ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

Thermal Stresses of a Thin Rectangular Plate: An Inverse Problem

Ritesh Ganar; A. A. Navlekar; S. H. Bagade and N. W. Khobragade
Department of Mathematics, MJP Educational Campus,
RTM Nagpur University, Nagpur 440 033, India

Abstract- ***This paper is concerned with inverse transient thermoelastic problem of a thin clamped rectangular plate in which we need to determine the temperature distribution, unknown temperature gradient, thermal stresses and deflection when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.***

KEY WORDS: **Thin rectangular plate, transient problem, inverse thermo elastic problem, deflection.**

I. INTRODUCTION

Khobragade et al. [1, 2] have derived thermal deflection of a thick clamped rectangular plate , Khobragade et al. [5, 6, 8-10] have investigated displacement function, temperature distribution and stresses of a thin rectangular plate and Khobragade et al. [11] have established displacement function , temperature distribution and stresses of a thick rectangular plate.

In the present paper, an attempt is made to determine the temperature distribution, unknown temperature gradient, thermal stresses and deflection of the plate occupying the space *D:* $\{(x,y,z)\in R^3 : 0\le x\le a, 0\le y\le b, 0\le z\le h\}$ with the known boundary conditions.

Finite Fourier sine transform and Laplace transform techniques are used to find the solution of the problem. Numerical estimate for the temperature distribution is obtained and depicted graphically.

II. STATEMENT OF THE PROBLEM

Consider a thin isotropic rectangular plate occupying the space *D.* The differential equation satisfied by the deflection $\omega(x,y,t)$ as Khobragade et al. [1] is

$$D\nabla^4\omega(x,y,t) = \frac{-\nabla^2 M_T(x,y,t)}{1-\nu} \tag{1}$$

where,

ν is the Poisson's ratio of the plate material ,

M_T denote the thermal momentum of the plate and

D denote the flexural rigidity,

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$$

The resultant thermal momentum M_T is defined as

$$M_T(x,y,t) = \alpha E\int_0^h zT(x,y,z,t)dz \tag{2}$$

Where α, *E* are the linear coefficient of thermal expansion of the material, and Young's modulus respectively.

Since the edge of the rectangular plate is fixed and clamped,

$$\omega = \frac{\partial^2\omega}{\partial x^2} = \frac{\partial^2\omega}{\partial y^2} = 0 \text{ at x = a and y = b} \tag{3}$$

The temperature of the plate at time t satisfying the differential equation as Nowacki [15] is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x,y,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{4}$$

where *k* is the thermal diffusivity of the material of the plate,

subject to the initial and boundary conditions**:**

$$T(x,y,z,0) = 0 \tag{5}$$

$$\left[T(x,y,z,t) + k_1\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = f_1(y,z,t) \tag{6}$$

$$\left[T(x,y,z,t) + k_2\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = f_2(y,z,t) \tag{7}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=0} = g_1(x,z,t) \tag{8}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=b} = g_2(x,z,t) \tag{9}$$

$$[T(x,y,z,t)]_{z=0} = 0 \tag{10}$$

$$[T(x,y,z,t)]_{z=\xi} = g(x,y,t) \tag{11}$$

$$[T(x,y,z,t)]_{z=h} = f(x,y,t) \text{ (Unknown)} \tag{12}$$

The displacement components u_x and u_y u_z in the x and y and z directions respectively as Tanigawa et al. [1] are

$$u_x = \int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu\frac{\partial^2 U}{\partial x^2}\right) + \lambda T\right]dx \tag{13}$$

$$u_y = \int_0^b\left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu\frac{\partial^2 U}{\partial y^2}\right) + \lambda T\right]dy \tag{14}$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

$$u_z = \int_0^h \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2} \right) + \lambda T \right] dz \qquad (15)$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and *U* (x,y,z,t) is the Airy's stress functions which satisfy the differential equation as Tanigawa et al. [1] is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)^2 U(x,y,z,t) = -\lambda E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$$

$$\times T(x,y,z,t) \qquad (16)$$

The stress components in terms of U(x, y, z, t) Tanigawa et al. [1] are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right] \qquad (17)$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} \right] \qquad (18)$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right] \qquad (19)$$

Equations (1) to (19) constitute the mathematical formulation of the problem under consideration.

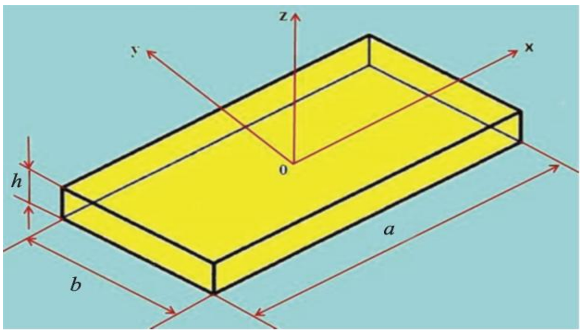

Fig 1: Geometry of the problem

III. SOLUTION OF THE PROBLEM

Applying Marchi-Fasulo, transform and finite Fourier cosine Transform and Fourier sine Transform to the equations one obtains.

$$\frac{d\overline{\overline{T}}^*}{dt} + \alpha q^2 \overline{\overline{T}}^* = \psi \qquad (20)$$

Where $q^2 = \mu_n^2 + \frac{m^2\pi^2}{b^2} + \frac{p^2\pi^2}{\xi^2}$

and

$$\psi = \alpha \left[\frac{p_n(a)}{k_2} \bar{f}_2^* - \frac{p_n(-a)}{k_1} \bar{f}_1^* + (-1)^m \bar{g}_2^* - \bar{g}_1^* \right.$$

$$\left. + (-1)^{p+1} \left(\frac{p\pi}{\xi} \right) \overline{\overline{g}}_3 + \frac{\overline{\overline{g}}^*}{k} \right].$$

Solution of equation (20) is given by

$$\overline{\overline{T}}^* = e^{-\alpha q^2 t} \left(\int_0^t \Psi e^{\alpha q^2 t'} dt' \right) \qquad (21)$$

Applying inversion of Fourier sine, Fourier cosine and Marchi-Fasulo transform we get the temperature distribution and unknown temperature gradient as

$$T(x,y,z,t) = \frac{4}{b\xi} \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P_n(x)}{\mu_n} \cos\left(\frac{m\pi y}{b}\right)$$

$$\times \sin\left(\frac{p\pi z}{\xi}\right) \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \qquad (22)$$

$$f(x,y,t) = \frac{4}{b\xi} \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P_n(x)}{\mu_n} \cos\left(\frac{m\pi y}{b}\right)$$

$$\times \sin\left(\frac{p\pi h}{\xi}\right) \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \qquad (23)$$

where *p*, *m*, *n* are the positive integers

Using equation (22) in equation (16) we get

$$U(x,y,z,t) = \frac{-4(1+\nu)a_t}{q^2 b\xi} \sum_{p=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P_n(x)}{\mu_n} \cos\left(\frac{m\pi y}{b}\right)$$

$$\times \sin\left(\frac{p\pi z}{\xi}\right) \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \qquad (24)$$

Using equation (24) in equations (13) to (15) we get

$$u_x = \frac{4}{b\xi} \int_{-a}^{a} \sum_{m,n,p} \left\{ \frac{(1+\upsilon)a_t}{E\mu_n q^2} \left[\left(\frac{m^2\pi^2}{b^2} - \frac{p^2\pi^2}{\xi^2} \right) P_n(x) - \upsilon P_n''(x) \right] + \frac{P_n(x)}{\mu_n} \right\}$$

$$\times \cos\left(\frac{m\pi y}{b}\right) \times \sin\left(\frac{p\pi z}{\xi}\right)$$

$$\times \left(e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \right) dx \qquad (25)$$

$$u_y = \frac{4}{b\xi} \int_0^b \sum_{m,n,p} \left\{ \frac{(1+\upsilon)a_t}{E\mu_n q^2} \left[\left(\frac{p^2\pi^2}{\xi^2} - \frac{\upsilon m^2\pi^2}{b^2} \right) P_n(x) - \upsilon P_n''(x) \right] + \frac{\lambda P_n(x)}{\mu_n} \right\}$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

$$\times \cos(\frac{m\pi y}{b}) \times \sin(\frac{p\pi z}{\xi})$$

$$\times \left(e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \right) dy \tag{26}$$

$$u_z = \frac{4}{b\xi} \int_0^{\xi} \sum_{m,n,p} \left\{ \frac{(1+\upsilon)a_t}{E\mu_n q^2} \left[\left(\frac{m^2\pi^2}{b^2} - \frac{\upsilon p^2\pi^2}{\xi^2} \right) P_n(x) - P_n''(x) \right] + \frac{\lambda P_n(x)}{\mu_n} \right\}$$

$$\times \cos(\frac{m\pi y}{b}) \times \sin(\frac{p\pi z}{\xi})$$

$$\times \left(e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \right) dz \tag{27}$$

Using equation (24) in equations (17) to (19) we get

$$\sigma_{xx} = \frac{4\pi^2(1+\nu)a_t}{q^2 b^3 \xi^3} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{(\xi^2 m^2 + p^2 b^2)}{\mu_n} P_n(x)$$

$$\times \cos(\frac{m\pi y}{b}) \times \sin(\frac{p\pi z}{\xi}) \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \tag{28}$$

$$\sigma_{yy} = \frac{-4(1+\nu)a_t}{q^2 b \xi^3} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{(\xi^2 P_n''(x) - p^2\pi^2 P_n(x))}{\mu_n}$$

$$\times \cos(\frac{m\pi y}{b}) \times \sin(\frac{p\pi z}{\xi}) \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \tag{29}$$

$$\sigma_{zz} = \frac{-4(1+\nu)a_t}{q^2 b^3 \xi} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{(b^2 P_n''(x) - m^2\pi^2 P_n(x))}{\mu_n}$$

$$\times \cos(\frac{m\pi y}{b}) \times \sin(\frac{p\pi z}{\xi}) \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \tag{30}$$

IV. DETERMINATION OF THERMAL DEFLECTION

Substituting the value of temperature distribution *T(x, y, z, t)* from equation (22) in equation (2), one obtains

$$M_T(x,y,t) = \frac{4\alpha_t E\xi}{\pi} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{(-1)^{p+1}}{p} \frac{P_n(x)}{\mu_n} \cos(\frac{m\pi y}{b})$$

$$\times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \tag{31}$$

We assume that the solution of equation (1) satisfying equation (3) as

$$\omega(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} c_{mn}(t) \cos(\frac{m\pi y}{b}) \tag{32}$$

Using the equations (31) and (32) in (1), one obtains

$$c_{mn}(t) = \frac{4\alpha_t E\xi}{m^4 \pi^5 b^6} \sum_{p=1}^{\infty} \frac{(-1)^{p+1}}{p} \left(b^2 P_n''(x) - m^2\pi^2 \right)$$

$$\times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \tag{33}$$

Substituting the value of $\omega_{mn}(t)$ in equation (32), one obtains the expression for thermal deflection as

$$\omega(x,y,t) = \frac{4\alpha_t E\xi}{b^6 \pi^5} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{(-1)^{p+1}}{m^4 p \mu_n}$$

$$\times \left(b^2 P_n''(x) - m^2\pi^2 \right) \cos(\frac{m\pi y}{b})$$

$$\times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \tag{34}$$

V. SPECIAL CASE AND NUMERICAL RESULTS

Set $g(x,y,t) = (1-e^{-t})(x^2 - ax)(y^2 - by)$,

a = 1,

b = 2,

h = 2,

t = 1sec

$\xi = 1.5$ and

k= 0.86

in equations (22) we get

$$T(x,y,z,t) = (1.33) \sum_{p=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{P_n(x)}{\mu_n} \cos(\frac{m\pi y}{2})$$

$$\times \sin(\frac{p\pi z}{1.5}) \times e^{-\alpha q^2 t} \int_0^t \psi e^{\alpha q^2 t'} dt' \tag{35}$$

VI. CONCLUSION

The temperature distribution and thermal deflection of a thin rectangular plate have been obtained, with the aid of finite Fourier sine transform and Laplace transform techniques when the stated boundary conditions are known. The results are obtained in the form of infinite series. The series solutions converge provided if we take sufficient number of terms in the series.

The expressions are represented graphically. The temperature distribution, and deflection that are obtained can be applied to the design of useful structures or

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 4, Issue 1, July 2014

machines in engineering applications.

ACKNOWLEDGEMENT

The authors are thankful to University Grant Commission, New Delhi for providing the partial financial assistance under major research project scheme.

REFERENCES

[1] Khobragade N. W., Payal Hiranwar, H. S.Roy and Lalsingh Khalsa: Thermal Deflection of a Thick Clamped Rectangular Plate, Int. J. of Engg. and Information Technology, vol. 3, Issue 1, pp. 346-348, (2013).

[2] Ghume Ranjana S and Khobragade, N. W: “Deflection of A Thick Rectangular Plate”, Canadian Journal on Science and Engg. Mathematics Research, Vol.3 No.2, pp. 61-64, (2012).

[3] Hamna Parveen and Khobragade, N. W: “Thermal Stresses Of A Thick Circular Plate Due To Heat Generation”, Canadian Journal on Science and Engg. Mathematics Research, Vol. 3 No. 2, pp. 65-69, (2012).

[4] Hamna Parveen; Navneet Kumar and Khobragade, N. W: “Thermal deflection of a thin circular plate with radiation”, African journal of mathematics and computer science research, vol.5 (4), 66-70, (2012).

[5] Roy, Himanshu and Khobragade, N.W: “Transient Thermoelastic Problem Of An Infinite Rectangular Slab”, Int. Journal of Latest Trends in Maths, Vol. 2, No. 1, pp. 37-43, (2012).

[6] Lamba, N.K; and Khobragade, N.W: “Thermoelastic Problem of a Thin Rectangular Plate Due To Partially Distributed Heat Supply”, IJAMM, Vol. 8, No. 5, pp.1-11, (2012).

[7] Gahane, T. T, Khalsa, L H and Khobragade, N.W: “Thermal Stresses in A Thick Circular Plate With Internal Heat Sources”, Journal of Statistics and Mathematics, Vol. 3, Issue 2, pp. 94-98, (2012).

[8] Patil V.B. and Khobragade, N.W: “Direct thermoelastic problem of heat conduction with internal heat generation and partially distributed heat supply in rectangular plate”, Canadian Journal of Science & Engineering Mathematics, Vol. 3, No.5, pp. 193-197, (2012).

[9] Sutar C. S. and Khobragade, N.W: “An inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate”, Canadian Journal of Science & Engineering Mathematics, Vol. 3, No.5, pp. 198-201, (2012).

[10] Roy H. S, Bagade S. H. and N.W.Khobragade: Thermal Stresses of a Semi infinite Rectangular Beam, Int. J. of Engg. And Information Technology, vol. 3, Issue 1, pp. 442-445, (2013).

[11] Jadhav, C.M; and Khobragade, N.W: “An Inverse Thermoelastic Problem of a thin finite Rectangular Plate due to Internal Heat Source”, Int. J. of Engg. Research and Technology, vol.2, Issue 6, pp. 1009-1019, (2013).

[12] Sneddon, I. N: The use of integral transform, Mc Graw Hill book co. (1974), chap.3.

[13] Khobragade, N.W: Thermo elastic analysis of a thick circular plate, Int. J. of Engg. And Information Technology, vol. 3, Issue 5, pp.94-100, (2013).

[14] Khobragade, N. W and Wankhede, P. C: An inverse unsteady-state thermoelastic problem of a thin rectangular plate, The Journal of Indian Academy of Mathematics, vol. 25, No. 2, (2003).

[15] Nowacki, W. Thermoelasticity, Addition- Wisely Publishing Comp. Inc. London, 1962.

[16] Tanigawa, Y. and Komatsubara, Y.: Thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field, Journal of Thermal Stresses, Vol. 20, pp. 517-524 (1997).

AUTHOR BIOGRAPHY

Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 16 years in the area of Boundary value problems (Thermoelasticity in particular) and Operations Research. Published more than 184 research papers in reputed journals. Fifteen students awarded Ph.D Degree and seven students submitted their thesis in University for award of Ph.D Degree under their guidance.

Dr. Sanjay Bagade, M.Sc. Ph.D Assistant Professor, Shiksha Mandal’s Janakidevi Bajaj College of Science Wardha.

Dr. A.A. Navlekar For being M.Sc, Ph.D in maths, he has been teaching since 2004 for 10 years at PM Paithan , BAM University, Aurangabad.

Mr. Ritesh Ganar M.Sc in maths, he is a research scholar of Maths, RTM Nagpur University, Nagpur and sucessfully handled different capacities.