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Thermal Stresses of a Thin Rectangular Plate With Internal Moving Heat Source

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Abstract- This paper deals with the study of heat conduction equation with internal moving heat source and to find the thermal stresses in very thin rectangular plate. There is a lot of work on finding the thermal stresses of rectangular plate with internal heat generation, but in this paper authors have been considered the internal moving point heat source. The solution of the problem has been found by using finite Marchi-Fasulo transform and finite Fourier sine transform techniques. The results are obtained in the form of infinite series.

Key Words: **Rectangular plate, internal moving heat source, Marchi-Fasulo transform thermal stresses.**

I. INTRODUCTION

Many authors considered the heat conduction equation with internal heat generation and find the solution by various methods. N.W.Khobragade and P.C.Wankhede [7] studied an inverse unsteady-state thermoelastic problem of a thin rectangular plate. Recently D.T.Solanke and M.H.Durge [1] have considered the heat conduction equation with internal moving heat source for a Neumann's thin rectangular plate and find the thermal stresses by using Green's theorem. Also D.T.Solanke and M.H.Durge [2] have determined the temperature distribution and thermal stresses in thin rectangular plate with moving line heat source taking second type boundary condition by using integral transform technique and Green's theorem.

In present paper, authors considered thermoelastic problem with second and third type boundary condition in thin rectangular plate occupying the region $D:-a\le x\le a,-b\le y\le b,0\le z\le h\}$.

The solution of the problem is obtained by using finite Marchi-Fasulo transform and finite Fourier sine transform techniques in the form of infinite series.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the region $D:-a\le x\le a,\ -b\le y\le b,0\le z\le h\}$ where $h<b<a,\ h$ is thickness which is very small. The plate is subjected to the motion of moving point heat source at the point $(0,y',z')$ which move it's place along x,y,z axes with constant velocity vector

$$\bar{v}=v_1 i+v_2 j+v_3 k$$

Where v_1,v_2,v_3 are component of velocity vector along x,y,z axes respectively. The temperature distribution of the rectangular plate is given by

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (1)$$

Where k is the thermal conductivity and α is thermal diffusivity of the material of the plate.
Consider an instantaneous moving point heat source at point $(0,y',z')$ and releasing its heat spontaneously at time t' Such volumetric moving heat source in rectangular coordinates is given by

$$g(x,y,z,t)=g_0\delta(x)\delta(y-y')\delta(z-z')\delta(t-t')$$

Hence equation (1) becomes

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{k}g_0\delta(x)\,\delta(y-y')\delta(z-z')\delta(t-t')$$

$$=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2)$$

Where $y'=v_2 t$ and $z'=v_3 t$, with initial condition

$$T(x,y,z,0)=0$$

(3)

And the boundary conditions are given by

$$\left[T(x,y,z,t)+k_1\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a}=0 \qquad (4)$$

$$\left[T(x,y,z,t)+k_2\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a}=0$$

(5)

$$\left[T(x,y,z,t)+k_3\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=b}=0$$

(6)

$$\left[T(x,y,z,t)+k_4\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=-b}=0$$

(7)

$$[T(x,y,z,t)]_{z=0}=f_1(x,y,t) \qquad (8)$$

$$[T(x,y,z,t)]_{z=h}=f_2(x,y,t) \qquad (9)$$

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Let us introduce a thermal stress function χ related to component of stress in the rectangular coordinates system as [6] is

$$\chi = \chi_c + \chi_p \quad (10)$$

where χ_c is the complementary solution and χ_p is particular solution.

χ_c and χ_p are governed by an equations,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_c = 0$$

(11)

and

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \chi_p = -\alpha E \Gamma .$$

(12)

Since plate is thin z is negligible and where $\Gamma = T - T_0$, T_0 is initial temperature.

Also component of stress functions are given by

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2} \quad (13)$$

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2}$$

(14)

$$\sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y}$$

(15)

The boundary condition is $\sigma_{yy} = 0$, $\sigma_{xy} = 0$ at $y = b$.

Equation (1) to (15) constitutes the statement of the problem.

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform two times and then finite Fourier sine transform to equation (2), also using given boundary conditions we get

$$-a_l^2 \bar{\bar{T}}^* - b_m^2 \bar{\bar{T}}^* - \frac{n^2\pi^2}{h^2}\bar{\bar{T}}^* + \frac{n\pi}{h}[(-1)^{n+1} f_2 + f_1]$$
$$+ \frac{1}{k} g_0 4 a_l a \cos(a_l a)\cos(b_m b)\delta(t-t')$$
$$\times \cos(d_n y')\sin\left(\frac{n\pi z'}{h}\right) = \frac{1}{\alpha}\frac{d\bar{\bar{T}}^*}{dt} \quad (16)$$

Solving above equation and using initial condition we get,

$$\bar{\bar{T}}^* = e^{-\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)t}$$
$$\times \left[\int e^{\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)t} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] \right.\right.$$
$$+ \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')$$
$$\left.\left. \times \delta(t-t')\sin\left(\frac{n\pi z'}{h}\right)\right\rangle dt + \Omega \right]$$

(17)

Where

$$\Omega = -\left[\int e^{\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)t} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] \right.\right.$$
$$+ \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')$$
$$\left.\left. \times \delta(t-t')\sin\left(\frac{n\pi z'}{h}\right)\right\rangle dt + T_1 \right]_{t=0}$$

Taking inversion of sine transform and Marchi-Fasulo transform, we get

$$T = \left(\frac{2}{h}\right) \sum_{l,m,n=1}^{\infty} \frac{P_l(x)}{\lambda_l}\frac{R_m(y)}{\mu_m}$$
$$\times \left[\int e^{\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)(t-t')} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] \right.\right.$$
$$+ \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')$$
$$\left.\left. \times \delta(t-t')\sin\left(\frac{n\pi z'}{h}\right)\right\rangle dt + T_1 \right] \sin\left(\frac{n\pi z}{h}\right)\Bigg\}$$

(18)

and

$$\Gamma = \left(\frac{2}{h}\right) \sum_{l,m,n=1}^{\infty} \frac{P_l(x)}{\lambda_l}\frac{R_m(y)}{\mu_m}$$
$$\times \left[\int e^{\alpha\left(a_l^2 + b_m^2 + \frac{n^2\pi^2}{h^2}\right)(t-t')} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] \right.\right.$$
$$+ \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')$$

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$$\times \delta(t-t')\sin\left(\frac{n\pi z'}{h}\right)\Big\rangle dt + T_1\Big]\sin\left(\frac{n\pi z}{h}\right)\Bigg\}$$

IV. DETERMINATION OF STRESS FUNCTIONS

Let the suitable form of χ_c satisfying (11) is given by

$$\chi_c = \sum_{l,m,n=1}^{\infty}\Bigg\{ y^2\left[c_1 e^{\frac{n\pi x}{a}} + c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + y^2\left[c_3 e^{\frac{n\pi x}{a}} + c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\Bigg\} \quad (19)$$

Let the suitable form of χ_c satisfying (12) is given by

$$\chi_p = \frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\sum_{l,m,n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\frac{R_m(y)}{\mu_m} \times\Bigg[\int e^{\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')}\Big\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] + \frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y') \times \delta(t-t')\sin\frac{n\pi z'}{h}\Big\rangle dt + T_1\Big]\sin\left(\frac{n\pi z}{h}\right)\Bigg\} \quad (20)$$

Therefore χ is given by

$$\chi = \sum_{l,m,n=1}^{\infty}\Bigg\{ y^2\left[c_1 e^{\frac{n\pi x}{a}} + c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + y^2\left[c_3 e^{\frac{n\pi x}{a}} + c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}$$

$$+\frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\times\sum_{l,m,n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\frac{R_m(y)}{\mu_m}\times\Bigg[\int e^{\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')}\Big\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right]$$

$$+\frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')$$

$$\times \delta(t-t')\sin\left(\frac{n\pi z'}{h}\right)\Big\rangle dt + T_1\Big]\sin\left(\frac{n\pi z}{h}\right)\Bigg\} \quad (21)$$

Using (21) in (13),(14),(15) we get

$$\sigma_{xx} = \sum_{l,m,n=1}^{\infty}\Bigg\{ 2\left[c_1 e^{\frac{n\pi x}{a}} + c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + 2\left[c_3 e^{\frac{n\pi x}{a}} + c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}$$

$$+\frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\times\Bigg\{\sum_{l,m,n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\frac{R_m(y)}{\mu_m}\times\Bigg[\int e^{\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')}\Big\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right]$$

$$+\frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')$$

$$\times \delta(t-t')\sin\left(\frac{n\pi z'}{h}\right)\Big\rangle dt + T_1\Big]\sin\left(\frac{n\pi z}{h}\right)\Bigg\}\Bigg\}_{yy} \quad (22)$$

$$\sigma_{yy} = \sum_{l,m,n=1}^{\infty}\Bigg\{ y^2\frac{n^2\pi^2}{a^2}\left[c_1 e^{\frac{n\pi x}{a}} + c_2 e^{\frac{-n\pi x}{a}}\right]\sin\left(\frac{n\pi z}{h}\right) + y^2\frac{n^2\pi^2}{a^2}\left[c_3 e^{\frac{n\pi x}{a}} + c_4 e^{\frac{-n\pi x}{a}}\right]\cos\left(\frac{n\pi z}{h}\right)\Bigg\}$$

$$+\frac{2\alpha E a^2 b^2}{h(a^2+b^2)}\times\Bigg\{\sum_{l,m,n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\frac{R_m(y)}{\mu_m}\times\Bigg[\int e^{\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')}\Big\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right]$$

$$+\frac{a}{k} g_0 4 a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y')$$

$$\times \delta(t-t')\sin\left(\frac{n\pi z'}{h}\right)\Big\rangle dt + T_1\Big]\sin\left(\frac{n\pi z}{h}\right)\Bigg\}\Bigg\}_{xx} \quad (23)$$

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$$\sigma_{xy} = \sum_{l,m,n=1}^{\infty} \left\{ 2y\frac{n\pi}{a}\left[c_1 e^{\frac{n\pi x}{a}} - c_2 e^{\frac{-n\pi x}{a}} \right] \sin\left(\frac{n\pi z}{h}\right) + 2y\frac{n\pi}{a}\left[c_3 e^{\frac{n\pi x}{a}} - c_4 e^{\frac{-n\pi x}{a}} \right] \cos\left(\frac{n\pi z}{h}\right) \right\}$$

$$+ \frac{2\alpha E a^2 b^2}{h(a^2+b^2)} \times \left\{ \sum_{l,m,n=1}^{\infty} \frac{P_1(x)}{\lambda_1}\frac{R_m(y)}{\mu_m} \times \left[\int e^{\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] + \frac{a}{k} g_0 4a_l b_m \cos(a_l a)\cos(b_m b)\cos(d_n y') \times \delta(t-t')\sin\left(\frac{n\pi z'}{h}\right) \right\rangle dt + T_1 \right] \sin\left(\frac{n\pi z}{h}\right) \right\}_{xy} \quad (24)$$

Using the boundary conditions $\sigma_{xy} = 0, \sigma_{yy} = 0$, at $y = b$ and equation (23) and (24) we get,

$$C_1 = -\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2} + \frac{a\gamma_2}{2bn\pi}\right]$$

$$C_2 = -\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2} + \frac{a\gamma_2}{2bn\pi}\right] e^{\frac{2n\pi x}{a}} + \frac{a\gamma_2}{2bn\pi} e^{\frac{n\pi x}{a}}$$

And $C_3 = C_4 = 0$

Where

$$\gamma_1 = \frac{2\alpha E a^2 b^2}{h(a^2+b^2)} \times \left\{ \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{P_1(x)}{\lambda_1}\frac{P_m(y)}{\lambda_m} \times \left[\int e^{\alpha\left(a_l^2+a_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] + \frac{a}{k} g_0 4a_l a_m \cos(a_l a)\cos(a_m b)\cos(a_n y') \times \delta(t-t')\sin\frac{n\pi z'}{h} \right\rangle dt + T_1 \right] \right\}_{xy} \Bigg\}_{y=b} \quad (25)$$

$$\gamma_2 = \frac{2\alpha E a^2 b^2}{h(a^2+b^2)} \times \left\{ \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{P_1(x)}{\lambda_1}\frac{P_m(y)}{\lambda_m} \times \left[\int e^{\alpha\left(a_l^2+a_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] + \frac{a}{k} g_0 4a_l a_m \cos(a_l a)\cos(a_m b)\cos(a_n y') \times \delta(t-t')\sin\frac{n\pi z'}{h} \right\rangle dt + T_1 \right] \right\}_{xy} \Bigg\}_{y=b} \quad (26)$$

Substituting the above values in (22) to (24), it gets

$$\sigma_{xx} = \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left\{ 2\left[-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2} + \frac{a\gamma_2}{2bn\pi}\right] e^{\frac{n\pi x}{a}} + \left[-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2} + \frac{a\gamma_2}{2bn\pi}\right] e^{\frac{n\pi x}{a}} + \frac{a\gamma_2}{2bn\pi} \right] \sin\left(\frac{n\pi z}{h}\right) \right] \right\}$$

$$+ \frac{2\alpha E a^2 b^2}{h(a^2+b^2)} \times \left\{ \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{P_1(x)}{\lambda_1}\frac{P_m(y)}{\lambda_m} \times \left[\int e^{\alpha\left(a_l^2+a_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] + \frac{a}{k} g_0 4a_l a_m \cos(a_l a)\cos(a_m b)\cos(a_n y') \times \delta(t-t')\sin\frac{n\pi z'}{h} \right\rangle dt + T_1 \right] \sin\left(\frac{n\pi z}{h}\right) \right\}_{yy} \quad (27)$$

$$\sigma_{yy} = \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \left\{ y^2 \frac{n^2\pi^2}{a^2}\left[-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2} + \frac{a\gamma_2}{2bn\pi}\right] e^{\frac{n\pi x}{a}} + \left[-\frac{1}{2}\left[\frac{a^2\gamma_1}{bn^2\pi^2} + \frac{a\gamma_2}{2bn\pi}\right] e^{\frac{n\pi x}{a}} + \frac{a\gamma_2}{2bn\pi} \right] \sin\left(\frac{n\pi z}{h}\right) \right] \right\}$$

$$+ \frac{2\alpha E a^2 b^2}{h(a^2+b^2)} \times \left\{ \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{P_1(x)}{\lambda_1}\frac{P_m(y)}{\lambda_m} \times \left[\int e^{\alpha\left(a_l^2+a_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')} \left\langle \frac{n\pi}{h}\left[(-1)^{n+1} f_2 + f_1\right] + \frac{a}{k} g_0 4a_l a_m \cos(a_l a)\cos(a_m b)\cos(a_n y') \right. \right.$$

$$\times\delta(t-t')\sin\frac{n\pi z'}{h}\Big\rangle dt+T_1\Big]\sin\left(\frac{n\pi z}{h}\right)\Big\}\Big\}_{xx} \quad (28)$$

$$\sigma_{xy}=\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Big\{2y\frac{n\pi}{a}\Big[-\frac{1}{2}\Big[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\Big]e^{\frac{n\pi x}{a}}$$

$$+\Big[\frac{1}{2}\Big[\frac{a^2\gamma_1}{bn^2\pi^2}+\frac{a\gamma_2}{2bn\pi}\Big]e^{\frac{n\pi x}{a}}-\frac{a\gamma_2}{2bn\pi}\Big]\sin\left(\frac{n\pi z}{h}\right)\Big\}\Big\}$$

$$+\frac{2\alpha Ea^2b^2}{h(a^2+b^2)}\times\Big\{\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_1(x)}{\lambda_1}\frac{P_m(y)}{\lambda_m}$$

$$\times\Big[\int e^{\alpha\left(a_l^2+a_m^2+\frac{n^2\pi^2}{h^2}\right)(t-t')}\Big\langle\frac{n\pi}{h}\big[(-1)^{n+1}f_2+f_1\big]$$

$$+\frac{a}{k}g_0 4a_l a_m\cos(a_l a)\cos(a_m b)\cos(a_n y')$$

$$\times\delta(t-t')\sin\frac{n\pi z'}{h}\Big\rangle dt+T_1\Big]\sin\left(\frac{n\pi z}{h}\right)\Big\}\Big\}_{xy} \quad (29)$$

V. SPECIAL CASE AND NUMERICAL RESULTS

Set

$$f_1(x,y,t)=(x-a)^2(x+a)^2(y-b)^2(y+b)^2(1-e^{-t})$$
$$f_2(x,y,t)=(x-a)^2(x+a)^2(y-b)^2(y+b)^2e^h(1-e^{-t})$$

a = 2, b = 4, h = 2, t = 1 sec in equation (18) we get

$$T(x,y,z,t)=\sum_{l,m,n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\frac{R_m(y)}{\mu_m}$$

$$\times\Big[\int e^{\alpha\left(a_l^2+b_m^2+\frac{n^2\pi^2}{4}\right)(1-t')}\Big\langle\frac{n\pi}{2}\big[(-1)^{n+1}f_2+f_1\big]$$

$$+\frac{2}{k}g_0 4a_l b_m\cos(2a_l)\cos(4b_m)\cos(d_n y')$$

$$\times\delta(t-t')\sin\left(\frac{n\pi z'}{2}\right)\Big\rangle dt+T_1\Big]\sin\left(\frac{n\pi z}{2}\right)\Big\} \quad (30)$$

VI. CONCLUSION

In this paper, the temperature distribution has been found by using the finite Marchi-Fasulo transform and the finite Fourier sine transform with internal moving heat point source. The results are obtained in the form of infinite series. Also I determined the thermal stress functions which are obtained in the form of infinite series.

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