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Thermal Stresses of a Thick Circular Plate due to Heat Generation

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Abstract- ***In this paper, an attempt has been made to study thermoelastic response of a thick circular plate occupying the space*** $D: 0 \le r \le a,\ -h \le z \le h,$ ***due to heat generation with radiation type boundary conditions. Here we apply integral transform techniques to find the thermoelastic solution.***

Keywords: **Thermo elastic Response, Circular Plate, Thermal Stresses, integral transform**

I. INTRODUCTION

Khobragade et al. [3 - 12] have derived temperature distribution, displacement function, thermal stresses and thermal deflection of a thick and thin circular plate. Further Khobragade et al. [13] have established displacement function, temperature distribution and stresses and deflection of a triangular plate.

This paper is concerned with transient thermoelastic problem of a thick circular plate occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ due to heat generation with radiation type boundary conditions.

II. STATEMENT OF THE PROBLEM

Consider thick circular plate of thickness $2h$ occupying the space $D: 0 \le r \le a,\ -h \le z \le h,$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r,z,t)$ as Nowacki [2] is

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=\left(\frac{1+\nu}{1-\nu}\right)\alpha_t T \qquad (1)$$

where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation as Noda [3] is

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\chi(r,z,t)=\frac{1}{k}\frac{\partial T}{\partial t} \qquad (2)$$

Subject to initial condition

$$M_t(T,1,0,0)=F(r,z) \quad 0 \le r \le a, -h \le z \le h. \qquad (3)$$

The boundary conditions are

$$\left.\begin{aligned} M_r(T,0,1,0)&=g_1(z,t)\\ M_r(T,0,1,a)&=g_2(z,t)\end{aligned}\right\},\quad -h \le z \le h,\ t>0 \qquad (4)$$

$$\left.\begin{aligned} M_z(T,1,k_1,h)&=f_1(r,t)\\ M_z(T,1,k_2,-h)&=f_2(r,t)\end{aligned}\right\},\quad 0 \le r \le a,\ t>0 \qquad (5)$$

where k is thermal diffusivity of material of the plate.

The displacement function in the cylindrical coordinate system are represented by Love's function as Khobragade [4] are

$$u_r=\frac{\partial\phi}{\partial r}-\frac{\partial^2 L}{\partial r\partial z} \qquad (6)$$

$$u_z=\frac{\partial\phi}{\partial z}+2(1-\nu)\nabla^2 L-\frac{\partial^2 L}{\partial z^2} \qquad (7)$$

The Love's function [14] must satisfy

$$\nabla^2\nabla^2 L=0 \qquad (8)$$

where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the thermoelastic displacement potential ϕ and Love's function L as Noda [3] are

$$\sigma_{rr}=2G\left\{\left(\frac{\partial^2\phi}{\partial r^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\nu\nabla^2 L-\frac{\partial^2 L}{\partial r^2}\right)\right\} \qquad (9)$$

$$\sigma_{\theta\theta}=2G\left\{\left(\frac{1}{r}\frac{\partial\phi}{\partial r}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left(\nu\nabla^2 L-\frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right)\right\} \qquad (10)$$

$$\sigma_{zz}=2G\left\{\left(\frac{\partial^2\phi}{\partial z^2}-\nabla^2\phi\right)+\frac{\partial}{\partial z}\left\{\left((z-\nu)\nabla^2 L-\frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \qquad (11)$$

$$\sigma_{rz}=2G\left\{\frac{\partial^2\phi}{\partial r\partial z}+\frac{\partial}{\partial r}\left\{\left((1-\nu)\nabla^2 L-\frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \qquad (12)$$

For traction free surface stress function

$\sigma_z=\sigma_{r\theta}=0$ at $z=\pm h$ for thick plate.

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Equations (1) to (12) constitute the mathematical formulation of the problem under consideration.

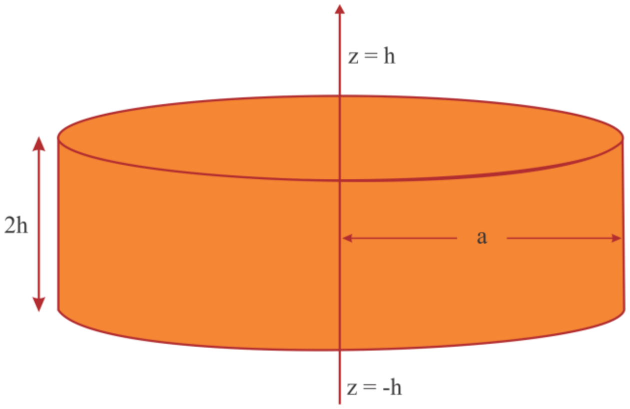

Fig. 1: The geometry of the problem

III. SOLUTION OF THE PROBLEM

Applying Hankel transform to the equation (2), we get

$$-\xi_m^2 T^*(\xi_m,z,t)+\frac{d^2T^*}{dz^2}(\xi_m,z,t)+\chi^*(\xi_m,z,t)=\frac{1}{k}\frac{dT^*}{dt} \qquad (13)$$

Again applying Marchi-Fasulo transform to above equation, we obtain

$$\frac{d\bar{T}^*}{dt}+kP^2\bar{T}^*=\Psi \qquad (14)$$

where

$$P^2=\xi_m^2+a_n^2$$

Equation (14) is a linear equation whose solution is given by

$$\bar{T}^*(\xi_m,n,t)=e^{-kP^2t}\int_0^t \Psi e^{kP^2t'}dt^1+Ce^{-kP^2t} \qquad (15)$$

Using (3), we get

$$C=\bar{F}^*(m,n)$$

Thus we have

$$\bar{T}^*(\xi_m,n,t)=e^{-kP^2t}\left[\int_0^t \Psi e^{kP^2t^1}dt^1+\bar{F}^*(m,n)\right] \qquad (16)$$

Applying inversion of Marchi-Fasulo transform and Hankel transform to the differential equation (16), we get

$$T(r,z,t)=\frac{2}{a^2}\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}e^{-kP^2t}$$

$$\times\left[\int_0^t \Psi\, e^{kP^2t'}dt'+\bar{F}^*(m,n)\right] \qquad (17)$$

This is the desired solution of the given problem.

Let us assume Love's function L, which satisfy condition (10) as

$$L(r,z)=\frac{2}{a^2}\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}\Omega \qquad (18)$$

where

$$\Omega=e^{-kP^2t}\left[\int_0^t \Psi e^{kP^2t'}dt'+\bar{F}^*(m,n)\right]$$

Using (1) and (17), we get displacement potential ϕ *as*

$$\phi=A\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n'(z)}{\lambda_n}\Omega+B(t)\right] \qquad (19)$$

where

$$A=\left(\frac{1+\nu}{1-\nu}\right)\frac{2\alpha_t}{a^2},$$

$$B(t)=\int e^{-kP^2t}\left(\int_0^t \Psi\, e^{-kP^2t'}dt'+\bar{F}^*(m,n)\right)dt$$

IV. DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (18) and (19) in equation (6), we get

$$u_r=A\sum_m\sum_n\frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n'(z)}{\lambda_n}\psi+B(t)\right]$$

$$-\frac{2}{a^2}\sum_m\sum_n\frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n'(z)}{\lambda_n}\Omega \qquad (20)$$

$$u_z=A\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n''(z)}{\lambda_n}\Omega+B(t)\right]$$

$$+2(1-\nu)\left[\frac{2}{a^2}\sum_m\sum_n\frac{\xi_m^2[J_1'(r\xi_m)+J_1(r\xi_m)]}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}\Omega\right]$$

$$+\frac{2(1-2\nu)}{a^2}\sum\sum\frac{J_0(r\xi_m)}{J_1(a\xi_m)^2}\frac{P_n''(z)}{\lambda_n}\Omega \qquad (21)$$

Substituting equations (18) and (19) in equations (9) to (12), we obtain

$$\sigma_{rr}=2G\left\{\frac{2(\nu-1)}{a^2}\sum_m\sum_n\frac{\xi_m^2 J_1'(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n'(z)}{\lambda_n}\Omega\right.$$

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$$+\frac{2}{a^2 r}\sum_m\sum_n \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n'(z)}{\lambda_n}\Omega$$

$$+\frac{2}{a^2}\sum_m\sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n'(z)}{\lambda_n}\Omega$$

$$-\frac{A}{r}\sum_m\sum_n \frac{\xi_m J_1(r\xi_m)}{J_1(a\xi_m)^2}\left[\frac{P_n'(z)}{\lambda_n}\Omega + B(t)\right]$$

$$\left.-A\sum_m\sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n'(z)}{\lambda_n}\Omega + B(t)\right]\right\} \quad (22)$$

$$\sigma_{\theta\theta} = 2G\left\{\frac{2\nu}{a^2}\sum_m\sum_n \frac{\xi_m^2 J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^1(z)}{\lambda_n}\Omega\right.$$

$$+\frac{2(\nu-1)}{a^2}\sum\sum \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^1(z)}{\lambda_n}A\Omega$$

$$+\frac{2\nu}{a^2}\sum_m\sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^{111}(z)}{\lambda_n}\Omega$$

$$-A\sum_m\sum_n \frac{J_0(r\xi_m)}{J_1(a\xi_m)^2}\left[\frac{P_n^1(z)}{\lambda_n}\Omega + B(t)\right]$$

$$\left.-A\sum_m\sum_n \xi_m^2 \frac{J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n^1(z)}{\lambda_n}\Omega + B(t)\right]\right\} \quad (23)$$

$$\sigma_{zz} = 2G\left\{\frac{(2-\nu)}{a^2}\sum\sum \frac{\xi_m^2 J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^1(z)}{\lambda_n}\Omega\right.$$

$$+\frac{2(2-\nu)}{a^2 r}\sum_m\sum_n \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^1(z)}{\lambda_n}\Omega$$

$$+\frac{2(1-\nu)}{a^2}\sum\sum \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^{111}(z)}{\lambda_n}\Omega$$

$$-A\sum\sum \frac{\xi_m^2 J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n^1(z)}{\lambda_n}\Omega + B(t)\right]$$

$$\left.-\frac{A}{r}\sum\sum \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n^1(z)}{\lambda_n}\Omega + B(t)\right]\right\} \quad (24)$$

$$\sigma_{rz} = 2G\left\{\frac{2(1-\nu)}{a^2}\sum\sum \frac{\xi_m^3 J_1^{11}(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P^1(z)}{\lambda_n}\Omega\right.$$

$$+\frac{2(-\nu)}{a^2}\sum\sum \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n^{11}(z)}{\lambda_n}\Omega$$

$$+\frac{2(1-\nu)}{a^2}\sum\sum \frac{\xi_m}{[J_1(a\xi_m)]^2}\left\{\frac{\xi_m r J_1^1(r\xi_m)}{r_2}\right\}\frac{P_n(z)}{\lambda_n}\Omega$$

$$\left.+A\sum\sum \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2}\left[\frac{P_n^4(z)}{\lambda_n}\Omega + B(t)\right]\right\} \quad (25)$$

Where

$$A = \left(\frac{1+\nu}{1-\nu}\right)\frac{2\alpha_t}{a^2},$$

$$\Omega = e^{-kp^2 t}\left[\int_0^t \Psi e^{-kp^2 t^1} dt^1 + \overline{F}^*(m,n)\right],$$

$$B(t) = \int \Omega \, dt$$

V SPECIAL CASE

Set $F(r,z) = z^2(1-r^2)$ (26)

Applying Marchi-Fasulo transform, are obtain

$$\overline{F}(r,n) = (1-r^2)\int_{-h}^{h} z^2 P_n(z) dz$$

$$\overline{F}(r,n) = (1-r^2)\Phi_n\left[\frac{2h^2\sin(a_n h)}{a_n} + \frac{4h\cos(a_n h)}{a_n^{\ 2}} - \frac{4\sin(a_n h)}{a_n^{\ 3}}\right]$$

Where

$$P_n(z) = Q_n\cos(a_n z) - W_n\sin(a_n z) \ ,$$

$$Q_n = a_n(\alpha_1+\alpha_2)\cos(a_n h) + (\beta_1-\beta_2)\sin(a_n h)$$

$$W_n = (\beta_1-\beta_2)\cos(a_n h) + a_n(\alpha_1-\alpha_2)\sin(a_n h)$$

Again on applying Hankel transform, we obtain

$$\overline{F}^*(m,n) = \Pi_n\left[\frac{a}{\xi_m}J_1(a\xi_m) - \frac{a(a^2\xi_m^2-4)}{\xi_m^3}J_1(a\xi_m) - \frac{2a^2}{\xi_m^2}J_0(a\xi_m)\right] \quad (28)$$

Where

$$\Pi_n = \Phi_n\left[\frac{2h^2\sin(a_n h)}{a_n} + \frac{4h\cos(a_n h)}{a_n^{\ 2}} - \frac{4\sin(a_n h)}{a_n^{\ 3}}\right]$$

And

$$\Phi_n = a_n(\alpha_1+\alpha_2)\cos(a_n h) + (\beta_1-\beta_2)\sin(a_n h).$$

Using equation (27) in equation (17), one obtains

$$T(r,z,t) = \frac{2}{a^2}\sum_m\sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}e^{-kp^2 t}$$

$$\times\left[\int_0^t \Psi e^{kp^2 t} \, dt^1 + \Pi_n\right]$$

$$\times\left(\frac{a}{\xi_m}J_1(a\xi_m)-\frac{a(a^2\xi_m^2-4)}{\xi_m{}^3}J_1(a\xi_m)-\frac{2a^2}{\xi_m^2}J_0(a\xi_m)\right) \tag{29}$$

V. NUMERICAL RESULTS

Set

$a=2, k=15.9\times10^6, t=1$ second in equation (29), we get

$$T(r,z,t)=\frac{2}{4}\sum_m\sum_n\frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2}\frac{P_n(z)}{\lambda_n}e^{-(15.9\times10^6)P^2t}$$

$$\times\left[\int_0^1\Psi e^{(15.9\times10^6)P^2t^1}dt^1+\Pi_n\left(\frac{2}{\xi_m}J_1(2\xi_m)-\frac{2(4\xi_m^2-4)}{\xi_m{}^3}J_1(2\xi_m)-\frac{2}{\xi_m^2}J_0(2\xi_m)\right)\right] \tag{30}$$

VI. CONCLUSION

In this article, the temperature distribution, displacement and thermal stresses of a thick circular plate are investigated with known boundary conditions. Finite integral transform techniques are used to obtain numerical results. The results are obtained in terms of Bessel's function in the form of infinite series.

Any particular cases of special interest can be assigned to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.

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AUTHOR BIOGRAPHY

Dr. N.W. Khobragade for being M.Sc in statistics and Maths, he attained Ph.D in both subjects. He has been teaching since 1986 for 28 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities.

At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems (Thermoelasticity in particular) and Operations Research. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and six students submitted their thesis in University for award of Ph.D Degree under their guidance.

Mrs. A.A. Kulkarni For being M.Sc in maths, she has been teaching since 1990 for 21 years at PCE of Maths, RTM Nagpur University, Nagpur and sucessfully handled different capacities.

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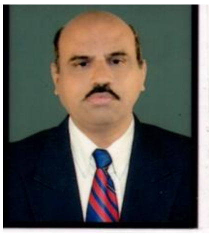
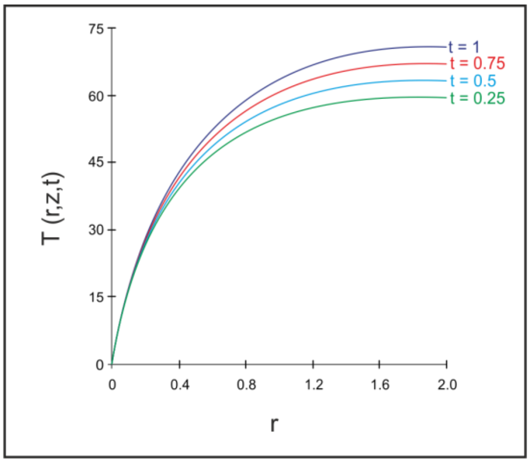
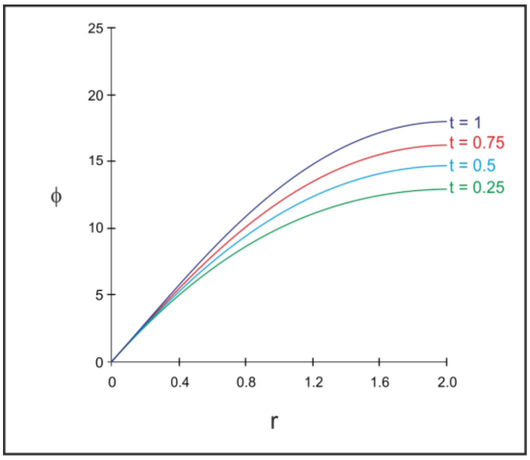


Dr. S.S.Singru M.Sc in Maths, he attained Ph.D in 2008. He has been teaching since 1988 for 26 years at Shri Dnyanesh Mahavidyalaya Nawargaon, and successfully handled different capacities.

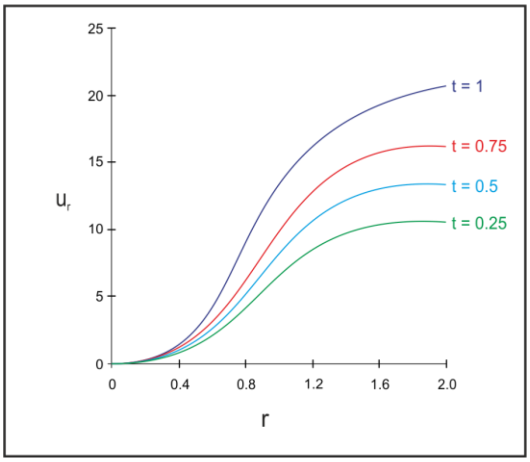
APPENDIX



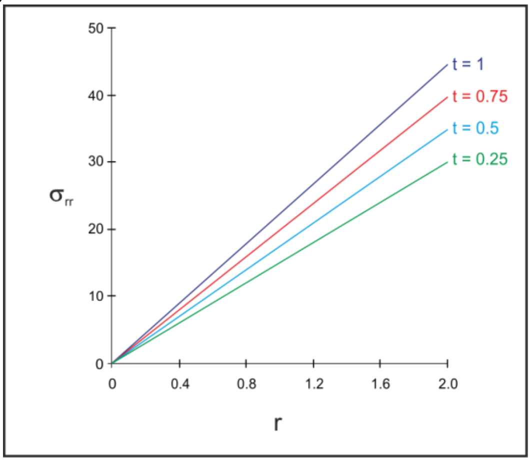
Graph 1: Graph of temperature distribution versus radius



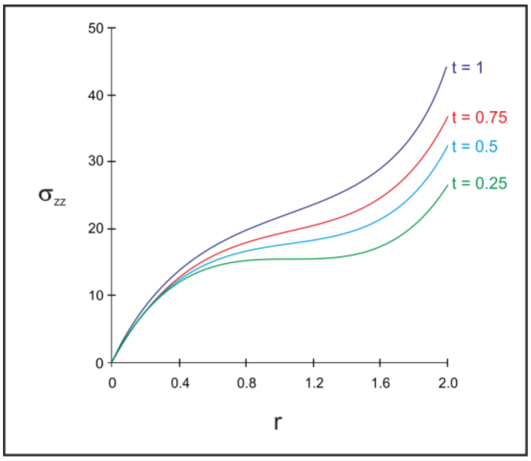
Graph 2: Graph of displacement function versus radius



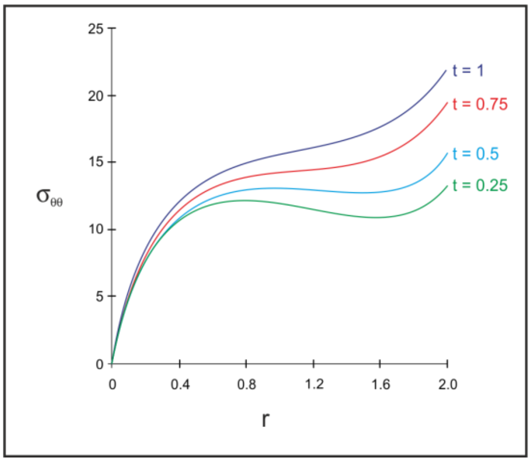
Graph 3: Graph of stress component versus radius



Graph 4: Graph of radial stresses versus radius



Graph 5: Graph of axial stresses versus radius



Graph 6: Graph of tangential stresses versus radius