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Thermal Stress Analysis of a Thick Hollow Cylinder

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Abstract-In this paper, an attempt has been made to study thermoelastic response of a direct thermoelastic problem of a hollow cylinder occupying the space $D: a \le r \le b,\ 0 \le z \le h,$ ***with radiation type boundary conditions. We apply integral transform technique to find the thermoelastic solution.***

Keywords: **Thermo elastic Response, hollow cylinder, Thermal Stresses, inverse problem.**

I. INTRODUCTION

Khobragade et al. [2-18] have investigated temperature distribution, displacement function, and stresses of a thin as well as thick hollow cylinder and Khobragade et al. [13] have established displacement function , temperature distribution and stresses of a semi-infinite cylinder.

In the present paper, an attempt is made to study the theoretical solution for a thermoelastic problem to determine the temperature distribution, displacement and stress functions of a hollow cylinder with boundary conditions occupying the space $D = \{(x, y, z) \in R^3 : a \le (x^2 + y^2)^{1/2} \le b,\ 0 \le z \le h\}$, where $r = (x^2 + y^2)^{1/2}$. A transform defined by Zgrablich et al. [2] is used for investigation which is a generalization of Hankel's double transform and used to treat the problem with radiation type boundaries conditions.

II. FORMULATION OF THE PROBLEM

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function $\phi(r, z, t)$ satisfying the differential equation as Khobragade [2] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right) a_t T \qquad (1)$$

with $\phi = 0$ at $r = a$ and $r = b$

where ν and a_t are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and $T(r, z, t)$ is the heating temperature of the cylinder at time t satisfying the differential equation as Khobragade [2] is

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

where $\kappa = K / \rho c$ is the thermal diffusivity of the material of the cylinder, K is the conductivity of the medium, c is its specific heat and ρ is its calorific capacity (which is assumed to be constant) respectively, subject to the initial and boundary conditions

$$M_t(T, 1, 0, 0) = F \quad \text{for all} \quad a \le r \le b,\ 0 \le z \le h \qquad (3)$$

$$M_r(T, 1, k_1, a) = F_1(z, t), \text{ for all } 0 \le z \le h,\ t > 0 \qquad (4)$$

$$M_r(T, 1, k_2, b) = F_2(z, t) \text{ for all } 0 \le z \le h,\ t > 0 \qquad (5)$$

$$M_z(T, 1, k_3, -h) = F_3(r, t) \text{ for all } a \le r \le b,\ t > 0 \qquad (6)$$

$$M_z(T, 1, k_4, h) = G(r, t) \text{ for all } a \le r \le b,\ t > 0 \qquad (7)$$

being:

$$M_{\vartheta}(f, \bar{k}, \bar{\bar{k}}, \$) = (\bar{k}\ f + \bar{\bar{k}}\ \hat{f})_{\vartheta = \$}$$

Where the prime (^) denotes differentiation with respect to ϑ, radiation constants are $\bar{k}$ and $\bar{\bar{k}}$ on the curved surfaces of the plate respectively.

The radial and axial displacement U and W satisfy the uncoupled thermoelastic equation as Khobragade [2] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1}\frac{\partial e}{\partial r} = 2\left(\frac{1+\nu}{1-2\nu}\right) a_t \frac{\partial T}{\partial r} \qquad (8)$$

$$\nabla^2 W + (1-2\nu)^{-1}\frac{\partial e}{\partial z} = 2\left(\frac{1+\nu}{1-2\nu}\right) a_t \frac{\partial T}{\partial z} \qquad (9)$$

Where,

$$e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial r} \qquad (10)$$

(2)

$$U = \frac{\partial \phi}{\partial r}, \qquad (11)$$

$$W = \frac{\partial \phi}{\partial z} \qquad (12)$$

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The stress functions are given by

$$\tau_{rz}(a, z, t) = 0, \; \tau_{rz}(b, z, t) = 0, \; \tau_{rz}(r, 0, t) = 0 \tag{13}$$

$$\sigma_r(a, z, t) = p_1, \sigma_r(b, z, t) = -p_o, \sigma_z(r, 0, t) = 0 \tag{14}$$

where p_1 and p_o are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations as Khobragade [2] are

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) \tag{16}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left(\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right) \tag{17}$$

$$\tau_{rz} = G\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right) \tag{18}$$

where $\lambda = 2G\nu/(1-2\nu)$ is the Lame's constant, G is the shear modulus and U, W are the displacement components.

Equations (1)-(18) constitute the mathematical formulation of the problem under consideration.

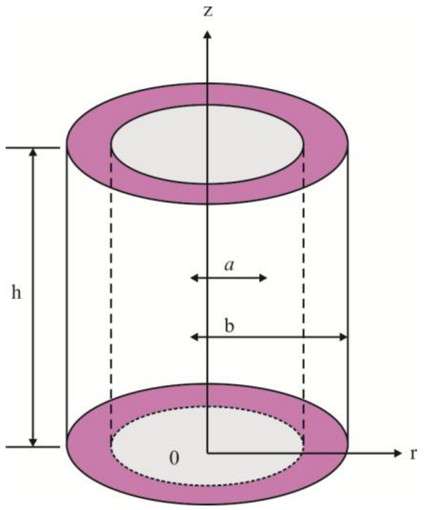

Fig 1: Geometry of the problem

III. SOLUTION OF THE OF THE PROBLEM

Applying transform defined in [2] to the equations (3), (4) and (6) over the variable r having $p = 0$ with responds to the boundary conditions of type (5) and taking the Laplace transform , one obtains

$$\bar{T}^*(n, z, s) = e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha P^2 t'} dt'\right] \tag{20}$$

where constants involved $\bar{T}^*(n, z, s)$ are obtained by using boundary conditions (6). Finally applying the inversion theorems of transform defined in [2] and inverse Laplace transform by means of complex contour integration and the residue theorem, one obtains the expressions of the temperature distribution $T(r, z, t)$ for heating processes as

$$T(r, z, t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{P_m(z)\, S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n \lambda_m} \times e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha P^2 t'} dt'\right] \tag{21}$$

Where n is the transformation parameter as defined in appendix, m is the Marchi-Fasulo transform parameter.

IV. DETERMINATION OF DISPLACEMENT AND STRESS FUNCTION

Substituting the value of temperature distribution from (21) in equation (1) one obtains the thermoelastic displacement function $\phi(r, z, t)$ as

$$\phi(r, z, t) = \frac{r^2 a_t}{4}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{P_m(z)\, S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n \lambda_m} \times e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha p^2 t'} dt'\right] \tag{22}$$

Using (22) in the equations (11) and (12) one obtains

$$U = \frac{k\, a_t(1+\nu)}{2\xi(1-\nu)}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\varphi_{nm}\left[2S_0(k_1, k_2, \mu_n r) + rS'_0(k_1, k_2, \mu_n r)\right] \tag{23}$$

$$W = \frac{r^2\, k\, a_t(1+\nu)}{2\xi(1-\nu)}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\lambda_m\, \varphi_{nm}\cot(\lambda_m z)\, S_0(k_1, k_2, \mu_n r) \tag{24}$$

Substitution the value of (22), (23) in (16) to (19) one obtains the stress functions as

$$\sigma_r = \frac{k\, a_t\,(1+\nu)}{2\xi(1-\nu)}\sum_{m,n=1}^{\infty}\varphi_{nm} \times\left[\begin{array}{l}(\lambda + 2G)\left(r^2 S''_0(k_1, k_2, \mu_n r)\right.\\ \left. + 4rS'_0(k_1, k_2, \mu_n r) + 2S_0(k_1, k_2, \mu_n r)\right)\end{array}\right] \times\lambda\left[2\, S_0(k_1, k_2, \mu_n r) + r\, S'_0(k_1, k_2, \mu_n r)\right] - r^2\lambda_m\, S_0(k_1, k_2, \mu_n r) \tag{25}$$

$$\sigma_z = -\frac{k\, a_t\,(1+\nu)}{2\xi(1-\nu)}\sum_{m,n=1}^{\infty}\varphi_{nm}\left[(\lambda + 2G)r^2\lambda_m^{\,2} S_0(k_1, k_2, \mu_n r) - \lambda\left(r^2 S''_0(k_1, k_2, \mu_n r) + 5rS'_0(k_1, k_2, \mu_n r) + 4S_0(k_1, k_2, \mu_n r)\right)\right] \tag{26}$$

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$$\sigma_\theta = \frac{k\,a_t(1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \varphi_{nm}$$
$$\times(\lambda+2G)[rS_0'(k_1,k_2,\mu_n r)+2S_0(k_1,k_2,\mu_n r)]$$
$$+\lambda\left[r^2 S_0''(k_1,k_2,\mu_n r)+4rS_0'(k_1,k_2,\mu_n r)+\left(2-r^2\lambda_m^2\right)S_0(k_1,k_2,\mu_n r)\right] \quad (27)$$

$$\tau_{rz} = \frac{k\,a_t G(1+\nu)}{\xi(1-\nu)} \sum_{m,n=1}^{\infty}(\lambda_m \varphi_{nm} \cot(\lambda_m z))$$
$$\times\left[r^2 S_0'(k_1,k_2,\mu_n r)+2rS_0(k_1,k_2,\mu_n r)\right] \quad (28)$$

V. SPECIAL CASE

Set $f(r,t)=(1-e^{-t})\,\delta(r-r_0)$ (29)

Applying finite transform defined in Marchi Zgrablich [2] to the equation (29) one obtains

$$\bar f(n,t)=(1-e^{-t})\, r_0\, S_0(k_1,k_2,\mu_n r_0) \quad (30)$$

Substituting the value of (29) in the equations (21) to (28) one obtains

$$T(r,z,t)=\frac{2\kappa}{\xi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\lambda_m \sin(\lambda_m z)}{\cos(\lambda_m \xi)}$$
$$\times\int_0^t (1-e^{-t})\, e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}dt'$$
$$\times r_0\, S_0(k_1,k_2,\mu_n r_0) S_0(k_1,k_2,\mu_n r) \quad (31)$$

$$G(r,t)=\frac{2\kappa}{\xi}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\lambda_m \sin(\lambda_m h)}{\cos(\lambda_m \xi)}$$
$$\times\int_0^t (1-e^{-t})\, e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}dt'$$
$$\times r_0\, S_0(k_1,k_2,\mu_n r_0) S_0(k_1,k_2,\mu_n r) \quad (32)$$

VII. NUMERICAL RESULTS

Set, a = 1, b = 2, h = 2,t = 1sec ξ=1.5 and k= 0.86 in equations (22) we get

$$G(r,t)=(1.15)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\lambda_m \sin(2\lambda_m)}{\cos(1.5\lambda_m)}$$
$$\times\int_0^1 (1-e^{-t'})\, e^{-0.86(\mu_n^2+\lambda_m^2)(1-t')}dt'$$
$$\times r_0\, S_0(k_1,k_2,\mu_n r_0) S_0(k_1,k_2,\mu_n r) \quad (33)$$

VIII. CONCLUSION

In this chapter, we modify the conceptual idea proposed by Khobragade et al [2] for hollow cylinder and the temperature distributions, displacement and stress functions at the edge $z=h$ occupying the region of the cylinder $a \le r \le b,\ 0 \le z \le h$ have been obtained with the known boundary conditions. We develop the analysis for the temperature field by introducing the transformation defined by Zgrablich et al, finite Fourier sine transform and Laplace transform techniques with boundaries conditions of radiations type. The series solutions converge provided we take sufficient number of terms in the series. Since the thickness of cylinder is very small, the series solution given here will be definitely convergent. Assigning suitable values to the parameters and functions in the series expressions can derive any particular case. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.

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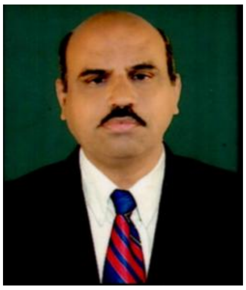

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