ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 3, September 2015

Thermal Stresses of Semi Infinite Rectangular Beam with Internal Heat Source: Steady-State Problem

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Abstract- ***This paper is concerned with inverse transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite rectangular beam when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem. The results are depicted graphically.***

Key Words: **Semi-infinite rectangular beam, inverse transient problem, Integral transform.**

I. INTRODUCTION

Khobragade et al. [2-7, 9] have investigated temperature distribution, displacement function, and stresses of a thin rectangular plate and **Khobragade et al**. [8] have established displacement function , temperature distribution and stresses of a semi-infinite rectangular beam.

In this Section, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a semi-infinite rectangular beam occupying the region D : $-a \le x \le a$;$0 \le y \le b$, $0 \le z \le \infty$ with known boundary conditions. Here Marchi-Fasulo transforms and Fourier cosine transform techniques have been used to find the solution.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space D: $a \le x \le a$; $0 \le y \le b$, $0 \le z \le \infty$. The displacement components u_x and u_y u_z in the x and y and z directions respectively as **Tanigawa et al.** [1] are

$$u_x = \int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\nu\frac{\partial^2 U}{\partial x^2}\right)+\lambda T\right]dx \tag{1}$$

$$u_y = \int_{0}^{b}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}-\nu\frac{\partial^2 U}{\partial y^2}\right)+\lambda T\right]dy \tag{2}$$

$$u_z = \int_{0}^{\infty}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-\nu\frac{\partial^2 U}{\partial z^2}\right)+\lambda T\right]dz \tag{3}$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z) is the Airy's stress functions which satisfy the differential equation as **Tanigawa et al.** [1] is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z) = -\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) T(x,y,z) \tag{4}$$

where T(x,y,z) denotes the temperature of a rectangular beam satisfy the following differential equation as Tanigawa et al. [1] is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z)}{k}=0 \tag{5}$$

where k is the thermal conductivity and α is the thermal diffusivity of the material, subject to the boundary conditions

$$\left[T(x,y,z)+k_1\frac{\partial T(x,y,z)}{\partial x}\right]_{x=a} = f_1(y,z) \tag{6}$$

$$\left[T(x,y,z)+k_2\frac{\partial T(x,y,z)}{\partial x}\right]_{x=-a} = f_2(y,z) \tag{7}$$

$$\left[\frac{\partial T(x,y,z)}{\partial y}\right]_{y=0} = f_3(x,z) \tag{8}$$

$$\left[\frac{\partial T(x,y,z)}{\partial y}\right]_{y=\xi} = f_4(x,z) \tag{9}$$

$$[T(x,y,z)]_{y=b} = G(x,z) \quad \text{(Unknown)} \tag{10}$$

$$[T(x,y,z)]_{Z=0} = 0 \tag{11}$$

$$[T(x,y,z)]_{Z=\infty} = 0 \tag{12}$$

The stress components in terms of U(x, y, z) **Tanigawa et al**. [1] are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right] \tag{13}$$

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ISO 9001:2008 Certified
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$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right] \tag{14}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right] \tag{15}$$

The equations (1) to (15) constitute the mathematical formulation of the problem under consideration.

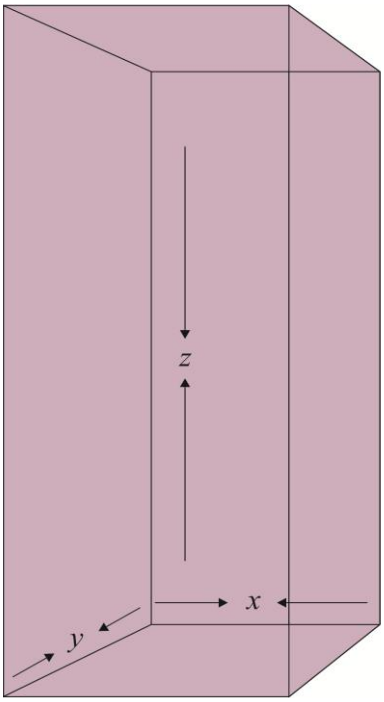

Fig 1: Geometry of the problem

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform and Fourier sine transform to the equation (5), we get

$$\frac{d\overline{T}^*}{dt} - q^2\overline{T}^* = \Psi \tag{16}$$

Where, $q^2 = \lambda_n^2 + p^2$

$$\psi = \frac{P_n(-a)}{k_2} f_2^* - \frac{P_n(a)}{k_1} f_1^* - \frac{\bar{g}^*}{k}$$

Equation (16) is a linear equation whose solution is given by

$$\overline{T}^* = Ae^{qy} + Be^{-qy} + F(y) \tag{17}$$

Where $F(y)$ is the P.I.

Using boundary conditions (8) and (9) we get

$$A = \frac{e^{-q\xi}\left(F'(0) - \bar{f}_3^*\right) + F'(\xi)}{2q\sinh(q\xi)}$$

$$B = \frac{e^{q\xi}\left(F'(0) - \bar{f}_3^*\right) + \bar{f}_4^* - F'(\xi)}{2q\sinh(q\xi)}$$

Substituting the values of A and B in equation (17) one obtains

$$\overline{T}^* = \frac{[\left(F'(0) - \bar{f}_3^*\right)\cosh\left(q(y-\xi)\right) + \left(\bar{f}_4^* - F'(\xi)\right)\cosh(qy)]}{q\sinh(q\xi)} + F(y) \tag{18}$$

Applying inverse Fourier sine transform and inverse Marchi-Fasulo transform on equation (18) we get,

$$T = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n} \times \int_0^{\infty}\left\{\frac{\left(F'(0) - \bar{f}_3^*\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-\xi) + \left(\bar{f}_4^* - F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)} + F(y)\right\} \times \sin(pz)\,dp \tag{19}$$

$$G = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n} \times \int_0^{\infty}\left\{\frac{\left(F'(0) - \bar{f}_3^*\right)\cosh\sqrt{p^2+\lambda_n^2}\,(b-\xi) + \left(\bar{f}_4^* - F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,b\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)} + F(b)\right\} \tag{20}$$

IV. AIRY'S STRESS FUNCTIONS

Substituting the value of temperature distribution (17) T(x,y,z) from (19) in equation (4) one obtains

$$U = \frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n} \int_0^{\infty}\left\{\frac{\left(F'(0) - \bar{f}_3^*\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-\xi) + \left(\bar{f}_4^* - F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)} + F(y)\right\} \times \sin(pz)\,dp \tag{20}$$

(18)

(14)

V. DISPLACEMENT COMPONENTS

Substituting the values of Airy's stress function from equation (20) in the equation (1) to (3), (15) one obtains

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 5, Issue 3, September 2015

$$u_x = \frac{-2\lambda}{\pi}\int_{-a}^{a}\int_{0}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-\xi)+\left(\bar{f}_4^{\,*}-F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)}\left[\left(\lambda_n^2\right)P_n(x)-\nu P_n''(x)\right]+\left[F''(y)-\left(P^2+1\right)F(y)\right]P_n(n)-\nu F(y)P_n''(n)\right\}\sin(pz)dp\,dx \quad (21)$$

$$u_y = \frac{2\lambda}{\pi}\int_{-a}^{b}\int_{0}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,(y-\xi)\right)+\left(\bar{f}_4^{\,*}-F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)}\left[\left((\nu+1)P^2+\nu\lambda_n^2+1\right)P_n(x)-\nu P_n''(x)\right]+\left[\left(P^2+1\right)F(y)+\nu F''(y)\right]P_n(n)-F(y)P_n''(n)\right\}\sin(pz)dp\,dy \quad (22)$$

$$u_z = \frac{-2\lambda}{\pi}\int_{0}^{\infty}\int_{0}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-\xi)+\left(\bar{f}_4^{\,*}-F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)}\left[\left((\nu-1)P^2+\lambda_n^2-1\right)P_n(x)+P_n''(x)\right]+\left[\left(\nu P^2-1\right)F(y)+\nu F''(y)\right]P_n(n)+F(y)P_n''(n)\right\}\times\sin(pz)dp\,dz \quad (23)$$

VI. DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z) from equation (20) in the equation (14) to (16) one obtain the stress functions as,

$$\sigma_{xn} = \frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_{0}^{\infty}\frac{P_n(x)}{\lambda_n}\left\{\lambda_n^2\,\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-\xi)+\left(\bar{f}_4^{\,*}-F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)}+F''(y)-P^2F(y)\right\}\sin(pz)dp \quad (24)$$

$$\sigma_{yy} = \frac{2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_{0}^{\infty}\frac{P_n(x)-P_n''(x)}{\lambda_n}\left[\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-\xi)+\left(\bar{f}_4^{\,*}-F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)}+F(y)\right]\sin(pz)dp \quad (25)$$

$$\sigma_{zz} = \frac{-2\lambda E}{\pi}\sum_{n=1}^{\infty}\int_{0}^{\infty}\frac{1}{\lambda_n}\left\{\frac{\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}\,(y-\xi)+\left(\bar{f}_4^{\,*}-F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)}\left[\left(P^2+\lambda_n^2\right)P_n(x)+P_n''(x)\right]+F''(y)P_n(x)+F(y)P_n''(x)\right\}\sin(pz)dp \quad (26)$$

Equations (20) and (21) are the required solutions.

VII. SPECIAL CASE

Set

$$f(x,y,z,t) = (x-a)^2(x+a)^2(z+e^{-z})(e^{y-t})$$

$$\overline{f}(n,y,z,t) = (z+e^{-z})(e^{y-t}) \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{a_n^{\,2}}\right]$$

Substituting this value in equation (20) we get

$$G = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}$$

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ISSN: 2277-3754

ISO 9001:2008 Certified

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Volume 5, Issue 3, September 2015

$$\int_0^\infty \left\{ \frac{\begin{matrix}\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2}(b-\xi) \\ \left(\bar{f}_4^{\,*}-F'(\xi)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,b\right)\end{matrix}}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)} + F(b) \right\} \sin(pz)dp \qquad (27)$$

VIII. NUMERICAL RESULTS

Set $a=2$, $k=0.86$, $b=3$, $\xi=2$, $t=1$ sec in the equations (20) to obtain

$$G=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}$$

$$\int_0^\infty \left\{ \frac{\begin{matrix}\left(F'(0)-\bar{f}_3^{\,*}\right)\cosh\sqrt{p^2+\lambda_n^2} \\ \left(\bar{f}_4^{\,*}-F'(2)\right)\cosh\left(\sqrt{p^2+\lambda_n^2}\,y\right)\end{matrix}}{\sqrt{p^2\lambda_n^2}\,\sinh\left(\xi\sqrt{p^2+\lambda_n^2}\right)} + F(3) \right\} \sin(pz)dp \qquad (28)$$

IX. MATERIAL PROPERTIES

The numerical calculations has been carried out for an Aluminum (pure) rectangular beam with the material properties as,

Density ρ =169 lb/ft ^{3}

Specific heat = 0.208 Btu/lbOF

Thermal conductivity K = 117 Btu/(hr. ftOF)

Thermal diffusivity α = 3.33 ft^2/hr.

Poisson ratio ν = 0.35

Coefficient of linear thermal expansion α_t = 12.84 x 10^{-6}1/F

Lame constant μ = 26.67

Young's modulus of elasticity E = 70G Pa

X. DIMENSIONS

The constants associated with the numerical calculation are taken as

Length of rectangular beam x = 4ft

Breath of rectangular beam y = 3 ft

Height of rectangular beam z = 10^{3}ft

XI. CONCLUSION

In this article, the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a semi-infinite rectangular beam have been obtained, when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and semi-infinite Fourier cosine transform techniques. The results are obtain in the form of infinite series in terms of Bessel's function and depicted graphically.

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