

Inverse Thermo elastic Problem of Semi Infinite Rectangular Beam due to Heat Generation

Ashwini Mahakalkar; Ranjana S. Ghume, N. W. Khobragade
 Department of Mathematics, MJP Educational Campus,
 RTM Nagpur University, Nagpur 440 033, India

Abstract- This paper is concerned with inverse transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite rectangular beam when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem. The results are depicted graphically.

KEY WORDS: Semi-infinite rectangular beam, inverse transient problem, Integral transform.

I. INTRODUCTION

Khobragade et al. [2-7, 9] have investigated temperature distribution, displacement function, and stresses of a thin rectangular plate and Khobragade et al. [8] have established displacement function, temperature distribution and stresses of a semi-infinite rectangular beam.

In this paper, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a semi-infinite rectangular beam occupying the region $D : -a \leq x \leq a ; 0 \leq y \leq b, 0 \leq z \leq \infty$ with known boundary conditions. Here Marchi-Fasulo transforms and Fourier cosine transform techniques have been used to find the solution.

II. STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space $D: a \leq x \leq a ; 0 \leq y \leq b, 0 \leq z \leq \infty$. The displacement components u_x and u_y u_z in the x and y and z directions respectively as Tanigawa et al. [1] are

$$u_x = \int_{-a}^a \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \lambda T \right] dx \quad (1)$$

$$u_y = \int_0^b \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \lambda T \right] dy \quad (2)$$

$$u_z = \int_0^\infty \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2} \right) + \lambda T \right] dz \quad (3)$$

where E, ν , and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and $U(x,y,z,t)$ is the Airy's stress functions which satisfy the differential equation as Tanigawa et al. [1] is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)^2 U(x, y, z, t) = -\lambda E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \times T(x, y, z, t) \quad (4)$$

where $T(x,y,z,t)$ denotes the temperature of a rectangular beam satisfy the following differential

equation as Tanigawa et al. [1] is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x, y, z, t)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (5)$$

Where k is the thermal conductivity and α is the thermal diffusivity of the material,

subject to initial condition

$$T(x, y, z, 0) = f(x, z, t) \quad (6)$$

The boundary conditions are

$$\left[T(x, y, z, t) + k_1 \frac{\partial T(x, y, z, t)}{\partial x} \right]_{x=a} = f_1(y, z, t) \quad (7)$$

$$\left[T(x, y, z, t) + k_2 \frac{\partial T(x, y, z, t)}{\partial x} \right]_{x=-a} = f_2(y, z, t) \quad (8)$$

$$[T(x, y, z, t)]_{y=0} = f_3(x, z, t) \quad (9)$$

$$[T(x, y, z, t)]_{y=\xi} = f_4(x, z, t) \text{ (Known)} \quad (10)$$

$$[T(x, y, z, t)]_{y=b} = G(x, z, t) \text{ (Unknown)} \quad (11)$$

$$\left. \frac{\partial T(x, y, z, t)}{\partial z} \right|_{z=0} = 0 \quad (12)$$

$$\left. \frac{\partial T(x, y, z, t)}{\partial z} \right|_{z=\infty} = h(x, y, t) \quad (13)$$

The stress components in terms of $U(x, y, z, t)$ Tanigawa et al. [1] are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right] \quad (14)$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} \right] \quad (15)$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right] \quad (16)$$

The equations (1) to (16) constitute the mathematical formulation of the problem under consideration.

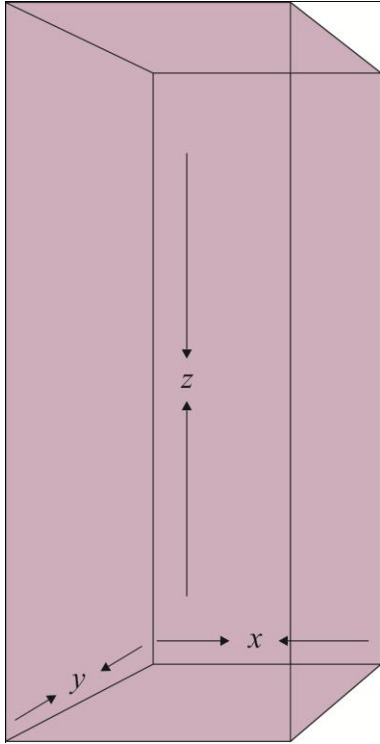


Fig 1: Geometry of the problem

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform, finite Fourier sine transform and Fourier cosine transform to the equations, we get

$$\frac{d\bar{T}}{dt} + \alpha q^2 \bar{T} = \frac{\alpha g}{k} + \Psi \quad (17)$$

This is a linear equation whose solution is given by

$$\bar{T}(m, n, \eta, t) = e^{-\alpha q^2 t} \left(\bar{f} + \int_0^t \left[\frac{\alpha g}{k} + \Psi \right] e^{\alpha q^2 t'} dt' \right)$$

where,

$$\Psi = \frac{P_n(a)}{k_1} f_1 - \frac{P_n(-a)}{k_2} f_2 + \frac{m\pi}{b} [(-1)^{m+1} f_4 + f_3] \quad (19)$$

Now, applying inversion of Fourier Cosine transform,

Fourier sine transform and finite Marchi-Fasulo transform to the equation (18), one obtains the expression for temperature distribution and unknown temperature gradient as

$$T(x, y, z, t) = \left(\frac{4\eta}{\xi\pi} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \Lambda(z) \quad (20)$$

$$G(x, z, t) = \left(\frac{4\eta}{\xi\pi} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin(pb) \Lambda(z) \quad (21)$$

where,

$$\Lambda(z) = \int_0^{\infty} B(t) \cos(\eta z) dz$$

$$B(t) = e^{-\alpha q^2 t} \left(\bar{f} + \int_0^t \left[\frac{\alpha g}{k} + \Psi \right] e^{\alpha q^2 t'} dt' \right)$$

$$p = \left(\frac{m\pi}{\xi} \right)$$

$$q^2 = \left(1 + \lambda_n^2 + \frac{m^2 \pi^2}{b^2} \right)$$

Equations (20) and (21) are the required solutions.

IV. AIRY'S STRESS FUNCTIONS

Substituting the value of temperature distribution T(x,y,z,t) from (20) in equation (4) one obtains

$$U(x, y, z, t) = - \left(\frac{4\eta\pi E}{\xi\pi} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \Lambda(z) \quad (22)$$

$$\text{where } \Lambda(z) = \int_0^{\infty} B(t) \cos(\eta z) dz$$

V. DISPLACEMENT COMPONENTS

Substituting the values of Airy's stress function from equation (22) in the equation (1) to (3), one obtains

$$u_x = - \left(\frac{4\eta\lambda}{\xi\pi} \right) \Lambda(z) \int_{-a}^a \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) (-p^2 \sin py) - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \eta^2 \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \right. \\ \left. - \nu \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n''(x)}{\lambda_n^2} \right) \sin py + \lambda \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \right\} dx \quad (23)$$

$$u_y = - \left(\frac{4\eta\lambda}{\xi\pi} \right) \Lambda(z) \int_0^b \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \eta^2 \left(\frac{P_n(x)}{\lambda_n} \right) \sin py - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n''(x)}{\lambda_n^2} \right) \sin py \right. \\ \left. + \nu \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) (-p^2 \sin py) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \right\} dy \quad (24)$$

$$u_z = -\left(\frac{4\eta\lambda}{\xi\pi}\right)\Lambda(z)\int_0^\infty \left\{ \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n''(x)}{\lambda_n^2}\right) \sin py + \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) (-p^2 \sin py) \right. \\ \left. + v \sum_{m=1}^\infty \sum_{n=1}^\infty \eta^2 \left(\frac{P_n(x)}{\lambda_n}\right) \sin py - \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \sin py \right\} dz \quad (25)$$

VI. DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z,t) from equation (22) in the equation (14) to (16) one obtain the stress functions as,

$$\sigma_{xx} = -\left(\frac{4\eta\lambda E}{\xi\pi}\right)\Lambda(z)\sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \left[\sum_{m=1}^\infty (-p^2 \sin py) - \eta^2 \sum_{m=1}^\infty \sin py \right]$$

$$\sigma_{yy} = -\left(\frac{4\eta\lambda E}{\xi\pi}\right)\Lambda(z)\sum_{m=1}^\infty \sin py \left[\sum_{n=1}^\infty \left(\frac{P_n''(x)}{\lambda_n^2}\right) - \eta^2 \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \right]$$

$$\sigma_{zz} = -\left(\frac{4\eta\lambda E}{\xi\pi}\right)\Lambda(z) \left[\sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n''(x)}{\lambda_n^2}\right) \sin py + \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) (-p^2 \sin py) \right] \quad (28)$$

VII. SPECIAL CASE

Set

$$f(x, y, z, t) = (x - a)^2 (x + a)^2 (z + e^{-z})(e^{y-t})$$

$$\bar{f}(n, y, z, t) = (z + e^{-z})(e^{y-t})$$

$$\times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right] \quad (29)$$

Substituting the above value in equations (20) to (28) one obtains

$$T(x, y, z, t) = \left(\frac{4\eta}{\xi\pi}\right) \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \sin py \Lambda(z)$$

$$\times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right]$$

$$(z + e^{-z})(e^{y-t}) \quad (30)$$

$$G(x, z, t) = \left(\frac{4\eta}{\xi\pi}\right) \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \sin(py) \Lambda(z)$$

$$\times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right]$$

$$(z + e^{-z})(e^{y-t}) \quad (31)$$

$$U(x, y, z, t) = -\frac{4\eta\pi E}{\xi\pi} \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \sin py \Lambda(z)$$

$$\times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right]$$

$$(z + e^{-z})(e^{y-t}) \quad (32)$$

$$u_x = -\frac{4\eta\lambda}{\xi\pi} \Lambda(z) \int_{-a}^a \left\{ \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) (-p^2 \sin py) - \sum_{m=1}^\infty \sum_{n=1}^\infty \eta^2 \left(\frac{P_n(x)}{\lambda_n}\right) \sin py \right. \\ \left. - v \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n''(x)}{\lambda_n^2}\right) \sin py + \lambda \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \sin py \right\} dx \quad (26)$$

$$\times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right]$$

$$(z + e^{-z})(e^{y-t}) \quad (33)$$

$$u_y = -\frac{4\eta\lambda}{\xi\pi} \Lambda(z) \int_0^b \left\{ \sum_{m=1}^\infty \sum_{n=1}^\infty \eta^2 \left(\frac{P_n(x)}{\lambda_n}\right) \sin py - \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n''(x)}{\lambda_n^2}\right) \sin py \right. \\ \left. + v \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) (-p^2 \sin py) + \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \sin py \right\} dy$$

$$\times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right]$$

$$(z + e^{-z})(e^{y-t}) \quad (34)$$

$$u_z = -\frac{4\eta\lambda}{\xi\pi} \Lambda(z) \int_0^\infty \left\{ \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n''(x)}{\lambda_n^2}\right) \sin py + \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) (-p^2 \sin py) \right. \\ \left. + v \sum_{m=1}^\infty \sum_{n=1}^\infty \eta^2 \left(\frac{P_n(x)}{\lambda_n}\right) \sin py - \sum_{m=1}^\infty \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \sin py \right\} dz$$

$$\times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right]$$

$$(z + e^{-z})(e^{y-t}) \quad (35)$$

$$\sigma_{xx} = -\frac{4\eta\lambda E}{\xi\pi} \Lambda(z) \sum_{n=1}^\infty \left(\frac{P_n(x)}{\lambda_n}\right) \left[\sum_{m=1}^\infty (-p^2 \sin py) - \eta^2 \sum_{m=1}^\infty \sin py \right]$$

$$\times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (41)$$

$$\sigma_{yy} = -\frac{4\eta\lambda E}{\xi\pi} \Lambda(z) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin py \left[\sum_{n=1}^{\infty} \left(\frac{P_n''(x)}{\lambda_n^2} \right) - \eta^2 \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \right] \times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (36)$$

$$u_x = -\left(\frac{2\eta\lambda}{\pi} \right) \Lambda(z) \int_{-a}^a \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) (-p^2 \sin py) - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} -\eta^2 \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \right. \\ \left. - v \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n''(x)}{\lambda_n^2} \right) \sin py + \lambda \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \right\} dx \times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (42)$$

$$\sigma_{zz} = -\frac{4\eta\lambda E}{\xi\pi} \Lambda(z) \left[\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n''(x)}{\lambda_n^2} \right) \sin py + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) (-p^2 \sin py) \right] \times \left[\frac{a_n \cos^2(a_n a) - \cos(a_n a) \sin(a_n a)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (37)$$

$$u_y = -\left(\frac{2\eta\lambda}{\pi} \right) \Lambda(z) \int_0^b \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \eta^2 \left(\frac{P_n(x)}{\lambda_n} \right) \sin py - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n''(x)}{\lambda_n^2} \right) \sin py \right. \\ \left. + v \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) (-p^2 \sin py) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \right\} dy \times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (43)$$

VIII. NUMERICAL RESULTS

Set $a = 2, k = 0.86, b = 3, \xi = 2, t = 1$ sec in the equations (30)- (38) to obtain

$$T(x, y, z, t) = \left(\frac{2\eta}{\pi} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \Lambda(z) \times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (39)$$

$$u_z = -\left(\frac{2\eta\lambda}{\pi} \right) \Lambda(z) \int_0^{\infty} \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n''(x)}{\lambda_n^2} \right) \sin py + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) (-p^2 \sin py) \right. \\ \left. + v \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \eta^2 \left(\frac{P_n(x)}{\lambda_n} \right) \sin py - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \right\} dz \times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (44)$$

$$G(x, z, t) = \left(\frac{2\eta}{\pi} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin(pb) \Lambda(z) \times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (40)$$

$$\sigma_{xz} = -\left(\frac{2\eta\lambda E}{\pi} \right) \Lambda(z) \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \left[\sum_{m=1}^{\infty} (-p^2 \sin py) - \eta^2 \sum_{m=1}^{\infty} \sin py \right] \times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2} \right] (z + e^{-z})(e^{y-t}) \quad (45)$$

$$U(x, y, z, t) = -\left(\frac{2\eta\pi E}{\pi} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \sin py \Lambda(z) \times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2} \right]$$

$$\sigma_{yy} = -\left(\frac{2\eta\lambda E}{\pi} \right) \Lambda(z) \sum_{m=1}^{\infty} \sin py \left[\sum_{n=1}^{\infty} \left(\frac{P_n''(x)}{\lambda_n^2} \right) - \eta^2 \sum_{n=1}^{\infty} \left(\frac{P_n(x)}{\lambda_n} \right) \right] \times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2} \right]$$

$$\times (z + e^{-z})(e^{y-t}) \quad (46)$$

$$\sigma_z = -\left(\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2 \sin py)\right]$$

$$\times \left[\frac{a_n \cos^2(2a_n) - \cos(2a_n) \sin(2a_n)}{a_n^2}\right]$$

$$\times (z + e^{-z})(e^{y-t}) \quad (47)$$

IX. MATERIAL PROPERTIES

The numerical calculations has been carried out for an Aluminum (pure) rectangular beam with the material properties as,

Density $\rho = 169 \text{ lb/ft}^3$

Specific heat = 0.208 Btu/lbOF

Thermal conductivity $K = 117 \text{ Btu/(hr. ftOF)}$

Thermal diffusivity $\alpha = 3.33 \text{ ft}^2/\text{hr.}$

Poisson ratio $\nu = 0.35$

Coefficient of linear thermal expansion $\alpha_t = 12.84 \times 10^{-6}/\text{F}$

Lame constant $\mu = 26.67$

Young's modulus of elasticity $E = 70 \text{ GPa}$

X. DIMENSIONS

The constants associated with the numerical calculation are taken as

Length of rectangular beam $x = 4\text{ft}$

Breath of rectangular beam $y = 3 \text{ ft}$

Height of rectangular beam $z = 10^3\text{ft}$

XI. CONCLUSION

In this article, the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a semi-infinite rectangular beam have been obtained, when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and semi-infinite Fourier cosine transform techniques. The results are obtain in the form of infinite series in terms of Bessel's function and depicted graphically.

ACKNOWLEDGEMENT

The authors are thankful to University Grant Commission, New Delhi for providing the partial financial assistance under major research project scheme.

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AUTHOR BIOGRAPHY



Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities.

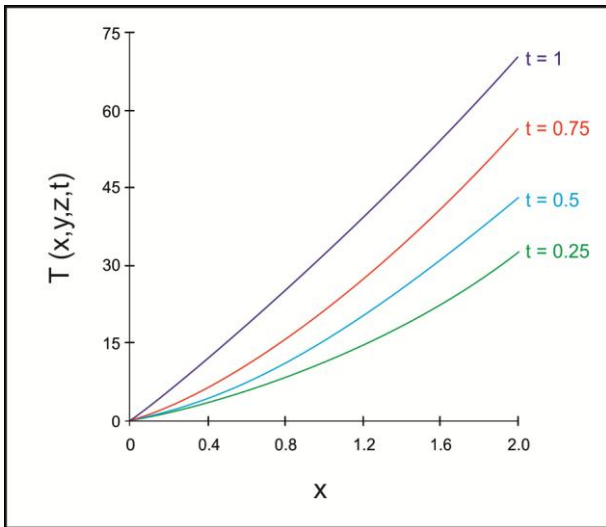
At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems (Thermo elasticity in particular) and Operations Research. Published more than 180 research papers in reputed journals.

Fourteen students awarded Ph.D Degree and six students submitted their thesis in University for award of Ph.D Degree under their guidance.

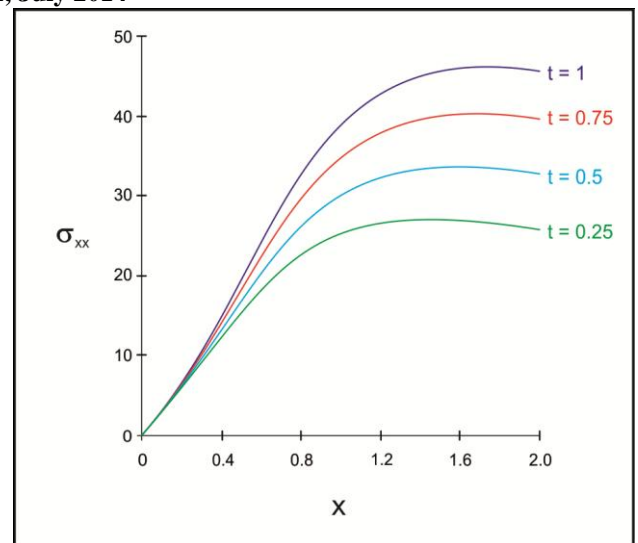


Ms. R. S. Ghume, M.Sc (Maths), research student Dept. of Maths, RTM Nagpur University, Nagpur

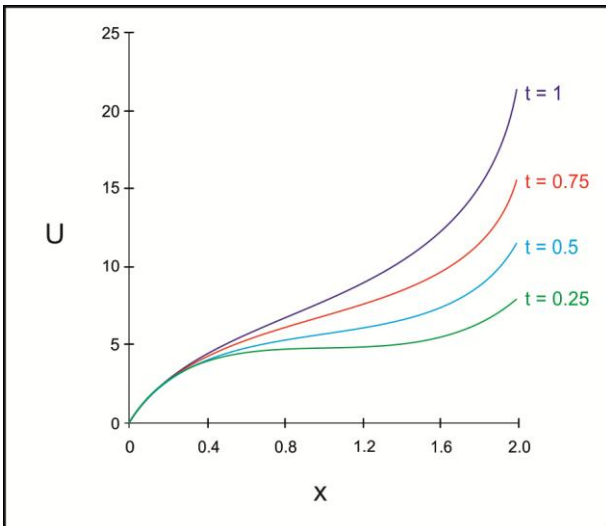
APPENDIX



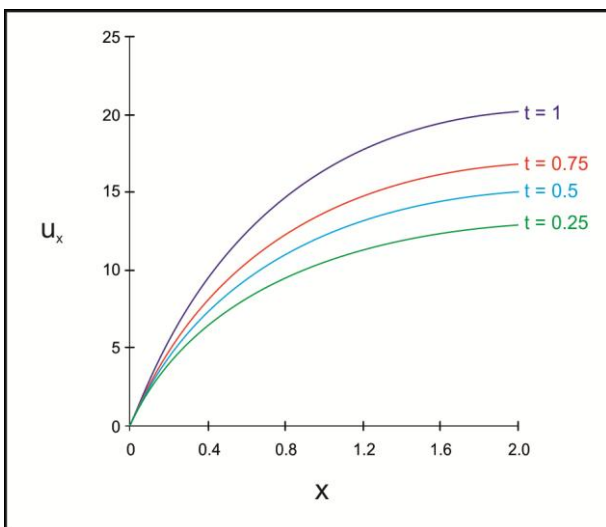
Graph 1: Temperature distribution vs. x



Graph 4: Stress distribution vs. x



Graph 2: Displacement function vs. x



Graph 3: Displacement components vs. x