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Analysis of Parameter Estimation Methods for Weibull Distribution and Interval Data

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Abstract*— In the absence of a system for continuous monitoring, the reliability data do not contain information about the precise time of occurrence of failures. Such data are interval (grouped) and censored. In these cases, problems arise in the application of mathematical apparatus. It is insufficient the accuracy of the results obtained in the parameters' estimations of the distribution law of time to failure. While for censored and complete data there is a number of works and studies, the interval data are less common in literature. In this paper we tested and analyzed methods for assessing the parameters of Weibull distribution with a view to their applicability in the analysis of interval data.

***Index Terms*— reliability, Weibull distribution, interval data.**

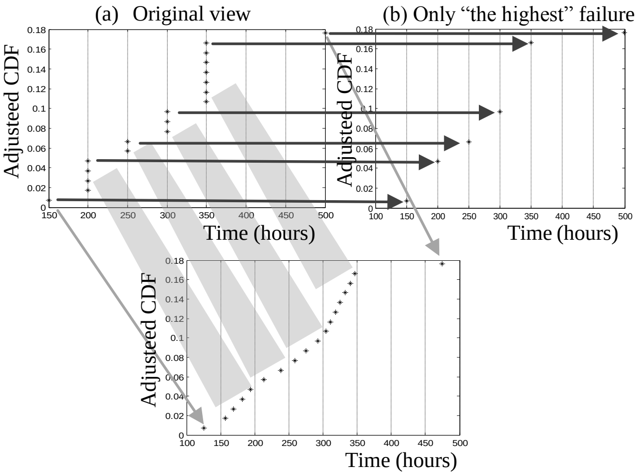
I. INTRODUCTION

In practice, the statistical evaluation of the reliability of electronic devices is performed by analyzing the data collected for failures that occurred during the operation or during reliability testing. Some of the factors influencing the accuracy of the results are the ways of control of the operating state and registration of moments at which failures occurred. At the presence of a system for continuous monitoring of all tested products, or all system units, any failure moment is recorded precisely. This increases the accuracy of the assessment of reliability indexes due to the avoiding of time uncertainty of failure data. In the process of data collection for reliability during operation, as well as during reliability tests, recording the precise moments of failures is difficult, sometimes impossible, to realize. In the information for the failures, the moment of occurrence is usually unknown, and data are censored (right and left censored), or interval data (inspection or grouped) [1]. The uncertainty can be due to the fact that the state of the devices is recorded only at the beginning and at the end of the test or observation time during normal operation. Additional difficulties emerge when studying and evaluating the reliability based on various types of data. While the problems related to working with censored (of different types) and complete data are well described and analyzed in literature [1]-[4], the appropriate practical ideas for processing interval data are less common.

Data collected in consecutive periodic inspections are named interval or grouped data. To simplify the analysis we assume that all items in a sample begin the tests at the same time, all devices are operable at the beginning of tests, and the inspection moments are previously known and are the same for all items in the sample. Then, the collected data contain information about the number of failures in each interval and the number of surviving devices after completion of the test.

The problem is how to evaluate the moments of failures within the *i*-th interval $[t_i,t_i+\Delta t_i)$ between two consecutive inspections where Δt_i is the duration of the interval. In his book [5] H. Rinne assumes that the failures have emerged in the middle of the interval at the time $t_i+\Delta t_i/2$, with maximum error $\pm\Delta t_i/2$. Thus, conclusions based on interval data provide additional uncertainty of results (in comparison to complete data), which is especially noticeable in small samples and small number of failures within an interval or the whole test.

In this paper we examine several methods of estimation of Weibull distribution parameters with a view to their applicability in the analysis of interval data. Some of these methods are only applicable when using complete data with precise failure times, and right censored data. This requires a reorganization of source data, which on the one hand increases the uncertainty in the final results, but on the other, simplifies the applied mathematical apparatus. With a sufficient degree of confidence, several ways to reorganization might be used:


(a) Original view
(b) Only “the highest” failure

(c) Uniform distribution

Fig.1. Transformation of interval data into complete data.

1. For each interval, only the failure with the highest statistical evaluation of the probability of failure is taken into account. This approach is proposed and successfully used by Sherwin [6].
2. Uniform distribution of failures within each interval.
3. Normal distribution within each interval.

For the purposes of our analysis we will call resulting from the transformation values of the failure times "transformed" data.

Fig.1 graphically presents the first and second way of reorganization, where adjusted CDF is point estimation of cumulative distribution function, or probability of failure, $F^{*}_{\mathrm{adj}}(t)$, usually calculated by Benard's approximation [7].

This article shows an approach to formulating the mathematical formalism by which to carry out processing of simulation and experimental data, and comparing the accuracy of the results obtained in the evaluation of the parameters of Weibull distribution, handling interval data.

II. ASSESSING THE WEIBULL DISTRIBUTION PARAMETERS FOR INTERVAL DATA USING THE METHOD OF MAXIMUM LIKELIHOOD

The methods which are traditionally used in the regression analysis of data from the reliability tests are the method Median Rank Regression (MRR), and the method of maximum likelihood (ML). In cases when interval data are available, only ML allows evaluation of the Weibull distribution parameters with no need to perform a data conversion [1]. This is achieved with the formulation of a likelihood function $L(t|\alpha,\beta)$ of such type [6]:

$$\begin{aligned} L(t\,|\,\alpha,\beta) &= \prod_{i=1}^{k} L_i(t\,|\,\alpha,\beta) = \prod_{i=1}^{k} \left(F_{iH}-F_{iL}\right)^{n_i} = \\ &= \prod_{i=1}^{k} \left[\left(1-\exp\left(-\left(t_{iH}/\alpha\right)^{\beta}\right)\right)-\left(1-\exp\left(-\left(t_{iL}/\alpha\right)^{\beta}\right)\right)\right]^{n_i} =, \\ &= \prod_{i=1}^{k} \left[\exp\left(-\left(t_{iL}/\alpha\right)^{\beta}\right)-\exp\left(-\left(t_{iH}/\alpha\right)^{\beta}\right)\right]^{n_i} \end{aligned} \qquad (1)$$

where k is the number of reporting intervals; n_i is the number of failures recorded in the i-th interval; n is the total number of registered failures, $\sum_{i=1}^{k} n_i = n$; F_{iH} and F_{iL} are the values of the probability of failure at the upper, respectively lower end of the i-th interval $[t_{iL},t_{iH})$; α and β are respectively the characteristic time and shape parameter of the Weibull distribution. We consider two-parametric form of Weibull distribution in our investigations. Equation (1) is only applicable to complete data and precise failure times.

With the exception of complete data, all other cases suggest the existence of given numbers of items which have preserved their working condition after completion of the tests. These devices determine the presence of right censored data. For each test, the likelihood function includes these data by adding another interval, $k+1$, beginning from the end of the test and finishing far ahead in time – enough time to assume that all "survivors" are failed. Starting point of this interval t_{k+1L} is the time of termination of the test t_{kH}, or $t_{k+1L}=t_{kH}$, and the end of the interval t_{k+1H} is regarded as infinity, $t_{k+1H}=\infty$. Then the probability of failure at the end of the interval will be $F_{k+1H}=1$, and the likelihood function takes this simplified form:

$$\begin{aligned} L_{k+1}(t\,|\,\alpha,\beta) &= \left(F_{k+1H}-F_{k+1L}\right)^{r} = \\ &= \left[1-\left(1-\exp\left(-\left(t_{k+1L}/\alpha\right)^{\beta}\right)\right)\right]^{r} = \left[\exp\left(-\left(t_{k+1L}/\alpha\right)^{\beta}\right)\right]^{r}, \end{aligned} \qquad (2)$$

where r is the number of survived items.

In a similar way one can consider the first interval in which the starting point is a value $t_{1L}=0$:

$$\begin{aligned} L_1(t\,|\,\alpha,\beta) &= \left(F_{1H}-F_{1L}\right)^{n_1} = \\ &= \left[\left(1-\exp\left(-\left(t_{1H}/\alpha\right)^{\beta}\right)\right)-\left(1-\exp\left(-\left(t_{1L}/\alpha\right)^{\beta}\right)\right)\right]^{n_1} = \cdot \\ &= \left[1-\exp\left(-\left(t_{1H}/\alpha\right)^{\beta}\right)\right]^{n_1} \end{aligned} \qquad (3)$$

Taking into account the above considerations, and resulting equations (2) and (3), the likelihood function $L(t|\alpha,\beta)$ can be shown as:

$$\begin{aligned} L(t\,|\,\alpha,\beta) &= \left[1-\exp\left(-\left(t_{1H}/\alpha\right)^{\beta}\right)\right]^{n_1} \cdot \left[\exp\left(-\left(t_{k+1L}/\alpha\right)^{\beta}\right)\right]^{r} \cdot \\ &\cdot \prod_{i=2}^{k} \left[\exp\left(-\left(t_{iL}/\alpha\right)^{\beta}\right)-\exp\left(-\left(t_{iH}/\alpha\right)^{\beta}\right)\right]^{n_i} \end{aligned} , \qquad (4)$$

where the total number N of tested devices is

$$N = r + \sum_{i=1}^{k} n_i \,.$$

To facilitate the further mathematical processing the logarithmic form of likelihood function $\pounds(t|\alpha,\beta)$ is obtained:

$$\begin{aligned} \pounds(t\,|\,\alpha,\beta) &= \ln\left(L(t\,|\,\alpha,\beta)\right) = n_1.\ln\left[1-e^{-\left(\frac{t_{1H}}{\alpha}\right)^{\beta}}\right] + \\ &+ r.\ln\left[e^{-\left(\frac{t_{k+1L}}{\alpha}\right)^{\beta}}\right] + \sum_{i=2}^{k} n_i.\ln\left[e^{-\left(\frac{t_{iL}}{\alpha}\right)^{\beta}} - e^{-\left(\frac{t_{iH}}{\alpha}\right)^{\beta}}\right] \end{aligned} . \qquad (5)$$

To assess the values of the parameters of Weibull distribution, partial differential equations of likelihood function on α and β are drawn. They form a system of two equations and its roots are estimates of the parameters of the distribution obtained by the method of maximum likelihood. After some simplifications and transformations, we obtain the final form of the equations:

$$\left|\begin{aligned} \varphi_1 &= n_1.\frac{t^{\beta}_{1H}}{B_1-1} - \sum_{i=2}^{k+1} n_i.t^{\beta}_{iL} + \sum_{i=2}^{k} n_i.\frac{t^{\beta}_{iH}-t^{\beta}_{iL}}{A_i-1} = 0 \\ \varphi_2 &= n_1.\frac{t^{\beta}_{1H}.\ln\left(\frac{t_{1H}}{\alpha}\right)}{B-1} - \sum_{i=2}^{k+1} n_i.t^{\beta}_{iL}.\ln\left(\frac{t_{iL}}{\alpha}\right) + \\ &+ \sum_{i=2}^{k} n_i.\frac{C_i}{A_i-1} = 0 \end{aligned}\right. \quad , n_{k+1}=r, \qquad (6)$$

where:

$$A_i = \exp\left[\left(t_{iH}/\alpha\right)^{\beta}-\left(t_{iL}/\alpha\right)^{\beta}\right], \qquad (7)$$

$$B_1 = \exp\left(t_{1H}/\alpha\right)^{\beta}, \qquad (8)$$

$$C_i = \left[t^{\beta}_{iH}.\ln\left(\frac{t_{iH}}{\alpha}\right) - t^{\beta}_{iL}.\ln\left(\frac{t_{iL}}{\alpha}\right)\right]. \qquad (9)$$

To solve the resulting system we use the method of Newton-Raphson (NR) for nonlinear systems on the iterative cycle of evaluation of the parameters α and β [8]:

$$\left.\begin{aligned}\hat{\alpha}_{i+1}&=\hat{\alpha}_i-\frac{\varphi_{1,i}\cdot\frac{\partial\varphi_{2,i}}{\partial\beta}-\varphi_{2,i}\cdot\frac{\partial\varphi_{1,i}}{\partial\beta}}{\frac{\partial\varphi_{1,i}}{\partial\alpha}\cdot\frac{\partial\varphi_{2,i}}{\partial\beta}-\frac{\partial\varphi_{1,i}}{\partial\beta}\cdot\frac{\partial\varphi_{2,i}}{\partial\beta}}\\ \hat{\beta}_{i+1}&=\hat{\beta}_i-\frac{\varphi_{2,i}\cdot\frac{\partial\varphi_{1,i}}{\partial\alpha}-\varphi_{1,i}\cdot\frac{\partial\varphi_{2,i}}{\partial\alpha}}{\frac{\partial\varphi_{1,i}}{\partial\alpha}\cdot\frac{\partial\varphi_{2,i}}{\partial\beta}-\frac{\partial\varphi_{1,i}}{\partial\beta}\cdot\frac{\partial\varphi_{2,i}}{\partial\beta}}\end{aligned}\right\},\qquad(10)$$

where the denominator $\left(\frac{\partial\varphi_{1,i}}{\partial\alpha}\cdot\frac{\partial\varphi_{2,i}}{\partial\beta}-\frac{\partial\varphi_{1,i}}{\partial\beta}\cdot\frac{\partial\varphi_{2,i}}{\partial\beta}\right)$ is the determinant of the Jacobian matrix for the system of non-linear equations $\{\varphi_1, \varphi_2\}$. $\hat{\alpha}_i$ and $\hat{\beta}_i$ are the estimates received of real values of characteristic life α and shape parameter β of Weibull distribution.

Equations (11)-(14) represent the partial derivatives of φ_1 and φ_2 on parameters α and β:

$$\frac{\partial\varphi_1}{\partial\alpha}=\frac{\beta}{\alpha^{1+\beta}}\cdot\left(n_1.B_1\left(\frac{t_{1H}^{\beta}}{B_1-1}\right)^2+\sum_{i=2}^{k}n_i.A_i.\left(\frac{t_{iH}^{\beta}-t_{iL}^{\beta}}{A_i-1}\right)^2\right),\qquad(11)$$

$$\frac{\partial\varphi_1}{\partial\beta}=\frac{n_1.t_{1H}^{\beta}}{B_1-1}\cdot\left[\ln(t_{1H})-\frac{\left(\frac{t_{1H}}{\alpha}\right)^{\beta}.\ln\left(\frac{t_{1H}}{\alpha}\right).B_1}{B_1-1}\right]-\sum_{i=2}^{k+1}n_i.t_{iL}^{\beta}.\ln(t_{iL})+,\qquad(12)$$
$$+\sum_{i=2}^{k}\frac{n_i}{A_i-1}\cdot\left[t_{iH}^{\beta}.\ln(t_{iH})-t_{iL}^{\beta}.\ln(t_{iL})-\frac{\left(t_{iH}^{\beta}-t_{iL}^{\beta}\right).D_i.A_i}{A_i-1}\right]$$

$$\frac{\partial\varphi_2}{\partial\alpha}=-\frac{1}{\alpha.}\left(\begin{aligned}&\frac{n_1.t_{1H}^{\beta}}{B_1-1}\cdot\left(1-\frac{\left(\frac{t_{1H}}{\alpha}\right)^{\beta}.\ln\left(\frac{t_{1H}}{\alpha}\right).\beta.B_1}{B_1-1}\right)+\sum_{i=2}^{k+1}n_i.t_{iL}^{\beta}+\\&+\sum_{i=2}^{k}\frac{n_i}{A_i-1}\cdot\left(t_{iL}^{\beta}-t_{iH}^{\beta}-\frac{C_i.\beta.\left(\left(\frac{t_{iL}}{\alpha}\right)^{\beta}-\left(\frac{t_{iH}}{\alpha}\right)^{\beta}\right)}{A_i-1}.A_i\right)\end{aligned}\right),\qquad(13)$$

$$\frac{\partial\varphi_2}{\partial\beta}=\frac{n_1.t_{1H}^{\beta}}{B_1-1}.\ln\left(\frac{t_{1H}}{\alpha}\right)\left(\ln(t_{1H})-\frac{\left(\frac{t_{1H}}{\alpha}\right)^{\beta}.\ln\left(\frac{t_{1H}}{\alpha}\right).B_1}{B_1-1}\right)-,\qquad(14)$$
$$-\sum_{i=2}^{k+1}n_i.t_{iL}^{\beta}.\ln(t_{iL}).\ln\left(\frac{t_{1L}}{\alpha}\right)+$$
$$+\sum_{i=2}^{k}\frac{n_i}{A_i-1}\cdot\left[\begin{aligned}&t_{iH}^{\beta}.\ln(t_{iH}).\ln\left(\frac{t_{iH}}{\alpha}\right)-\\&-t_{iL}^{\beta}.\ln(t_{iL}).\ln\left(\frac{t_{iL}}{\alpha}\right)-\frac{C_i.D_i.A_i}{A_i-1}\end{aligned}\right]$$

where

$$D_i=\left(\frac{t_{iH}}{\alpha}\right)^{\beta}.\ln\left(\frac{t_{iH}}{\alpha}\right)-\left(\frac{t_{iL}}{\alpha}\right)^{\beta}.\ln\left(\frac{t_{iL}}{\alpha}\right).\qquad(15)$$

Since solving the partial derivatives by non-linear functions often is a difficult task, a solution of (6) by presenting the partial derivatives by the method of secants [7] was presented for the engineering reliability analysis:

$$\frac{\partial\varphi_i}{\partial\alpha}=\frac{\varphi_i(\alpha+\delta.\alpha,\beta)-\varphi_i(\alpha,\beta)}{\delta.\alpha},\ i=\{1,2\},\qquad(16)$$

$$\frac{\partial\varphi_i}{\partial\beta}=\frac{\varphi_i(\alpha,\beta+\delta.\beta)-\varphi_i(\alpha,\beta)}{\delta.\beta},\ i=\{1,2\},\qquad(17)$$

where δ is the convergence speed.

When the interval data is converted to transformed data, the evaluations of the Weibull distribution parameters are obtained by the equations, described by R. Abernethy [6]:

$$\xi=\frac{\sum_{i=1}^{n}t_i^{\hat{\beta}}.\ln(t_i)}{\sum_{i=1}^{n}t_i^{\hat{\beta}}}-\frac{1}{r}.\sum_{i=1}^{r}\ln(t_i)-\frac{1}{\hat{\beta}}=0,\qquad(18)$$

$$\hat{\alpha}=\left[\frac{\sum_{i=1}^{n}t_i^{\hat{\beta}}}{r}\right]^{\frac{1}{\hat{\beta}}}.\qquad(19)$$

According to the method NR, a partial derivative of ξ on β is derived:

$$\frac{\partial\xi}{\partial\beta}=\frac{\sum_{i=1}^{n}t_i^{\hat{\beta}}.\ln(t_i)^2}{\sum_{i=1}^{n}t_i^{\hat{\beta}}}-\frac{\left(\sum_{i=1}^{n}t_i^{\hat{\beta}}.\ln(t_i)\right)^2}{\left(\sum_{i=1}^{n}t_i^{\hat{\beta}}\right)^2}+\frac{1}{\hat{\beta}^2},\qquad(20)$$

and the evaluation of the shape parameter β is obtained, through iteration procedure:

$$\hat{\beta}_{i+1}=\hat{\beta}_i-\frac{\xi_i}{\frac{\partial\xi_i}{\partial\beta}}.\qquad(21)$$

Then by using equation (19) the estimates of characteristic time α is calculated.

In this case the calculation procedure can be simplified by applying of a modified method of secants [8]:

$$\hat{\beta}_{i+1}=\hat{\beta}_i-\frac{\delta.\beta_i.\xi_i(\beta)}{\xi(\beta_i+\delta.\beta_i)-\xi(\beta_i)}.\qquad(22)$$

III. SIMULATION EXPERIMENTS FOR EXAMINATION AND ANALYSIS OF THE APPLICABILITY OF THE STUDYING METHODS WHEN WORKING WITH INTERVAL DATA

For the purpose of Weibull reliability analysis, we developed a set of computational modules in MATLAB, which performs multiple tasks: generation of pseudorandom data with Weibull distribution, resulting in various types of data - complete, right censored, interval data; and failure times which are precisely known or not; preparation of data for graphical presentation; Weibull parameter estimation by several methods; obtaining additional statistical information, etc.

Using the developed computational package, we have explored methods ML and MRR upon presentation of interval data in original form or converted into transformed data and applying a different method of calculation:

1. ML for interval and right censored data and calculation by Newton-Raphson method (NR).
2. ML for interval data and right censored data and calculation by the modified secant method [8].
3. ML for precise failure times and right censored data, and calculation by method NR.
4. ML for precise failure times and right censored data, and calculation by the modified secant method.
5. MRR analysis in terms of X (XonY), where $X = \ln(t)$ and $Y = \ln(-\ln(1 - F(t)))$ are terms of the linear transformation of the Weibull law on X:

$$F(t) = 1 - \exp\left(-(t/\alpha)^{\beta}\right) \rightarrow X = (1/\beta)\cdot.Y + \ln(\alpha). \qquad (23)$$

6. MRR analysis in terms of Y (YonX):

$$F(t) = 1 - \exp\left(-(t/\alpha)^{\beta}\right) \rightarrow Y = \beta.X - \beta.\ln(\alpha). \qquad (24)$$

7. ML for interval and right censored data shifted half interval left, and calculation by the method NR.
8. ML for interval and right censored data shifted half interval left, and calculation with modified secant method.

The purpose of the experiment was to assess the effectiveness of the methods in a different interpretation of the interval data and different calculation methods. This analysis provides the basis for selecting the most appropriate method and mathematical apparatus for each case, or combination of methods for greater confidence.

It is assumed in the simulations that all survived devices remained in the test sample until the end of the tests. To limit the amount of received information, and for generalization of the conclusions, without losing the accuracy of the final results, we assume that all simulated realizations have the same parameters of the law of distribution.

The parameters of the generated pseudorandom data are within the following limits:
$(\{\alpha,\ \beta\})=(\{500,1.2\},\{500,3\},\{1000,2\},\{1000,5\})$; n=2000; m=\{5,15,30,50,100\};T=\{200,500\}; i=\{5,10\},

where n is the number of trials, m is the sample size, T is duration of tests, and i is number of intervals.

In addition to their authentic form of interval data, the generated simulation data are converted and used in the following variations:

-all “failures” are located at the end of the intervals;

-all “failures” are located in the middle of the intervals;

-“failures” are uniformly distributed in the intervals and the resulting times are considered as the precise times of “failures”.

We used data, prepared that way, to obtain the estimates of parameters α and β by investigated methods. Following a statistical analysis, the methods have been evaluated according to the following criteria:

1. Fraction of correct results - the calculated values of parameters that can represent a proper assessment.
2. Fraction of wrong results - results without physical meaning or too far away from the expected values.
3. The average and standard deviation of the parameter values.
4. Fraction of correct results and parameters’ average, depending on the number of registered failures.
5. Fraction of correct results within ± 1%, ± 5%, ± 10%, or ± 20% of the actual value of the parameters individually and as a couple:
6. Size of the confidence intervals.

IV. RESULTS AND INFERENCES

The analysis confirms the conclusions formulated by Abernethy [6] Meeker [9] and others, on the applicability and specificity of the observed methods. We found that some aspects of the study have not been a subject of examination by the scientific community. Table I and Table II represent part of obtained results, and abbreviation “NaN” means results without physical meaning. Here are some important conclusions that we formulated after analyzing the collected information from simulation experiments:

1. We found out that the reliability and accuracy of the estimation of the distribution parameters can be significantly Improved, as the majority of the results are up to 10% deviation. These results are achieved by using the method ML for interval data and shifting by half an interval to the left, and the calculation method NR. It was found, however, that at the same time the size of the confidence intervals of characteristic time α arose of up to 10 %.
2. Using the method ML, the obtained estimations of the shape parameter β, are more accurate compared to the accuracy of the estimates of α, particularly in studies with a small number of recorded failures. In such cases the accuracy of the results could be increased by application of methods and approaches which, by analogy with the Bayesian approach, estimate the parameters of the distributions by combining the data obtained from the tests and the presence of a priori knowledge about the reliability of devices with similar functions, structure and properties. Such method is the Weibayes method [10].
3. Comparison of the results obtained by the method ML, in combination with method NR or methods of secants for calculations, gives the advantage of method NR in respect of accuracy. It is important to note that the differences were not very big, so that in need for alleviated and more convenient to field using mathematical apparatus, the application of the method of secant is acceptable.

Table I. Sample of data obtained about some of the observed implementation of MLE.

MLE interval data / NR						MLE interval data shifted / NR						MLE interval data shifted / method of secants						Number of failures
α	α90% conf. int		β	β90% conf. int.		α	α90% conf. int		β	β90% conf. int.		α	α90% conf. int.		β	β90% conf. int.		
NaN	NaN	NaN	NaN	NaN	NaN	542,54	408,73	720,17	1,33	0,55	3,23	544,98	410,11	724,19	1,33	0,55	3,22	1
498,03	378,86	654,69	1,15	0,51	2,62	NaN	NaN	NaN	0,03	0,02	0,07	NaN	NaN	NaN	-0,69	NaN	NaN	2
527,53	238,61	1166,28	0,75	0,26	2,14	535,62	426,06	673,35	1,49	0,62	3,60	555,75	429,67	718,83	1,39	0,57	3,35	1
527,53	238,61	1166,28	0,75	0,26	2,14	535,62	426,06	673,35	1,49	0,62	3,60	555,75	429,67	718,83	1,39	0,57	3,35	1
955,70	186,64	4893,71	0,67	0,20	2,29	NaN	NaN	NaN	1,90	1,38	2,61	603,19	151,34	2404,17	1,07	0,34	3,42	1
575,16	357,50	925,35	0,99	0,39	2,56	626,28	361,85	1083,95	0,86	0,33	2,20	620,33	396,96	969,38	0,99	0,40	2,45	1
314,22	221,42	445,92	1,16	0,47	2,85	485,52	370,71	635,90	1,12	0,46	2,68	606,83	469,14	784,92	1,06	0,45	2,53	4
955,70	186,64	4893,71	0,67	0,20	2,29	NaN	NaN	NaN	1,90	1,38	2,61	603,19	151,34	2404,17	1,07	0,34	3,42	1
575,16	357,50	925,35	0,99	0,39	2,56	626,28	361,85	1083,95	0,86	0,33	2,20	620,33	396,96	969,38	0,99	0,40	2,45	1
0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0
492,82	391,86	619,79	1,13	0,55	2,33	501,77	401,15	627,62	1,13	0,55	2,30	623,30	502,76	772,74	1,06	0,53	2,16	3
575,16	357,50	925,35	0,99	0,39	2,56	626,28	361,85	1083,95	0,86	0,33	2,20	620,33	396,96	969,38	0,99	0,40	2,45	1
584,71	372,17	918,62	1,04	0,41	2,66	606,04	386,71	949,79	1,01	0,40	2,52	608,50	393,42	941,14	1,03	0,41	2,55	1
492,82	391,86	619,79	1,13	0,55	2,33	501,77	401,15	627,62	1,13	0,55	2,30	623,30	502,76	772,74	1,06	0,53	2,16	3
10645,82	NaN	NaN	-8,35	NaN	NaN	354,47	252,51	497,59	1,27	0,54	2,99	643,23	487,71	848,33	1,04	0,48	2,29	2
332,07	232,93	473,41	1,31	0,53	3,21	445,79	333,11	596,60	1,19	0,53	2,68	648,72	493,66	852,49	1,04	0,48	2,28	2
0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0
0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0
519,40	400,66	673,34	1,13	0,50	2,55	524,18	407,02	675,07	1,12	0,49	2,57	338,00	255,99	446,28	1,23	0,51	2,98	2

Table II. Results obtained for method ML and interval data, with method NR for calculation, and simulation parameters: *n*=2000; *α*=500; *β*=1,2; *T*=200; *i*=5.

(a) Statistical estimates and distribution of received values by number of failures.

	Sample size	Correct results	Results out of range	Wrong results	Average	Standard deviation	Average	Standard deviation	Distribution of wrong results depending on numbers of failures in each test						Distribution of correct results depending on numbers of failures in each test					
	m				α	σα	β	σβ	0	1	2-3	4-7	8-20	21-40	0	1	2-3	4-7	8-20	21-40
Absolute value	5	1617	363	20	566,87	137,69	1,06	0,19	0	0	20	0	0	0	0	750	830	37	0	0
	15	1921	18	61	541,74	113,15	1,12	0,25	0	0	10	51	0	0	0	94	587	1166	4	0
	30	1935	0	65	551,09	190,10	1,19	0,49	0	0	0	27	5	0	0	2	29	617	705	0
	50	1944	0	56	533,86	93,90	1,20	0,32	0	0	0	4	40	0	0	0	0	33	1745	58
	100	1966	0	34	531,35	112,83	1,23	0,33	0	0	0	0	4	30	0	0	0	0	77	1889
Representation as a fraction	5	0,99	0,18	0,01	566,87	137,69	1,06	0,19	NaN	0,00	0,02	0,00	NaN	NaN	NaN	1,00	0,98	1,00	NaN	NaN
	15	0,97	0,01	0,03	541,74	113,15	1,12	0,25	NaN	0,00	0,02	0,04	0,00	NaN	NaN	1,00	0,98	0,96	1,00	NaN
	30	0,97	0,00	0,03	551,09	190,10	1,19	0,49	NaN	0,00	0,00	0,04	0,01	NaN	NaN	1,00	1,00	0,96	0,99	NaN
	50	0,97	0,00	0,03	533,86	93,90	1,20	0,32	NaN	NaN	NaN	0,11	0,02	0,00	NaN	NaN	NaN	0,89	0,98	1,00
	100	0,98	0,00	0,02	531,35	112,83	1,23	0,33	NaN	NaN	NaN	NaN	0,05	0,02	NaN	NaN	NaN	NaN	0,95	0,98

(b) Distribution of correct results by their difference from original value.

	Sample size	Number of correct results for α in a range of:				Number of correct results for α in a range of:				Number of correct results for {α,β} in a range of:			
	m	1%	5%	10%	20%	1%	5%	10%	20%	1%	5%	10%	20%
Absolute value	5	220	546	835	1237	91	383	821	1330	0	210	671	1073
	15	173	721	1018	1527	86	688	1142	1670	0	393	800	1402
	30	211	818	1149	1457	137	935	1361	1670	29	592	974	1345
	50	223	855	1201	1574	186	1126	1502	1706	23	674	1067	1436
	100	254	969	1421	1740	269	1413	1674	1794	48	831	1306	1654
Representation as a fraction	5	0,14	0,34	0,52	0,76	0,06	0,24	0,51	0,82	0,00	0,13	0,41	0,66
	15	0,09	0,38	0,53	0,79	0,04	0,36	0,59	0,87	0,00	0,20	0,42	0,73
	30	0,11	0,42	0,59	0,75	0,07	0,48	0,70	0,86	0,01	0,31	0,50	0,70
	50	0,11	0,44	0,62	0,81	0,10	0,58	0,77	0,88	0,01	0,35	0,55	0,74
	100	0,13	0,49	0,72	0,89	0,14	0,72	0,85	0,91	0,02	0,42	0,66	0,84

(c) Average values of estimated parameters.

		Average value of correct results due to number of failures					
	m	0	1	2-3	4-7	8-20	21-40
α	5	NaN	642,27	501,71	500,11	NaN	NaN
β		NaN	0,97	1,15	1,12	NaN	NaN
α	15	NaN	584,48	578,08	522,87	498,15	NaN
β		NaN	1,07	1,03	1,16	1,12	NaN
α	30	NaN	533,66	480,17	633,28	503,03	NaN
β		NaN	1,07	1,04	1,22	1,16	NaN
α	50	NaN	NaN	NaN	597,56	531,57	500,58
β		NaN	NaN	NaN	1,06	1,21	1,14
α	100	NaN	NaN	NaN	NaN	698,87	524,52
β		NaN	NaN	NaN	NaN	1,24	1,23

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