

Interior Thermo elastic Solution of a Hollow Cylinder

Ranjana S. Ghume, Ashwini Mahakalkar, N. W. Khobragade
 Department of Mathematics, MJP Educational Campus,
 RTM Nagpur University, Nagpur 440 033, India

Abstract- In this paper, an attempt has been made to study thermoelastic response of an inverse thermoelastic problem of a hollow cylinder occupying the space $D: a \leq r \leq b, 0 \leq z \leq h$, with radiation type boundary conditions. We apply integral transform technique to find the thermoelastic solution.

Keywords: Thermo elastic Response, hollow cylinder, Thermal Stresses, inverse problem.

I. INTRODUCTION

Khobragade et al. [2-18] have investigated temperature distribution, displacement function, and stresses of a thin as well as thick hollow cylinder and Khobragade et al. [13] have established displacement function, temperature distribution and stresses of a semi-infinite cylinder.

In the present paper, an attempt is made to study the theoretical solution for a thermoelastic problem to determine the temperature distribution, displacement and stress functions of a hollow cylinder with boundary conditions occupying the space

$$D = \{(x, y, z) \in R^3 : a \leq (x^2 + y^2)^{1/2} \leq b, 0 \leq z \leq h\},$$

where $r = (x^2 + y^2)^{1/2}$. A transform defined by Zgrablich et al. [2] is used for investigation which is a generalization of Hankel's double radiation finite transform and used to treat the problem with radiation type boundaries conditions.

II. FORMULATION OF THE PROBLEM

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function $\phi(r, z, t)$ satisfying the differential equation as Khobragade [2] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu} \right) a_t T \quad (1)$$

with $\phi = 0$ at $r = a$ and $r = b$

where ν and a_t are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and $T(r, z, t)$ is the heating temperature of

the cylinder at time t satisfying the differential equation as Khobragade [2] is

$$\kappa \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] = \frac{\partial T}{\partial t} \quad (2)$$

where $\kappa = K / \rho c$ is the thermal diffusivity of the material of the cylinder, K is the conductivity of the medium, C is its specific heat and ρ is its calorific capacity (which is assumed to be constant) respectively, subject to the initial and boundary conditions

$$M_t(T, 1, 0, 0) = 0 \quad \text{for all } a \leq r \leq b, \quad 0 \leq z \leq h \quad (3)$$

$$M_r(T, 1, \bar{k}_1, a) = F_1(z, t), \quad \text{for all } 0 \leq z \leq h, \quad t > 0 \quad (4)$$

$$M_r(T, 1, \bar{k}_2, b) = F_2(z, t) \quad \text{for all } 0 \leq z \leq h, \quad t > 0 \quad (5)$$

$$M_z(T, 1, 0, 0) = F_3(r, t) \quad \text{for all } a \leq r \leq b, \quad t > 0 \quad (6)$$

$$M_z(T, 1, 0, \xi) = f(r, t) \quad \text{for all } a \leq r \leq b, \quad t > 0 \quad (7)$$

$$M_z(T, 1, 0, h) = G(r, t) \quad \text{for all } a \leq r \leq b, \quad t > 0 \quad (8)$$

being:

$$M_{\mathcal{G}}(f, \bar{k}, \bar{k}, \mathcal{G}) = (\bar{k} f + \bar{k} \hat{f})_{\mathcal{G}=\mathcal{G}}$$

where the prime (\wedge) denotes differentiation with respect to \mathcal{G} , radiation constants are \bar{k} and \bar{k} on the curved surfaces of the plate respectively. The radial and axial displacement U and W satisfy the uncoupled thermoelastic equation as Khobragade [2] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1} \frac{\partial e}{\partial r} = 2 \left(\frac{1+\nu}{1-2\nu} \right) a_t \frac{\partial T}{\partial r} \quad (9)$$

$$\nabla^2 W + (1-2\nu)^{-1} \frac{\partial e}{\partial z} = 2 \left(\frac{1+\nu}{1-2\nu} \right) a_t \frac{\partial T}{\partial z} \quad (10)$$

where

$$e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial r} \quad (11)$$

$$U = \frac{\partial \phi}{\partial r}, \quad (12)$$

$$W = \frac{\partial \phi}{\partial z} \quad (13)$$

The stress functions are given by

$$\tau_{rz}(a, z, t) = 0, \tau_{rz}(b, z, t) = 0, \tau_{rz}(r, 0, t) = 0 \quad (14)$$

$$\sigma_r(a, z, t) = p_i, \sigma_r(b, z, t) = -p_o, \sigma_z(r, 0, t) = 0 \quad (15)$$

where P_i and P_o are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations as Khobragade [2] are

$$\sigma_r = (\lambda + 2G) \frac{\partial U}{\partial r} + \lambda \left(\frac{U}{r} + \frac{\partial W}{\partial z} \right) \quad (16)$$

$$\sigma_z = (\lambda + 2G) \frac{\partial W}{\partial z} + \lambda \left(\frac{\partial U}{\partial r} + \frac{U}{r} \right) \quad (17)$$

$$\sigma_\theta = (\lambda + 2G) \frac{U}{r} + \lambda \left(\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z} \right) \quad (18)$$

$$\tau_{rz} = G \left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} \right) \quad (19)$$

where $\lambda = 2G\nu/(1-2\nu)$ is the Lamé's constant, G is the shear modulus and U, W are the displacement components. Equations (1)-(19) constitute the mathematical formulation of the problem under consideration.

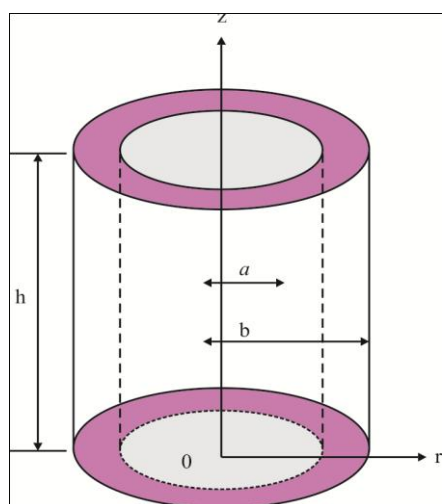


Fig 1: Geometry of the problem

III. SOLUTION OF THE OF THE PROBLEM

Applying transform defined in [2] to the equations (3), (4) and (6) over the variable r having $p = 0$ with responds to the boundary conditions of type (5) and taking the Laplace transform, one obtains

$$\bar{T}^*(n, z, s) = \bar{f}(n, s) \frac{\cosh[(\mu_n^2 + (s/\kappa))^{1/2} z]}{\sinh[(\mu_n^2 + (s/\kappa))^{1/2} \xi]} \quad (20)$$

where constants involved $\bar{T}^*(n, z, s)$ are obtained by using boundary conditions (6). Finally applying the inversion theorems of transform defined in [2] and inverse Laplace transform by means of complex contour integration and the residue theorem, one obtains the expressions of the temperature distribution $T(r, z, t)$ for heating processes as

$$T(r, z, t) = \frac{2\kappa}{\xi} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \varphi_{nm} S_0(\bar{k}_1, \bar{k}_2, \mu_n r) \quad (21)$$

$$G(r, t) = \frac{2\kappa}{\xi} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \varphi_{nm}' S_0(\bar{k}_1, \bar{k}_2, \mu_n r) \quad (22)$$

where

$$\varphi_{nm} = \frac{\lambda_m \sin(\lambda_m z)}{\cos(\lambda_m \xi)} \int_0^t \bar{f}(n, t') e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} dt', \quad (23)$$

$$\varphi_{nm}' = \frac{\lambda_m \sin(\lambda_m h)}{\cos(\lambda_m \xi)} \int_0^t \bar{f}(n, t') e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} dt' \quad (24)$$

Where n is the transformation parameter as defined in appendix, m is the Fourier sine transform parameter.

IV. DETERMINATION OF DISPLACEMENT AND STRESS FUNCTION

Substituting the value of temperature distribution from (21) in equation (1) one obtains the thermoelastic displacement function $\phi(r, z, t)$ as

$$\phi(r, z, t) = \frac{r^2 k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \varphi_{nm} S_0(k_1, k_2, \mu_n r) \quad (25)$$

Using (25) in the equations (11) and (12) one obtains

$$U = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \varphi_{nm} [2S_0(k_1, k_2, \mu_n r) + rS_0'(k_1, k_2, \mu_n r)] \quad (26)$$

$$W = \frac{r^2 k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \lambda_m \varphi_{nm} \cot(\lambda_m z) S_0(k_1, k_2, \mu_n r) \quad (27)$$

Substitution the value of (26), (27) in (16) to (19) one obtains the stress functions as

$$\sigma_r = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \frac{\varphi_{nm}}{C_n} \left[(\lambda + 2G)(r^2 S_0''(k_1, k_2, \mu_n r) + 4r S_0'(k_1, k_2, \mu_n r) + 2S_0(k_1, k_2, \mu_n r)) \right. \\ \left. \times \lambda [2S_0(k_1, k_2, \mu_n r) + r S_0'(k_1, k_2, \mu_n r)] - r^2 \lambda_m S_0(k_1, k_2, \mu_n r) \right] \quad (28)$$

$$\sigma_z = -\frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\varphi_{nm}}{C_n} \right) \left[(\lambda + 2G)r^2 \lambda_m^2 S_0(k_1, k_2, \mu_n r) - \lambda (r^2 S_0''(k_1, k_2, \mu_n r) + 5r S_0'(k_1, k_2, \mu_n r) + 4S_0(k_1, k_2, \mu_n r)) \right] \quad (29)$$

$$\sigma_\theta = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\varphi_{nm}}{C_n} \right) \left[(\lambda + 2G)[r S_0'(k_1, k_2, \mu_n r) + 2S_0(k_1, k_2, \mu_n r)] + \lambda [r^2 S_0''(k_1, k_2, \mu_n r) + 4r S_0'(k_1, k_2, \mu_n r) + (2 - r^2 \lambda_m^2) S_0(k_1, k_2, \mu_n r)] \right] \quad (30)$$

$$\tau_{rz} = \frac{k a_t G(1+\nu)}{\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\lambda_m \varphi_{nm} \cot(\lambda_m z)}{C_n} \right) [r^2 S_0'(k_1, k_2, \mu_n r) + 2r S_0(k_1, k_2, \mu_n r)] \quad (31)$$

V. SPECIAL CASE

$$\text{Set } f(r, t) = (1 - e^{-t}) \delta(r - r_0) \quad (32)$$

Applying finite transform defined in Marchi Zgrablich [2] to the equation (32) one obtains

$$\bar{f}(n, t) = (1 - e^{-t}) r_0 S_0(k_1, k_2, \mu_n r_0) \quad (33)$$

Substituting the value of (32) in the equations (21) to (31) one obtains

$$T(r, z, t) = \frac{2\kappa}{\xi} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \frac{\lambda_m \sin(\lambda_m z)}{\cos(\lambda_m \xi)} \times \int_0^t (1 - e^{-t'}) r_0 S_0(k_1, k_2, \mu_n r_0) e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} S_0(k_1, k_2, \mu_n r) dt' \quad (34)$$

$$G(r, t) = \frac{2\kappa}{\xi} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \frac{\lambda_m \sin(\lambda_m h)}{\cos(\lambda_m \xi)} \times \int_0^t (1 - e^{-t'}) r_0 S_0(k_1, k_2, \mu_n r_0) e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} S_0(\bar{k}_1, \bar{k}_2, \mu_n r) dt'$$

$$\phi(r, z, t) = \frac{r^2 k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \frac{\lambda_m \sin(\lambda_m z)}{\cos(\lambda_m \xi)} \times \int_0^t (1 - e^{-t'}) r_0 S_0(k_1, k_2, \mu_n r_0) e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} S_0(k_1, k_2, \mu_n r) dt' \quad (35)$$

$$U = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \frac{\lambda_m \sin(\lambda_m z)}{\cos(\lambda_m \xi)} \times \int_0^t (1 - e^{-t'}) r_0 S_0(k_1, k_2, \mu_n r_0) e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} dt' [2S_0(k_1, k_2, \mu_n r) + r S_0'(k_1, k_2, \mu_n r)] \quad (37)$$

$$\sigma_r = \frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \frac{\lambda_m \sin(\lambda_m z)}{C_n \cos(\lambda_m \xi)} \int_0^t (1 - e^{-t'}) r_0 S_0(k_1, k_2, \mu_n r_0) e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} dt' \times [(\lambda + 2G)(r^2 S_0''(k_1, k_2, \mu_n r) + 4r S_0'(k_1, k_2, \mu_n r) + 2S_0(k_1, k_2, \mu_n r)) \times \lambda [2S_0(k_1, k_2, \mu_n r) + r S_0'(k_1, k_2, \mu_n r)] - r^2 \lambda_m S_0(k_1, k_2, \mu_n r)] \quad (38)$$

$$\sigma_z = -\frac{k a_t (1+\nu)}{2\xi(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\lambda_m \sin(\lambda_m z)}{C_n \cos(\lambda_m \xi)} \int_0^t (1 - e^{-t'}) r_0 S_0(k_1, k_2, \mu_n r_0) e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} dt' \right) \times [(\lambda + 2G)r^2 \lambda_m^2 S_0(k_1, k_2, \mu_n r) - \lambda (r^2 S_0''(k_1, k_2, \mu_n r) + 5r S_0'(k_1, k_2, \mu_n r) + 4S_0(k_1, k_2, \mu_n r))] \quad (39)$$

VI. CONVERGENCE OF THE SERIES SOLUTION

In order for the solution to be meaningful the series expressed in equations (21) and (22) should converge for all $a \leq r \leq b$ and $0 \leq z \leq h$, and we should further investigate the conditions which has to be imposed on the functions $f(r, t)$ so that the convergence of the series expansion for $T(r, z, t)$ is valid. The temperature equation (21) can be expressed as

$$T(r, z, t) = \frac{2\kappa}{\xi} \sum_{n=1}^{M'} \frac{1}{C_n} \sum_{m=0}^{M'} \varphi_{nm} S_0(\bar{k}_1, \bar{k}_2, \mu_n r) \quad (40)$$

We impose conditions so that $T(r, z, t)$ converge in some generalized sense to $g(r, s)$ as $t \rightarrow 0$ in the transform domain. Taking into account of the asymptotic behaviors of μ_n , $S_0(k_1, k_2, \mu_n r)$, and C_n as given in Marchi Zgrablich [2], it is observed that the series expansion for $T(r, z, t)$ will be convergent by one term approximation as

$$\{\phi'_{nm}\} = \int_0^t \bar{f}(n, t') \left\{ e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} \right\} dt' = O \left\{ 1 / (\mu_n^2 + \lambda_m^2)^\kappa \right\}$$

$$, \kappa > 0 \tag{41}$$

Here $\bar{f}(r, t')$ in equation (32) can be chosen as one of the following functions or their combination involving addition or multiplication of constant, $\sin(\omega t)$, $\cos(\omega t)$, $\exp(kt)$, or polynomials in t . Thus, $T(r, z, t)$ is convergent to a limit both $\{T(r, z, t)\}_{r=b}$ as convergence of a series for $r=b$ implies convergence for all $r \leq b$ at any value of z . But for an exact solution would require the use of an infinite number of terms in the equations.

In the present solution only the first 5 terms of the transcendental equation

$$J_p(\alpha, \mu a) Y_p(\beta, \mu b) - J_p(\alpha, \mu a) Y_p(\beta, \mu b) = 0$$

and for other infinite series are used. The effects of truncating of numbers are brought out by the comparison table for solutions of different functions for 2 and 5 terms. It is evident from the table that the convergence is rapid for the temperature distribution during cooling process, radial and axial stress and somewhat slower in the case of other stresses as well as temperature distribution during heating process, while it is estimated from table that the possible error from the table is less than 2 percent.

Table 1. Convergence of solution as the number of terms used in the equation are increased from 2 to 5.

Functions	2 terms	5 terms
$T(r, z, t)$	-4.35298	-6.7712
σ_r	-285.18	-180.967
σ_z	285.66	170.496
σ_θ	69.3182	60.9464
τ_{rz}	-5.90491	-6.35717

VII. NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computation we consider material properties of low carbon steel (AISI 1119), which can be used for medium duty shafts, studs, pins, distributor cams, cam shafts, and universal joints having mechanical and thermal properties

$$\kappa = 13.97 [\mu m / s^2] \quad \nu = 0.29,$$

$$\lambda = 51.9 [W / (m - K)] \text{ and } a_t = 14.7 \mu m / m - ^0 C .$$

Setting the physical parameter with $a = 0.5$, $b = 1$ and $h = 3$.

VIII. CONCLUSION

In this chapter, we modify the conceptual idea proposed by Khobragade et al [2] for hollow cylinder and the temperature distributions, displacement and stress functions at the edge $z = h$ occupying the region of the cylinder $a \leq r \leq b$, $0 \leq z \leq h$ have been obtained with the known boundary conditions. We develop the analysis for the temperature field by introducing the transformation defined by Zgrablich et al, finite Fourier sine transform and Laplace transform techniques with boundaries conditions of radiations type. The series solutions converge provided we take sufficient number of terms in the series. Since the thickness of cylinder is very small, the series solution given here will be definitely convergent. Assigning suitable values to the parameters and functions in the series expressions can derive any particular case. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.

ACKNOWLEDGEMENT

The authors are thankful to University Grant Commission, New Delhi for providing the partial financial assistance under major research project scheme.

REFERENCES

- [1] Noda, N; Hetnarski, R.B; Tanigawa, y: Thermal Stresses, second edition Taylor & Francis, New York (2003), 260.
- [2] Gahane, T.T, Varghese, V. and Khobragade, N. W (2009): "Transient Thermoelastic Problem of a cylinder with heat sources", Int. J Latest Trend Math, Vol. 2 No. 1, pp. 25-36, 2012, UK.
- [3] Raut, G.N., Varghese, V. and Khobragade, N. W. (2009): "On the plane strain and plane stress solutions of uniformly heated functionally graded solid cylinder or disc problems", Advances in Math. Sci. and Appl., vol. 19, No.1, 403-413, Japan.
- [4] Kamdi, D. B, Khobragade, N. W, and Durge, M. H (2009): "Transient Thermoelastic Problem for a Circular Solid Cylinder with Radiation", Int. Journal of Pure and Applied Maths, vol. 54, No. 3, 387-406, Academic Publication.
- [5] Kulkarni, Ashwini A and Khobragade, N. W (2012): "Thermal stresses of a finite length hollow cylinder", Canadian Journal on Science and Engg. Mathematics Research, Vol. 3 No. 2, pp. 52-55, Canada.
- [6] Warbhe, M. S and Khobragade, N.W (2012): "Numerical Study Of Transient Thermoelastic Problem Of A Finite Length Hollow Cylinder", Int. Journal of Latest Trends in Maths, Vol. 2, No. 1, pp. 4-9, UK.
- [7] Lamba, N.K; and Khobragade, N.W (2011): "Analysis of Coupled Thermal Stresses in a Axisymmetric Hollow Cylinder", Int. Journal of Latest Trends in Maths, Vol. 1, No. 2, UK.

[8] Lamba, N.K, Walde, R. T and Khobragade, N.W (2012): “Stress functions in a hollow cylinder under heating and cooling processes”, Journal of Statistics and Mathematics. Vol. 3, Issue 3, pp. 118-124, BIO INFO Publication (Impact Factor 4.47).

[9] Gahane, T. T and Khobragade, N.W (2012): “Transient Thermoelastic Problem Of A Semi-infinite Cylinder With Heat Sources”, Journal of Statistics and Mathematics, Vol. 3, Issue 2, pp. 87-93, BIO INFO Publication. (Impact Factor 4.47).

[10] Hiranwar Payal C and Khobragade, N.W (2012): “Thermoelastic Problem Of A Cylinder With Internal Heat Sources”, Journal of Statistics and Mathematics, Vol. 3, Issue 2, pp. 87-93, BIO INFO Publication. (Impact Factor 4.47).

[11] Dange, W. K; Khobragade, N.W, and Durge, M. H (2012): “Thermal Stresses of a finite length hollow cylinder due to heat generation”, Int. J. of Pure and Appl. Maths, (accepted).

[12] N. W. Khobragade, Lalsingh Khalsa and Mrs Ashwini Kulkarni: Thermal Deflection of a Finite Length Hollow Cylinder due to Heat Generation, Int. J. of Engg. And Information Technology, vol. 3, Issue 1, pp. 372-375, (2013).

[13] Anjali C. Pathak, Payal Hiranwar, Lalsingh Khalsa and N. W. Khobragade (2013): Thermoelastic Problem of a Semi Infinite Cylinder with Internal Heat Sources, Int. J. of Engg. And Information Technology, vol. 3, Issue 1, USA (Impact Factor 1.895).

[14] R.T. Walde, Anjali C. Pathak and N.W. Khobragade (2013): Thermal Stresses of a Solid Cylinder with Internal Heat Source, Int. J. of Engg. And Information Technology, vol. 3, Issue 1, pp. 407-410, USA (Impact Factor 1.895).

[15] Jadhav, C.M; and Khobragade, N.W (2013): “An Inverse Thermoelastic Problem of finite length thick hollow cylinder with internal heat sources”, Advances in Applied Science Research, 4(3); 302-314.

[16] Khobragade, N.W (2013): Thermal stresses of a hollow cylinder with radiation type conditions, Int. J. of Engg. And Information Technology, vol. 3, Issue 5, pp.25-32, USA (Impact Factor 1.895).

[17] Khobragade, N.W (2013): Thermo elastic analysis of a solid circular cylinder, Int. J. of Engg. And Information Technology, vol. 3, Issue 5, pp. 155-162, USA (Impact Factor 1.895).

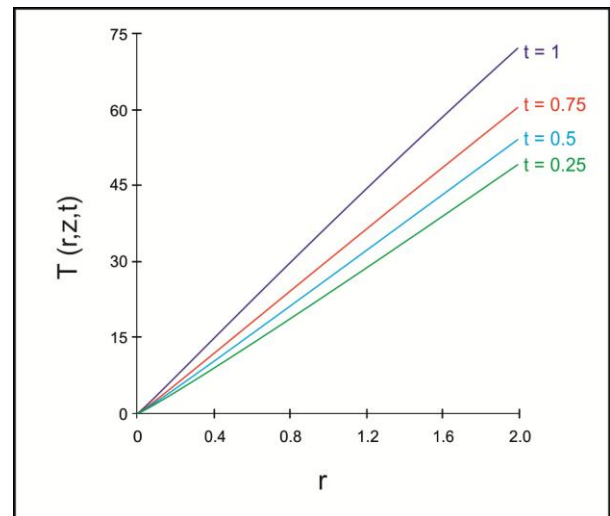
[18] Khobragade, N.W (2013): Thermo elastic analysis of a thick hollow cylinder with radiation conditions, Int. J. of Engg. And Information Technology, vol. 3, Issue 4, pp.380-387, USA (Impact Factor 1.895).

(Thermoelasticity in particular) and Operations Research. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and six students submitted their thesis in University for award of Ph.D Degree under their guidance.

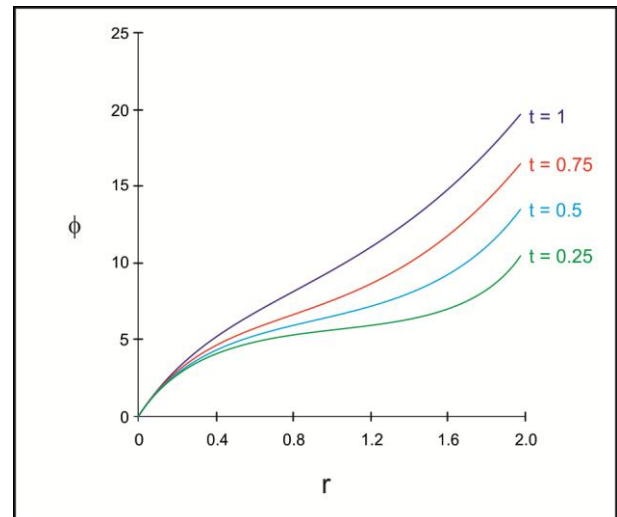


Ms. R. S. Ghume, M.Sc (Maths), research student
Dept. of Maths, RTM Nagpur University, Nagpur.

APPENDIX



Graph 1: Temperature distribution vs. r

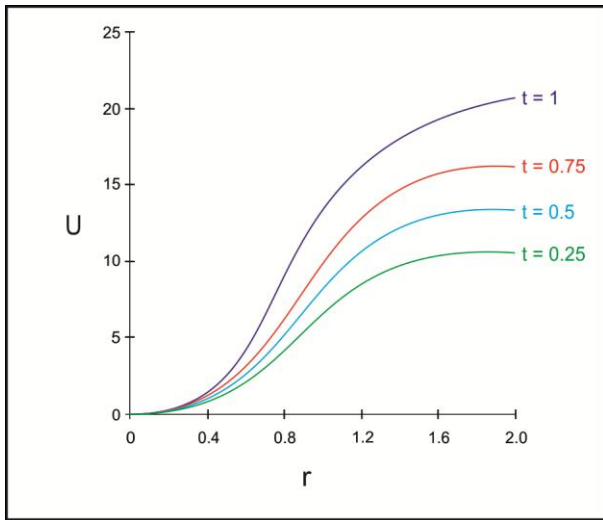


Graph 2: Distribution function vs. r

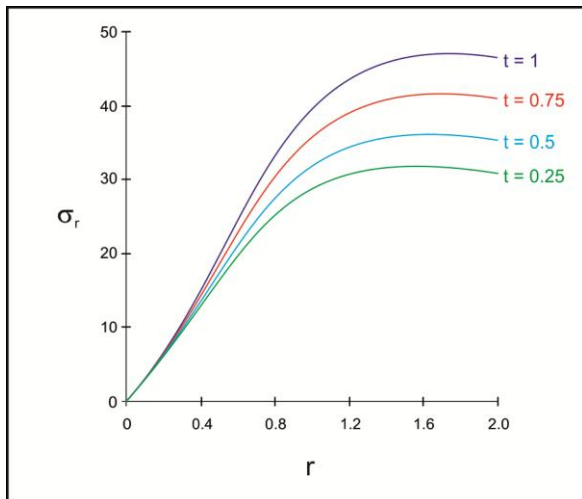
AUTHOR BIOGRAPHY



Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems



Graph 3: Thermal stress components vs. r



Graph 4: Radial stresses vs. r