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# Steady- State Thermoelastic Problem of a Thin Rectangular Plate due to Heat Generation

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***Abstract-This paper is concerned with steady state thermoelastic problem of a thin clamped rectangular plate in which we need to determine the temperature distribution, displacement and thermal stresses when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.***

***Key Words:* Thin rectangular plate, steady state problem, direct thermo elastic problem.**

## I. INTRODUCTION

Khobragade et al. [1, 2] have derived thermal deflection of a thick clamped rectangular plate , Khobragade et al. [5, 6, 8-10] have investigated displacement function, temperature distribution and stresses of a thin rectangular plate and Khobragade et al. [11] have established displacement function , temperature distribution and stresses of a thick rectangular plate.

In this paper, an attempt is made to determine the temperature distribution, displacement function and thermal stresses at any point of the plate occupying the space D: $\{-a \le x \le a, -b \le y \le b, -h \le z \le h\}$ with the known boundary conditions. Finite Marchi-Fasulo transform technique is used to find the solution of the problem.

## II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space D: $-a \le x \le a, -b \le y \le b, -h \le z \le h$. The displacement components $u_x$ and $u_y$ $u_z$ in the x and y and z directions respectively as Tanigawa et al. [1] are

$$u_x = \int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\nu\frac{\partial^2 U}{\partial x^2}\right)+\lambda T\right]dx \tag{1}$$

$$u_y = \int_{-b}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-\nu\frac{\partial^2 U}{\partial y^2}\right)+\lambda T\right]dy \tag{2}$$

$$u_z = \int_{-h}^{h}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-\nu\frac{\partial^2 U}{\partial z^2}\right)+\lambda T\right]dz \tag{3}$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z) is the Airy's stress functions which satisfy the differential equation as Tanigawa et al. [1] is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z) = -\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\times T(x,y,z) \tag{4}$$

where T(x,y,z) denotes the temperature of a rectangular plate satisfy the following differential equation as Tanigawa et al. [1] is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z)}{k}=0 \tag{5}$$

where k is the thermal conductivity and $\alpha$ is the thermal diffusivity of the material,

subject to the initial and boundary conditions:

$$T(x,y,z)+k_1\left[\frac{\partial T(x,y,z)}{\partial x}\right]_{x=-a} = f_1(y,z) \tag{6}$$

$$T(x,y,z)+k_2\left[\frac{\partial T(x,y,z)}{\partial x}\right]_{x=a} = f_2(y,z) \tag{7}$$

$$\left[T(x,y,z)+k_3\frac{\partial T(x,y,z)}{\partial y}\right]_{y=-b} = f_3(x,z) \tag{8}$$

$$\left[T(x,y,z)+k_4\frac{\partial T(x,y,z)}{\partial y}\right]_{y=b} = f_4(x,z) \tag{9}$$

$$[T(x,y,z)]_{z=-h} = f_5(x,y) \tag{10}$$

$$[T(x,y,z)]_{z=h} = f_6(x,y) \tag{11}$$

The stress components in terms of U(x, y, z) as Tanigawa et al. [1] are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right] \tag{12}$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}\right] \tag{13}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right] \tag{14}$$

Equations (1) to (14) constitute the mathematical formulation of the problem under consideration.

## III. SOLUTION OF THE PROBLEM

By applying Marchi-Fasulo transform w.r.to x and y successively to the equation (5), we get

$$\left(\frac{\partial^2 \bar{\bar{T}}}{\partial z^2}\right)-\mu_p\bar{\bar{T}}=\psi \tag{15}$$

Where,

$$\mu_p = \left(\lambda_n^{\,2}+\mu_m^{\,2}\right)$$

$$\Psi = \frac{P_l(a)}{k_1}f_2-\frac{P_l(-a)}{k_2}f_1+\frac{Q_m(b)}{k_4}f_4-\frac{Q_m(-b)}{k_3}f_3+\frac{g}{k}$$

Equation (15) is a second order differential equation whose

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solution is given by

$$\overline{\overline{T}} = Ae^{\mu_p z} + Be^{-\mu_p z} + P.I$$

where , $P.I = \frac{1}{D^2 - \mu_p}\psi$

$$\overline{\overline{T}} = Ae^{\mu_p z} + Be^{-\mu_p z} + \Phi \tag{16}$$

where, $\Phi = P.I$

Using equation (10 ) and (11 ) in equation(16) we get

$$\overline{\overline{T}} = \frac{[f_5 + \Phi_{-h}]}{\sinh(2\mu_p h)}\sinh\mu_p(z-h) + \frac{[f_6 + \Phi_h]}{\sinh(2\mu_p h)}\sinh\mu_p(z+h) \tag{17}$$

Using the inverses oF Marchi-Fasulo transform to equation (17), one obtains the expression for temperature distribution as

$$T(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega \tag{18}$$

where m, n are the positive integers,

$$\Omega = \frac{[f_5 + \Phi_{-h}]}{\sinh(2\mu_p h)}\sinh\mu_p(z-h) + \frac{[f_6 + \Phi_h]}{\sinh(2\mu_p h)}\sinh\mu_p(z+h)$$

Substituting equation (18) in equation (4) we get

$$U = -\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega \tag{19}$$

Substituting equation (19) in equations (1)- (3), the displacement components are obtained as

$$u_x = \lambda\left\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{Q_m(y)}{\mu_m}\Omega - \frac{Q_m''(y)}{\mu_m}\Omega - \frac{Q_m(y)}{\mu_m}\Omega''\right]\frac{1}{\lambda_n} \times \int_{-a}^{a}P_n(x)dx + \vartheta\frac{P'_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n}\Lambda\right\} \tag{20}$$

$$u_y = \lambda\left\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P_n(x)}{\lambda_n}\Omega - \frac{P''_n(x)}{\lambda_n}\Omega - \frac{P_n(x)}{\lambda_n}\Omega''\right]\frac{1}{\mu_m} \times \int_{-b}^{b}Q_m(y)dy + \vartheta\frac{P'_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n}\Omega\right\} \tag{21}$$

$$u_z = \lambda\left\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P'_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n} - \frac{P''_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n} - \frac{P'_n(x)}{\lambda_n}\frac{Q_m''(y)}{\mu_n}\right] \times \int_{-h}^{h}\Omega dz + \vartheta\frac{P'_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n}\Omega'\right\} \tag{22}$$

## IV. DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z) from equation (19) in the equations (12) to (14) one obtain the expression for stress functions as,

$$\sigma_{xx} = -\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P_n(x)}{\lambda_n}\frac{Q''_m(y)}{\mu_m}\Omega + \frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega''\right] \tag{23}$$

$$\sigma_{yy} = -\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P''_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega + \frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega''\right] \tag{24}$$

$$\sigma_{zz} = -\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P''_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega + \frac{P_n(x)}{\lambda_n}\frac{Q''_m(y)}{\mu_m}\Omega\right] \tag{25}$$

## V. SPECIAL CASE

Set $f_5(x,y) = (x^2 + ax)(y^2 + ay)(-h)$,

$$f_6(x,y) = (x^2 + ax)(y^2 + ay)(h)$$

$$\overline{\overline{f}}_5(x,y) = \left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n^{\ 2}}\right] \times \left[\frac{b_m\cos^2(b_m b) - \cos(b_m b)\sin(b_m b)}{b_m^{\ 2}}\right] \times (-h) \tag{26}$$

$$\overline{\overline{f}}_6(x,y) = \left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n^{\ 2}}\right] \times \left[\frac{b_m\cos^2(b_m b) - \cos(b_m b)\sin(b_m b)}{b_m^{\ 2}}\right] \times (h) \tag{27}$$

Substituting these values in equation (18) we get

$$T(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega_1 \tag{28}$$

## VI. NUMERICAL RESULTS

Set $a$ = 1m, b = 2m, h = 2m,t = 1sec and k= 0.86 in equation (28), we obtain

$$T(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}$$

$$\frac{\left[\left[\frac{a_n\cos^2(a_n) - \cos(a_n)\sin(a_n)}{a_n^{\ 2}}\right] \times \left[\frac{b_m\cos^2(2b_m) - \cos(2b_m)\sin(2b_m)}{b_m^{\ 2}}\right] \times (-2) + Z_{-2}\right]}{\sinh 4\mu_p}\sinh\mu_p(z-2)$$

$$+\frac{\left[\left[\frac{a_n\cos^2(a_n) - \cos(a_n)\sin(a_n)}{a_n^{\ 2}}\right] \times \left[\frac{b_m\cos^2(2b_m) - \cos(2b_m)\sin(2b_m)}{b_m^{\ 2}}\right] \times (2) + Z_2\right]}{\sinh 4\mu_p}\sinh\mu_p(z+2) \tag{29}$$

## VII. CONCLUSION

The temperature distribution, displacement function and thermal stresses of a thin rectangular plate have been obtained, with the aid of finite Marchi-Fasulo transform technique when the stated boundary conditions are known. The results are obtained in the form of infinite series. The series solutions converge provided if we take sufficient number of terms in the series.

The results that are obtained can be applied to the design

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of useful structures or machines in engineering applications.

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