer set semantics [2, 3]. ASP is considered in the late 1990s as a new programming paradigm [4]. Answer set programming languages has been used to solve many real life application problems, among them, production configuration [5], decision support for NASA shuttle controllers [6], synthesis of multiprocessor systems [7], reasoning tools in biology [8, 9], team building [10], composition of Renaissance music [11], and many more. A number of solvers have been proposed, such as: smodels [12, 13, 14], dlv [15], cmodels [16], assat [17, 18], and clasp [19].

In this paper, we show an application of Answer Set Programming to real life job shop scheduling problem occurring in a factory. The job shop scheduling problem (JSSP) is a very important practical problem. Efficient methods of solving it can have major effects on profitability and product quality. However, the JSSP is considered a member among the worst class of NP complete problems [1]. In general, the difficulty of the general JSSP makes it very hard for conventional search based methods to find near optima in reasonable time.

The JSSP is to schedule jobs on different machines minimizing the total time spent to complete all jobs. Given an n-jobs and m-machines, each job comprises a set of operations which must each be done on a different machine for different specified processing time, in a given job-dependent order. Each job must be processed in an uninterrupted fashion or a non-preemptive scheduling environment. It is not necessarily for a job to visit all the machines. The job can visit a subset of the existing set of machines. Each job has a release time and a due time to complete. The release time of a job is the arrival time for that job. The due time is the time that the job must be completed.

Example 1: assume we have 3-jobs J1, J2 and J3 and three machines: m1, m2, and m3. Table 1, shows the pre-specified order of operation for each job on the machines. The pair (m, t) specifies the processing time for a particular operation on each machine. For example, to complete job J1, it must completes three operations in the following order: visit machine M1 for 7 unit of time, then machine M3 for 8 unit of time, then machine M2 for 10 unit of time. The table also, shows the due time and release time for each job.

<table>
<thead>
<tr>
<th>Jobs</th>
<th>Operations</th>
<th>Release Time</th>
<th>Due Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m, t)</td>
<td>(m, t)</td>
<td></td>
</tr>
<tr>
<td>J1</td>
<td>1, 7</td>
<td>3, 8</td>
<td>2, 10</td>
</tr>
<tr>
<td>J2</td>
<td>2, 6</td>
<td>1, 4</td>
<td>3, 12</td>
</tr>
<tr>
<td>J3</td>
<td>1, 8</td>
<td>2, 8</td>
<td>3, 7</td>
</tr>
</tbody>
</table>

A legal schedule is a schedule of job sequences on each machine such that each job's operation order is preserved, a machine can process at most one operation at one time, and different operations of the same job are not simultaneously processed on different machines. The problem is to minimize the total time elapsed between the beginning of the first operation and the completion of the last operation (this is called the make span). Other measures of schedule quality exist, but shortest make span is the simplest and most widely used criteria.

Several methods used to solve the JSSP using B&B [22], simulated annealing [23], tabu search [24, 25, 26] and genetic algorithms [27, 28, 29, 30, 32], practical swarm optimization [31]. However, up to our knowledge, there is no answer set program implementation for the JSSP.

In this paper, we first give a brief overview of Answer Set Programming and its semantics. We then, present the job shop scheduling problem formally. After that, we present the solution to the job shop scheduling problem under Answer Set Programming. Finally, we present experimental results and conclusion.
II. BRIEF OVERVIEW OF ANSWER SET PROGRAMMING

We briefly recall the basics about ASP. An ASP-program is a collection of rules of the form:

\[ a_0 \leftarrow a_1, ..., a_n \text{ not } a_{n+1}, ..., a_m \]  \hspace{1cm} (1)

Where, each \( a_i \) is an atom. The head of the rule is a positive atom which is the left hand side of the clause in (1). The body of the rule is composed of literals (a literal is an atom or its negation, denoted by \( \text{not } a \)) which is on the right side of the clause in (1). A rule without body is a fact. A rule without head is a constraint. Also, the rules can be positive (\( n > 0 \)); negative (\( n > 0 \)) or both (\( m > 0 \) and \( n > 0 \)). The symbol \( \text{not} \) stands for default negation, also known as negation as failure.

If \( P \) is a ground, positive program (no negation as failure used), a unique answer set is defined as the smallest set of literals constructed from the atoms occurring in program \( P \) (minimal model). The last definition can be extended to any ground program \( P \) containing negation by considering the retract of \( P \) with respect to a set of atoms \( X \) obtained by the Gelfond-Lifschitz’s operator \([1]\). The retract, \( PX \), of \( P \) relative to \( X \) for all rules in (1) in an ASP \( P \) is the set of rules:

\[ a_0 \leftarrow a_1, ..., a_n \text{ not } a_{n+1}, ..., a_m \]

where \( a_{n+1}, ..., a_m \notin X \). Then \( PX \) is a program without the negation \( \text{not} \). Then \( X \) is an answer set for \( P \) if \( X \) is an answer set for \( PX \).

Once a program is described as an ASP \( P \), its solutions, if any, are represented by the answer set of \( P \). One important difference between ASP semantics and other semantics is that a logic program may have several answer sets or may have no answer set at all.

Answer Set Programming is a totally declarative language. ASP programs are not algorithms describing how to solve the problem; the program is just a formal description of the problem. The solution is completely found by the solver. An ASP solver requires grounded programs as input, and that is why before searching the answer set or solutions, the program is grounded by a preprocessor. Actually there are many ASP’s solvers such as: smodels, dlv, assat, cmodels, and clasp. The computation of answer sets is done in two phases: (i) grounding of the logic program \( P \); that is eliminating variables to obtain a propositional program ground \( P \). (ii) Computation of answer sets on the propositional program ground \( P \).

III. JOB SHOP SCHEDULING PROBLEM FORMULATION

Job shop scheduling is an optimization problem in which jobs are assigned to resources at particular times. There are several formulations for the JSSP, we have adopted the one presented in [32] as follows:

Given a set of \( n \)-jobs \( J = \{j_1, j_2, ..., j_n\} \), and \( m \)-machines \( M = \{M_1, ..., M_m\} \). Let \( n_j \) be the number of operations of job \( j \). Denote \( O_{jkq} \) the operation \( k \) of job \( j \) to be processed on machine \( q \). \( T_{jkq} \) and \( P_{jkq} \) be the start time and processing time of operation \( O_{jkq} \) respectively. Denote \( r_j \) and \( d_j \) the release time (earliest start time) and due time (latest ending time) of job \( j \). Let \( S_j \) denote the set of operation pairs \( (O_{jkq}, O_{jqr}) \) of job \( j \), where \( O_{jqr} \) must be processed before \( O_{jkq} \). Let \( R_q \) be the set of operations \( O_{jq} \) to be processed on machine \( q \). Our goal is to schedule all jobs on \( m \)-machines, while trying to minimize the completion time or the make span. The make span is the total time of the schedule (that is, when all the jobs have finished processing). Given a schedule \( U \), the completion time for a job \( j \) is \( C_{Uj} = \text{max}(T_{jkq} + P_{jkq}) \) for all \( k \in \{1, ..., n_j\}, j \in J \) and \( q \in M \). the makespan of a schedule \( S \) is the maximum completion time over all jobs in schedule \( U \): \( C_{Umax} = \text{max}_{j \in J}(T_{jkq} + P_{jkq}) \), \( k \in \{1, ..., n_j\} \) and \( q \in M \). The job shop scheduling problem is represented as follows:

Minimize \( C_{Umax} \), where, \( C_{Umax} = \text{max}_{j \in J}(T_{jkq} + P_{jkq}) \)

Subject to:

\[ T_{jkq} - T_{jqw} >= P_{jqw}, \text{ where } (O_{jkq}, O_{jqw}) \in S_j, k, w \in \{1, ..., n_j\}, j \in J \text{ and } q \in M. \]  \hspace{1cm} (2)

\[ T_{jqw} - T_{jqk} >= P_{jqk} \text{ or } T_{jqk} - T_{jqw} >= P_{jqw} \]  \hspace{1cm} (3)

where \( O_{jqk}, O_{jqw} \in R_q \), \( i, j \in J, q \in M, k \in \{1, ..., n_j\} \) and \( w \in \{1, ..., n_i\} \).

\[ r_j <= T_{jkq} <= d_j - P_{jkq}, \text{ where } j \in J, k \in \{1, ..., n_j\}, \text{ and } q \in M. \]  \hspace{1cm} (4)

Equation (2) represents the sequence constraint; Equation (3) represents resource constraints in a disjunctive format; and equation (4) represents the release and deadline time constraints.

IV. PROBLEM DESCRIPTION AND RESOLUTION IN ASP

In this section, we describe job shop scheduling problem in the language of gringo which is the grounder for the answer set programming solver. Initially, all operations of each job are in a waiting state. If an operation is selected for processing then the operation is in the working state. Once the operation processing is completed the operation state is in the complete state. Similarly, initially all machines are in the empty state. If a machine is selected for processing an operation, then the machine is in a busy state. If the machine completes processing the operation, then the machine will be in the empty state again. Five conditions need to be satisfied:

N1: When a machine selects an operation for a job, the machine must not be busy.
N2: Each operation of a job selected for processing must be in non complete state.
N3: each operation of a job selected for processing must be completed before the due time.
N4: Each operation of a job selected for processing must be after the release time for that job.
N5: The operation processing order must be preserved.
A. Constructing the data module $D_1$ of ASP

This module defines an instance of the JSSP. This module consists of a list of jobs, operations and machine. The jobs list is defined as a fact of the following form:

```
job(jobName, releaseTime, dueTime).
```

The operations list for each job is defined as a fact of the following form:

```
operation(Job, operation, processTime, machine).
```

To define the order of processing of operations in each job, the following operation dependency fact is defined:

```
dep(jobName, operation1, operation2).
```

This facts means that ‘operation1’ depends on ‘operation2’ in a particular job. This means ‘operation2’ must be completed first then ‘operation1’ can be processed.

The machines list available is defined as a fact of the following form:

```
machine(machineName).
```

B. The job shop scheduling preparation module $D_2$

This module defines new predicate that will simplify and speeding up finding the answer set models of the JSSP. In this module we assume that the total schedule time is ‘n’ and it is specified by the user. This module consists of the following rules:

- The first group consists of determining operation dependency. It is suffice to write the following rules:
  
  \[
  \begin{align*}
  \text{dependent}(J, O) &:: \text{dep}(J, O, O1). \\
  \text{nonDependent}(J, O) &:: \text{operation}(J, O, P, M), \text{not dependent}(O).
  \end{align*}
  \]

  The first rule defines all operations that are dependent on some other operations, i.e. operations that cannot be processed until some operation completes its execution. The rule defines that operation ‘O’ of job ‘J’ is dependent on some other operation. The second rule defines the operations that are not dependent on other operations, i.e. operations that are at the beginning of each job. Therefore, we have two kinds of operations; the non-dependent operations and the dependent operations.

- The second group consists of one rule that finds the total time to execute all operations for each job. This is done in ASP as:
  
  \[
  \text{totalTimeJob}(J, T) :: T = \# \text{sum} \{ P, O: \text{operation}(J, O, P, M), \text{job}(J, R, D). \}
  \]

  This rule uses the aggregates ‘sum’ to sum all processing time for all operation of a particular job J.

- The third group consists of finding the earliest and latest possible start time of each operation in each job. The start time is a range of possible times. Assume we have a job j with the following operations-processing time pair: (O₁, t₁), …, (Oₖ, tₖ). In addition, assume that the operations are ordered in the same order listed, i.e. O₂ depends on O₁, O₃ depends on O₂, …, Oₖ depends on Oₖ₋₁. Assume also that the total time of all operations processing time for a particular job is $T_j = \sum_{i=1}^{k} p_ji$, which computed from totalTimeJob(J, T). This time T is the minimum time needed to finish all operations of that job j. Let the total schedule time specified by the user is n. Then the earliest starting time of operation O₁ is zero (since it does not depend on any operation). The maximum possible starting time of operation O₁ is from 0 to n-T₁. The n-T₁ is because operation O₁ cannot start at n-T₁+1, since it needs a minimum of T₁-time unit to complete. Similarly, the earliest possible operation time for O₂ may start from p₁ (which is after completing O₁) to a maximum of n-T₁+p₁. For operation O₃, the earliest possible start time is p₁+p₂ and the latest possible start time is n-T₁+p₁+p₂. The following set of rules will do that:

  \[
  \begin{align*}
  \text{op}(J, O, P, M, R, D) &:: \text{operation}(J, O, P, M), \text{notDependent}(J, O), \text{totalTimeJob}(J, T), E = n-T, \text{job}(J, R, D), E >= D. \\
  \text{op}(J, O, P, M, S, E) &:: \text{operation}(J, O, P, M), \text{dep}(J, O, O1), \text{op}(J, O1, P1, M1, S1, E1), \text{totalTimeJob}(J, T), S = T1+P1, E = n-T+S, \text{job}(J, R, D), E < D. \\
  \text{op}(J, O, P, M, S, D) &:: \text{operation}(J, O, P, M), \text{dep}(J, O, O1), \text{op}(J, O1, P1, M1, S1, E1), \text{totalTimeJob}(J, T), S = T1+P1, E = n-T+S, \text{job}(J, R, D), E >= D.
  \end{align*}
  \]

  The rules determines the earliest and latest possible times [S, E] to execute an operation ‘O’ of job ‘J’; with processing time ‘P’ on machine ‘M’. The first two rules are for non-dependent operations. The rules compare the due time for the job with possible time range and select the earlier time. The second two rules determine the earliest and latest possible times [S, E] to execute dependent operations. It computes the earliest and latest possible times for the operation from the dependent operation and compares the times with the due time of the job. These four rules handle the constraint N3 and N4 that are listed in section IV.

- The final rule is used to define the possible time range for each job; as follows:

  \[
  \text{jobTimeSteps}(J, O, P, M, T) :: \text{op}(J, O, P, M, S, E), \text{time}(M, T), T >= S, T <= E.
  \]
Note, the rules defined in this module are all facts and it accelerates the search for a solution significantly.

C. The job shop scheduling solver module D3

This module describes solving the job shop scheduling problem. We are mainly interested in finding a schedule of processing all operations of all jobs in the shortest make span.

The module D3 will contain fluent busy(M, T) – “machine M is in a busy state at time T”, fluent complete(J, O, T) – “operation O of job name J is in a complete state at time T”. One type of action – ‘select’, will be used.

The transition diagram of D3 will be described by group of axioms:

- The first group defines the executability conditions for actions. We have one action "select(J, O, P, M, T)" which means selecting an operation O with a processing time P of job J to be executed on machine M at time T. The rules are as follows:

  0 { select(J, O, P, M, T) : op(J, O, P, M, S, E),
    T>=S, T<=E } 1 : - machine(M), time(M, T),
    not busy(M, T), avail(M, T).

  The action "select" selects one operation among the operations available to execute on a machine. The rule is a choice rule that is bounded by 0 and 1. This means selecting an operation at any time for a machine is arbitrary. It means further that an action is either selected or not selected. This rule is the generate rule that will generates all possible schedules. The rule will execute when the machine 'M' is not busy at time 'T' ('not busy(M, T)') and there is available jobs to select from 'avail(M, T)'. The rules 'busy(M, T)' and 'avail(M, T)' will be explained later. The 'not busy(M, T)' in the body of the rule handles the N1 constraint listed in section IV.

- The second group contains causal laws describing direct effect of actions. For example it is sufficient to have the rules:

  complete(J, O, T, P),
  op(J, O, P, M, T),
  time(M, T),
  T+P > time(M, T),
  T > time(M, T),
  dep(J, O, O1),
  not complete(J, O1, T),
  select(J, O, P, M, T),

  The first rule says that if an action "select" selects an operation for processing in a machine; then the operation will be in a complete state after passing the operation's processing time. The second rule says that a machine 'M' will be busy all the time step in which the operation is processed on the machine from T+1 to T+P+1.

- Two auxiliary rules are needed as follows:

  selectedMachine(M, T) :- select(J, O, P, M, T),
  jobTimeSteps(J, O, P, M, T),

  This first rule is trivial. It states that a machine is selected when it has been selected for processing some operation of a job at some time T. The second rule checks the availability of operations of a job at each time T.

- The third group of rules are constraints that eliminate unwanted answer set models, which are defined as follows:

  :- select(J, O, P, M, T), complete(J, O, T),

  :- select(J, O, P, M, T),
  operation(J, O, P, M), time(M, T),
  job(J, R, D), T+P > D.

  :- select(J, O, P, M, T),
  operation(J, O, P, M), time(M, T),
  job(J, R, D), T < R.

  :- select(J, O, P, M, T),
  dep(J, O, O1),
  not complete(J, O1, T),
  jobTimeSteps(J, O, P, M, T),
  op(J1, O1, P1, M, S1, E1), J!=J1,
  not complete(J1, O1, T), T+P > E1, T<=S1.

  The first constraint states that an operation should not be selected if it is already in a complete state. This is constraint N2 of section IV. The second constraint states that an operation should not pass the due time of the job (this is constraint N3 of section IV). The third constraint states that an operation should not be selected for processing before its arrival time or release time (this constraint N4 of section IV). The fourth constraint states that an operation should not be selected if its pre-operation is not in a complete state (i.e. order of operations is preserved). This is constraint N5 of section IV. The fifth constraint rejects the selection of an operation 'O' of a job 'J' that will cause another operation 'O1' of job 'J1' (J is different than J1) on the same machine 'M' cannot execute in its possible time range. However, the second and third constraints are redundant and can be eliminated since they are handled by the 'op/6' rules.

- The fourth group consists of rules that make sure that all operations of all jobs are completed. This can be written as:

  finish(J, O) :-
  operation(J, O, P, M, S, E),
  complete(J, O, T),
  time(M, T).

  :- not finish(J, O), operation(J, O, P, M).

  The first rule finds all operations of a job that are complete. The second rule is a constraint that rejects answer set models that includes non complete operations.

- To find the schedule of minimum make span, the following optimization rules are added:

  makespan(X) :- X =
  #max (T+P:select(J,O,P,M,T)).
#minimize \{X: \text{makespan}(X)\}.

The first rule returns the makespan for the schedule produced. It uses the aggregate max of clingo to find the maximum time to complete the schedule. The second rule finds the minimum make span among all schedules produced by the module D_3. It uses the optimization statement "minimize" of gringo to find the schedule with the minimum make span.

To complete the definition of the transition diagram of the domain, we need to specify what fluents do not change as the results of actions. This is a famous Frame Problem from [34] where the authors suggested solving it by formalizing the Inertia Axiom which says that "things tend to stay as they are". This is a typical default which can be easily represented in answer set programs. In our particular case, it will have the form:

\[
\text{complete}(J, O, T+1) : \neg \text{complete}(J, O, T), \ 
\text{ap}(J, O, P.M, S, E), \text{time}(T), T < n, T > S.
\]

V. EXPERIMENTAL RESULTS

Our experiments were designed to assess the performance of each of the ASP on job shop scheduling problems. We used the ten scheduling problems produced by Taillard with 7 jobs and 7 machines (7x7). Each of these problems consists of forty-nine operations to be scheduled subject to sequencing restrictions and resource capacity constraints. The operations are grouped into seven jobs of seven operations each. Operations within each job must be performed in order. Further, each job requires one of seven resources and each resource can be used by at most one job at a time.

Table (3) shows running the answer set program on several instances of the job shop scheduling problem. The ASP was run on a Intel core 2 due laptop with 1.2 GHz processor and 4GB RAM is used.

| Table II. Experimental results of ASP |
|-----------------|--------------------|-------------------|----------------|
| Problem Instance | CPU time in seconds | Shortest makespan found by ASP | Known shortest makespan |
| 3x3 Example 1    | 0.094              | 42                | 42             |
| 4x4 Instance     | 2.29               | 272               | 272            |
| 5x5 Instance     | 4.96               | 333               | 333            |
| 6x6: 6x6         | 0.45               | 55                | 55             |
| 7x7 instance1    | 47.00              | 590               | 590            |
| 7x7 instance2    | 36.44              | 558               | 558            |
| 7x7 instance3    | 54.77              | 605               | 605            |
| 7x7 instance4    | 59.51              | 671               | 671            |

Note that, the problems with large value of time steps can have big influence on the program’s performance when employing Answer Set programming as solution method, since the number of answer set candidates that needs to be checked is heavily dependent on the number of time steps. For larger problems such as 10x10 and 20x5 the cpu time is large (more than 10 hours).

VI. CONCLUSION

In this paper, we present an approach that uses ASP to represent the job shop scheduling problem to produce optimal plans. Job shop scheduling is known to be a hard problem. We have proposed to investigate and evaluate the capabilities of ASP to job shop scheduling problem. ASP is expressive enough to represent the constraint of the job shop scheduling problem. The paper also shows the expressive use of the aggregates and optimization sentences defined in the 'clingo' solver. Job shop scheduling problem can be a killer application of ASP when the time step increases and solving other job shop scheduling problems is an interesting future work.

Although we cannot solve the general case of the job shop scheduling problem satisfactorily at the moment, we note that the solution methodology proposed in our program could be very useful for further development in future work. In conclusion, ASP as a declarative programming language has been shown to be an elegant and highly maintainable approach for solving the job shop scheduling problem, but we have to admit that there is still work to do in order to obtain a competitive and robust solver. We need to investigate other special purpose constraints, heuristics, neighborhood structures (e.g. neighborhoods that each neighbor of feasible schedule is locally optimal [35]) and problem decomposition that can speed up the algorithm. For example, we can use the shortest processing time first (SPT) or shortest total processing time (STPT). Further research may focus on Flow job Shop Scheduling Problem (FJSSP) and Open Job Shop Scheduling problem (OJSSP).

REFERENCES


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