**Abstract**—The main objective of this study is to obtain and compare the performance of the standard Bayesian estimators of the reliability function $R(t)$ of the Pareto type I distribution under Generalized square error loss function in addition of Quadratic loss function, with informative and non-informative prior, with assuming that, the scale parameter, $\alpha$, is known. Estimators are compared empirically using Monte Carlo simulation by employing the Integral mean squares error (IMSE).

**Index Terms**—Pareto distribution; Maximum likelihood estimator, Reliability function, Generalized square error loss function, Quadratic loss function.

**I. INTRODUCTION**

The Pareto distribution was introduced (Pareto, 1897) as a model for the distribution of income. In addition to economics, its models in several different forms are now being used in a wide range of fields such as insurance, business, engineering, survival analysis, reliability and life testing.[6] The probability density function of Pareto type I distribution is defined as following [8]:

$$f(x, \theta) = \frac{\theta x^\theta}{(x + \alpha)^{\theta+1}} \quad t \geq \alpha, \quad \alpha > 0, \quad \theta > 0$$  \hspace{1cm} (1)

Where $t$ is a random variable, $\theta$ and $\alpha$ are the shape and scale parameters respectively. The cumulative distribution function of Pareto distribution type I, is given by:

$$F(x) = 1 - \left(\frac{x}{\alpha}\right)^\theta \quad t \geq \alpha, \alpha > 0, \quad \theta > 0$$  \hspace{1cm} (2)

The reliability function is given by:

$$R(t) = P(t > t)$$

$$= \int_{t}^{\infty} f(t, \alpha, \theta) \, dt$$

$$= \int_{t}^{\infty} \frac{\theta x^\theta}{u^{\theta+1}} \, du = \theta \alpha^\theta \left[\frac{1}{\theta t^\theta}\right]$$

$$R(t) = \left(\frac{\alpha}{t}\right)^\theta \quad t \geq \alpha, \quad \theta > 0, \quad \alpha > 0$$  \hspace{1cm} (3)

The Pareto distribution belongs to the exponential family of distribution as the density function (1) can be written as:

$$f(x, \theta) = \theta e^{\theta x - (\theta + 1)\ln t} = \theta e^{-\ln t} e^{-\theta \ln \left(\frac{1}{\alpha}\right)}$$

Hence,

$$a(\theta) = \theta, \quad b(\theta) = e^{-\ln t}, \quad c(x) = -\theta, \quad d(\theta) = \ln \left(\frac{1}{\alpha}\right)$$

Therefore, statistic $P = \sum_{i=1}^{n} \ln \left(\frac{1}{\alpha}\right)$ is a complete sufficient statistic for $\theta$, and it is easy to show that, $P$ is distributed as Gamma distribution with parameters $n$ and $\theta$.

**II. MAXIMUM LIKELIHOOD ESTIMATORS (MLE)**

Let $(t_1, \ldots, t_n)$ be the set of $n$ random lifetime from Pareto type I distribution with parameters $\theta$ and $\alpha$.

The likelihood function is

$$L(\theta; t_1, t_2, \ldots, t_n) = \prod_{i=1}^{n} f(t_i; \theta)$$

$$L(t_1, \ldots, t_n; \theta) = \frac{\theta^n \alpha^n \pi^n}{\prod_{i=1}^{n} t_i^{\theta+1}} = \theta^n \alpha^n \pi^n e^{-(\theta + 1)\sum \ln t}$$

$$= \theta^n \pi^n \alpha^n e^{-(\theta + 1)\sum \ln t} \quad \hspace{1cm} (4)$$

Taking the logarithm for the likelihood function, so we get the function

$$\ln L(\theta; t_1, \ldots, t_n) = n \ln \theta + n \theta \ln \alpha - (\theta + 1) \sum_{i=1}^{n} \ln t_i$$

The partial derivative for the log-likelihood function, with respect to $\theta$ and then, equating to zero is

$$\frac{\partial[\ln L(\theta; t_1, \ldots, t_n)]}{\partial \theta} = n - n \theta \ln \alpha - \sum_{i=1}^{n} \ln t_i = 0$$

Hence, the MLE of $\theta$ denoted by $\hat{\theta}_{\text{ML}}$ is

$$\hat{\theta}_{\text{MLE}} = \frac{n}{\sum_{i=1}^{n} \ln t_i - n \ln \alpha} = \frac{n}{\sum_{i=1}^{n} \ln \left(\frac{1}{\alpha}\right)}$$  \hspace{1cm} (5)

Since the maximum likelihood estimator is invariant and one to one mapping [4], the Maximum likelihood estimator of Reliability function $R(t)_{\text{MLE}}$ well be

$$R(t)_{\text{MLE}} = \left(\frac{\alpha}{t}\right)^\theta$$  \hspace{1cm} (6)
III. STANDARD BAYES ESTIMATOR

In this section, we used a two loss functions as following

A. Generalized Square Error Loss Function (GS)

Al-Nasser and Saleh (2006) suggested the Generalized square error loss function in estimating the scale parameter and the Reliability function for Weibull distribution, which introduced as follows [11]:

\[ L_1(\theta, \hat{\theta}) = \left( \frac{\theta - \hat{\theta}}{\theta} \right)^2 \quad \text{for} \quad k = 0, 1, 2, 3, \ldots \]  

(7)

Where, \( a_k \) (\( j = 0, 1, 2, 3, \ldots, k \)) is a constant.

B. Quadratic Loss Function (QLF)

The Quadratic loss function which is asymmetric loss function defined for the positive values of the parameter. The Quadratic loss function \( L_2(\theta, \hat{\theta}) \) defined as follows: [5]

\[ L_2(\theta, \hat{\theta}) = \left( \frac{\theta - \hat{\theta}}{\theta} \right)^2 \]  

(8)

C. Bayes Estimator under Jeffry Prior Information

Let us assume that \( \theta \) has non-informative prior density defined as using Jeffrey prior information \( g(\theta) \) which is given by [2]:

\[ g_1(\theta) = c \theta^{\alpha-1} \exp\left(-\frac{\theta}{\beta}\right) \]

Where \( I(\theta) \) represents Fisher information which is defined as follows [8]:

\[ I(\theta) = -nE(\frac{\partial^2 \ln f(t)}{\partial \theta^2}) \]

Hence,

\[ g_1(\theta) = b \sqrt{n} \exp\left(-\frac{\theta^2}{2n}\right) \]  

(9)

After substitution into (9), we get

\[ g_1(\theta) = \frac{b}{\theta} \sqrt{n} \]

The posterior density function is

\[ h_1(\theta|t_1, \ldots, t_n) = \frac{g_1(\theta)L(\theta|t_1, \ldots, t_n)}{\int_0^{\infty} g_1(\theta)L(\theta|t_1, \ldots, t_n) d\theta} \]

(10)

Where \( \alpha \) is a constant.

\[ h_1(\theta|t_1, \ldots, t_n) = \frac{\theta^{-\alpha-1}e^{-\theta\beta}}{\Gamma(n)\beta^n} \]

Hence, the posterior density function of \( \theta \) with Jeffrey prior will be

\[ h_1(\theta|t_1, \ldots, t_n) = \frac{\theta^{n-1}e^{-\theta\beta}}{\Gamma(n)} \]  

(11)

The posterior density is recognized as the density of the Gamma distribution

\[ \theta \sim \text{Gamma}(n, \beta) \]

Bayes estimator under Generalized square error loss function: Recall that, the Generalized square error loss function (GS) is:

\[ L_1(\hat{\theta}, \theta) = (a_0 + a_1 \hat{\theta} + \cdots + a_k \hat{\theta}^k)(\hat{\theta} - \theta)^2 \]

Then, the Risk function under the generalized square error loss function is denoted by \( R_{GS}(\hat{\theta}, \theta) \)

\[ R_{GS}(\hat{\theta}, \theta) = E[L_1(\hat{\theta}, \theta)] = \int_0^\infty L_1(\hat{\theta}, \theta) h_1(\theta) d\theta \]

Taking the partial derivative for \( \beta \) with respect to \( \beta \) and setting it equal to zero yields

\[ \beta = \frac{a_0 E(\theta|t) + a_1 E(\theta^2|t) + \cdots + a_k E(\theta^k|t)}{a_0 + a_1 E(\theta|t) + \cdots + a_k E(\theta^k|t)} \]

(12)

Since \( \theta \sim \Gamma(n, \beta) \) and \( E(\theta) = \frac{n}{\beta} \), \( \text{var}(\theta) = \frac{n}{\beta^2} \)

\[ \theta = \frac{a_0 E(\theta|t) + a_1 E(\theta^2|t) + \cdots + a_k E(\theta^k|t)}{a_0 + a_1 E(\theta|t) + \cdots + a_k E(\theta^k|t)} \]

(13)

Now, we can find the Bayes estimator of Reliability function under generalized square error loss function by two methods.
1. Using the probability density function of the parameter θ[1]

According to this method of obtaining the Bayes estimator for the reliability function using Jeffery prior under generalized square error loss function will be

\[
R(t) = \frac{a_0 E[R(t)|\theta] + a_1 E[(R(t))^1|\theta] + \ldots + a_k E[(R(t))^k|\theta]}{a_0 + a_1 E[R(t)|\theta] + \ldots + a_k E[(R(t))^k|\theta]} \quad (14)
\]

The mth moment of R(t) can be found as follows

\[
E[(R(t))^m|\theta] = \int_0^\infty (R(t))^m h_1(\theta|t) \, d\theta \quad (15)
\]

\[
E[(R(t))^m|\theta] = \frac{p^n}{\Gamma(n)} \left( \frac{\ln(R(t)|\theta)}{\ln(\frac{a}{t})} \right)^{n-1} e^{-\left(\frac{p}{\ln(R(t)|\theta)}\right)} \quad (22)
\]

Now, substituting (21) and (22) into (20) given:

\[
\pi(R(t)|\theta) = \frac{p^n}{\Gamma(n)} \left( \frac{\ln(R(t)|\theta)}{\ln(\frac{a}{t})} \right)^{n-1} e^{-\left(\frac{p}{\ln(R(t)|\theta)}\right)} \quad (23)
\]

After substituting (23) into (24), we can find the mth moment of R(t) as follows

\[
E[(R(t))^m|\theta] = \int_0^\infty (R(t))^m \pi(R(t)|\theta) \, dR(t) \quad (24)
\]

2. Using the subsequent probability density function for the reliability function [7]

The estimation of Bayes to the reliability function can be found using loss functions and rely on subsequent probability density function for the reliability function R(t), which can be found through the subsequent probability density function parameter \( \theta \) using the style of the conversion through the relationship reliability function R(t) for parameter \( \theta \) is done as follows:

\[
R(t) = \frac{a^\theta}{t^\theta} \Rightarrow \ln R(t) = \theta \ln(\frac{a}{t}). \quad \text{Hence}
\]

\[
\theta = \frac{\ln R(t)}{\ln(\frac{a}{t})}
\]

So, the subsequent probability density function for reliability function can be obtained as following:

\[
\pi(R(t)|\theta) = h_2(\theta)|\theta| \quad (19)
\]

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Bayes estimator under Quadratic Loss function

Now, we derive Bayes estimator using Quadratic Loss function, where

$$l_2(\hat{\theta}, \theta) = \left( \frac{\theta - \hat{\theta}}{\theta} \right)^2 \left( 1 - \frac{\hat{\theta}}{\theta} \right)^2$$

The Risk function under the Quadratic Loss function is denoted by $R_Q(\hat{\theta}, \theta)$, where

$$R_Q(\hat{\theta}, \theta) = E(1 - \frac{\hat{\theta}}{\theta})^2 = \int_0^\infty (1 - \frac{\hat{\theta}}{\theta})^2 h_1(\theta | \xi) \, d\theta$$

Taking the partial derivative for the $R_Q$, with respect to $\hat{\theta}$ and then, equating to zero is:

$$\frac{\partial R_Q(\hat{\theta}, \theta)}{\partial \hat{\theta}} = 2 \int_0^\infty \left( 1 - \frac{\hat{\theta}}{\theta} \right) \left( -\frac{1}{\theta} \right) h_1(\theta | \xi) \, d\theta = 0$$

$$\Rightarrow \hat{\theta} = \frac{E(\frac{1}{\theta})}{E(\frac{1}{\theta^2})}$$

$$E\left(\frac{1}{\theta}\right) = \int_0^\infty \frac{p^n \theta^{n-1} e^{-\theta \xi}}{\Gamma(n)} \, d\theta = \frac{p}{(n-1)}$$

$$E\left(\frac{1}{\theta^2}\right) = \int_0^\infty \frac{p^n \theta^{n-2} e^{-\theta \xi}}{\Gamma(n)} \, d\theta = \frac{p^2}{(n-1)(n-2)}$$

Substituting into (27) gives

$$\hat{\theta}_{Q} = \frac{1}{\sqrt{\frac{p}{(n-1)}}} = \frac{n-2}{p}$$

So, the Bayesian estimation of Reliability function under Quadratic loss function with Jeffery's prior

$$\hat{R}(t) = \frac{E\left(\frac{1}{R(t)}\right)}{E\left(\frac{1}{R(t)^2}\right)}$$

### D. Bayes Estimator under Exponential Prior Distribution

Assuming that $\theta$ has informative prior as Exponential distribution, which takes the following form [1]:

$$g_\lambda(\theta) = \frac{e^{-\theta}}{\lambda}, \quad \theta > 0, \lambda > 0$$

Since, the posterior distribution of $\theta$ is:

$$h_\lambda(\theta | \xi) = \frac{L(t_1, \ldots, t_n | \theta) g_\lambda(\theta)}{\int_0^\infty L(t_1, \ldots, t_n | \theta) g_\lambda(\theta) \, d\theta}$$

$$h_\lambda(\theta | \xi) = \frac{\theta^n e^{-(\theta+1)\xi} \frac{1}{\lambda} e^{-\frac{\theta}{\lambda}}}{\int_0^\infty \theta^n e^{-(\theta+1)\xi} \frac{1}{\lambda} e^{-\frac{\theta}{\lambda}} \, d\theta}$$

$$h_\lambda(\theta | \xi) = \frac{(P + 1)^{n+1} \theta^{n-1} e^{-(\theta+1)\xi}}{\Gamma(n+1)}$$
Bayes Estimator under Generalized Square Error Loss Function

To obtain the Bayes Estimator using Generalized Square Error Loss Function (GS), we have

\[ \theta \sim \Gamma \left( n + 1, \frac{1}{3} \right), \quad \text{with:} \]

\[ E(\theta) = \frac{n + 1}{P + \frac{1}{3}}, \quad \text{var}(\theta) = \frac{n + 1}{\left( P + \frac{1}{3} \right)^2} \]

\[ E(\theta^k) = \frac{(n + k)(n + k - 1) \ldots (n + 1)}{\left( P + \frac{1}{3} \right)^k} \quad (35) \]

After substituting into generalized square error loss function (12), we get

\[ \hat{\theta}_{GS} = \left( \frac{n + 1}{P + \frac{1}{3}} + \frac{n + 1}{P + \frac{1}{3}} + \ldots + \frac{(n + k)(n + k - 1) \ldots (n + 1)}{\left( P + \frac{1}{3} \right)^{k-1}} \right) \]

So, the Bayes estimator of \( \theta \) under generalized square error loss function denoted by \( \hat{\theta}_{GS} \) is

\[ \hat{\theta}_{GS} = \left( \frac{\Gamma(n + 2 + j)}{(P + \frac{1}{3})^{j+1} \Gamma(n + 1)} \right)^{\frac{1}{j}} \]

\[ \hat{\theta}_{GS} = \left( \frac{\Gamma(n + 1 + j)}{(P + \frac{1}{3})^{j+1} \Gamma(n + 1)} \right)^{\frac{1}{j}} \quad (36) \]

From (12) we get, we can find the estimator of Reliability function under Generalized Square Error loss function:

\[ \hat{R}(t)_{GS} = \frac{\sum_{j=1}^{k} a_j \Gamma(n + 2 + j)}{(P + \frac{1}{3})^{j+1} \Gamma(n + 1)} \]

Bayesian estimation of Reliability function under Quadratic loss function with Exponential prior is obtained according to (27), therefore,

\[ \hat{R}(t)_{EQ} = \frac{n - 1}{(P + \frac{1}{2})^2} \]

Bayes Estimator under Quadratic loss function:

To obtain the Bayes Estimator under Quadratic loss function, we have:

\[ E \left( \frac{1}{\theta} | t \right) = \int_0^{\frac{1}{\theta}} h_2(\theta | t) d\theta = \frac{(P + \frac{1}{2})^2}{n} \frac{\sqrt{(P + \frac{1}{2})^2 - \theta^2}}{\Gamma(n)} \left( P + \frac{1}{2} \right) d\theta \]

\[ E \left( \frac{1}{\theta} | t \right) = \left( P + \frac{1}{2} \right)^2 \frac{\sqrt{(P + \frac{1}{2})^2 - \theta^2}}{\Gamma(n)} \left( P + \frac{1}{2} \right) d\theta \]

After substituting into (37), we get the Bayes estimator of \( R(t) \) under Generalized Square Error loss function denoted by \( R(t)_{GS} \) as

\[ \hat{R}(t)_{GS} = \left( \frac{\Gamma(n + 2 + j)}{(P + \frac{1}{3})^{j+1} \Gamma(n + 1)} \right)^{\frac{1}{j}} \]

\[ \hat{R}(t)_{GS} = \left( \frac{\Gamma(n + 1 + j)}{(P + \frac{1}{3})^{j+1} \Gamma(n + 1)} \right)^{\frac{1}{j}} \]

Bayesian estimation of Reliability function under Quadratic loss function with exponential prior is obtained according to (27), therefore,

\[ \hat{R}(t)_{EQ} = \frac{n - 1}{(P + \frac{1}{2})^2} \]


After substituting into (43), the Bayes estimator for the R(t) of Pareto distribution under Quadratic loss function with exponential prior, denoted by \( \hat{R}(t)_{EQ} \) is:

\[
\hat{R}(t)_{EQ} = \frac{P + \frac{1}{\lambda} + 2 \ln\left(\frac{\alpha}{\lambda}\right)}{P + \frac{1}{\lambda} + \ln\left(\frac{\alpha}{\lambda}\right)}
\]

IV. SIMULATION STUDY

We generated \( L = 2500 \) samples of size \( n = 20, 50, \) and \( 100 \) to represent small, moderate and large sample sizes from Pareto distribution with different values of the shape parameter \( \theta = 0.5, 1.5, \) and \( 2.5, \) with scale parameter \( \alpha = 1, 1.4, \) The scale parameter \( \alpha \) and \( \lambda \) of Exponential prior are \( \alpha = 1.5, 3 \) and assuming the values of \( \theta = 0.5, 1.5, \) and \( 2.5, \) with scale parameter \( \alpha = 1, 1.4, \) the performance of Bayes estimator under Generalized Squared error loss function when \( k = 1, 2 \) with exponential prior (\( \lambda = 0.5, 3 \)) is the best comparing to other estimators for all sample sizes. Followed by Bayes estimator under Generalized Squared error loss function when \( k = 1, 2 \) with exponential prior (\( \lambda = 0.5, 3 \)) for all sample size.

- The results in tables (2), (3) when \( \theta = 1.5, 2.5, 3 \) show that, Bayes estimator under Quadratic loss function with exponential prior (\( \lambda = 0.5, 3 \)) is the best comparing to other estimators for all sample sizes, followed by Bayes estimator under Generalized Squared error loss function when \( k = 2 \) with exponential prior (\( \lambda = 3 \)).

- On the other hand, using exponential prior with small value of \( \lambda = 0.5 \) is more appropriate than using exponential prior with relatively large value of \( \lambda = 3 \) with small values of \( \theta \), for all sample sizes.

REFERENCES


Table (1): IMSE Values for Different Estimators of Reliability Function of Pareto Distribution when $\theta = 0.5$

<table>
<thead>
<tr>
<th>Estimator</th>
<th>$n$</th>
<th>$\alpha = 1$</th>
<th>$\alpha = 1.4$</th>
<th>$\alpha = 1$</th>
<th>$\alpha = 1.4$</th>
<th>$\alpha = 1$</th>
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<tr>
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<td>0.0040446</td>
<td>0.0021093</td>
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<td>50</td>
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<td>0.0023994</td>
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<td>0.0032582</td>
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<td>BE(GS2) $\theta = 0.5$</td>
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### Table (2): IMSE Values for Different Estimators of Reliability Function of Pareto Distribution when $\theta = 1.5$

<table>
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### Table (3): IMSE Values for Different Estimators of Reliability Function of Pareto Distribution when $\theta = 2.5$

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