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A spectral model for the turbulent kinetic energy in a stratified shear flow

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***Abstract:** In this paper we attempt to derive an exact solution for the equation governing the kinetic energy spectrum, forwarded by Vinnichenko et al.[1] in a stably stratified shear flow. A simple empirical modelis developed for the turbulent kinetic energy spectrum E(k) of weakly-stratified turbulent shear flow.First a scheme for the existing model based on the Pao's (1965) turbulence energy cascade process hasbeen discussed thoroughly. Then we put forward the physical hypothesis for the spectral energy transfer hypothesis due to Obukhov in its modified form. The reduced spectral equation is solved in its exact form.The heat flux spectrum and the production term due to mean velocity shear are modelled approximatelyin each of the cases. In the next section we treat the spectral case in which the model is reduced to itssimple form, as obtained for the non-stratified turbulence phenomena. Spectral characteristics of suchturbulent energy spectrum are investigated in the wave number ranges $k_b \ll k \ll k_d$ and $k \gg k_d$. Effectof anisotropy due to shear and stratification is also demonstrated through simple computations. In thelast section we discussed the results that lead to some conclusive remarks.*

Nomenclature

k : Wave number
E(k) : Turbulent energy spectrum
ε : Rate of total energy dissipation
ν : Coefficient kinematic viscosity
W(k) : Represent the transfer of energy through the hierarchy of eddies
τ (k) : Spectrum of turbulent friction stress
H(k) : Spectrum of vertical heat flux
$\beta(=g/\bar{T})$: Buoyancy parameter
g : Acceleration due to gravity
$\bar{u}$: Mean velocity
u, w: Longitudinal and vertical velocity fluctuationcomponents
$\bar{T}$: Mean temperature
$\frac{d\bar{u}}{dz}$: Mean velocity gradient
$\frac{d\bar{T}}{dz}$: Mean temperature gradient
α : Ratio between the coefficients of turbulent mixing forheat and momentum
P: Kolmogoroff constant
L0: A length comparable with the scale of flow as awhole
$b\left[=\left(\frac{d\bar{u}}{dz}\right)^2-\alpha\beta\frac{d\bar{T}}{dz}\right]$: A dimensional parameter
$k_b\left(=b^{3/4}\varepsilon^{-1/2}\right)$: A defined wave number
$k_d\left[=\left(\varepsilon/\nu^3\right)^{1/4}\right]$: Kolmogoroff wave number

I. INTRODUCTION

Atmospheric motions are in general turbulent Kellogg's [2] observations of smoke puffs also suggests that the atmosphere is at least weakly turbulent everywhere. The turbulent motions in the atmosphere exist in many different scales. The motions in the large eddies may be approximated quasi-horizontal [3], whereas the vertical velocities are appreciable in the smaller eddies.

In the case of homogeneous turbulent shear,Tchen (cf. Hinze [4]) has proposed a different mechanism for the production of energy by the mean shear. According to Tchen, two cases may arise. In the first case, the vorticity of the basic stream is small compared with the vorticity of the turbulent motion. In the second case the vorticity of the basic stream is comparable with that of the turbulent motion. The intersections between the two vorticities are termed weak and strong respectively, in the first and second case.Gisina [5] has extended the above idea of Tchen to the case of thermally stratified turbulent shear flow. Panchev and Syrakov[6, 7] have investigated the spectral characteristics of thermally stratified turbulent flow respectively, with no shear and shear, based on the physically idea about the natural realisation of the degree of interaction between mean and turbulent fields.

In the present paper, we examine the spectral characteristics of the turbulent energy for the small scale motions in an inertial sub range, defined by $k_b \ll k \ll k_d$ and $k \gg k_d$,taking into account the effect of anisotropy due to shear and stratification. In modelling, the spectral transfer of energy, spectral flux of turbulent friction stress and spectral heat flux we take some extended view of the semi-empirical concept about the mechanism of turbulence and depend as well on dimensional reasoning's. We confine ourselves to the case of weak interaction between mean and turbulent fields, and postulate in turn that the spectral characteristics of turbulent energy depend on k, ε, ν and a parameter b, defined by

$$b\left[=\left(\frac{d\bar{u}}{dz}\right)^2-\alpha\beta\frac{d\bar{T}}{dz}\right].$$

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II. OBUKHOVHYPOTHESIS AND ITS MODIFICATION

We integrate first the spectral equation relations of the energy transfer spectrum W(k, t)and the turbulent kinetic energy spectrum E(k, t) between the limits of k, e.g., from 0 tok, we obtain (Panchev, 1971).

$$\frac{\partial}{\partial t}\int_0^k E(k',t)dk' = \int_0^k W(k',t)dk' - 2\nu\int_0^k E(k',t)dk' \quad (1)$$

Where $k \geq k_0 = L_0^{-1}$. That is in effect, we consider the universal equilibrium range land for $k \geq k_0$. E(k,t) is the equilibrium spectrum. According to Vinnichenko,

$$\int_0^\infty E(k',t)dk' \leq \int_0^k E(k',t)dk' \quad (2)$$

and $\frac{\partial}{\partial t}\int_0^k E(k',t)dk' \approx \frac{\partial}{\partial t}\int_0^\infty E(k',t)dk' =-$ ε =constant

(3)

Thus when $k \geq k_0$, we may write

$$\varepsilon = W(k,t) + 2\nu\int_0^k E(k',t)dk' \quad (4)$$

We now need a relation between W(k, t) and E(k, t) based on some intuitive physicalpicture of the energy transfer mechanism across the wave numbers. It was Obukhov[8]who first proposed an assumption which is read as

$$\int_0^k W(k',t)dk' = -\alpha_0[2\int_0^k k'^2 E(k',t)dk']^{\frac{1}{2}}\int_k^\infty E(K'',t)dk'' \quad (5)$$

Where α_0 is an universal constant.Obukhov obtained a reduced form forE(k, t) as $E(k,t) \sim k^{-5/3}$ but at large values of k, the solution decreases at slower rate than the continuation of $k^{-5/3}$ curve until at a certain cut off value of k, then E(k, t) is still finite. Since some of the consequences of Obukhov's theory are physically unlikely, Ellison (1962)[9] introduced a modification of this theory by assuming that the Reynolds stresses of the micro-component were largely determined by the extreme large-scale disturbances in this component (with wave numbers approaching k) such that

$$F(k,t) = -2\alpha_0\left[\int_0^k k'^2 E(k',t)dk'\right]^{1/2} . \int_k^\infty E(k',t)dk' \quad (6)$$

In other words, it was assumed that the mean Reynolds stresses of the macro-componentcan be approximately expressed in terms of the values of E(k) and k alone.In that case, dimensional analysis shows that they should be proportional to kE(k),and hence

$$W(k,t) = \alpha_E\left\{\int_0^k k'^2 E(k',t)dk'\right\}^{1/2} .kE(k) \quad (7)$$

According to this form, the Reynolds stress interacting with the mean rate of shear or Vorticity of the wave numbers smaller than k is entirely restricted to k. From the physical point of view, the assumption that W(k) is independent of the values of E(k0) with $k' > k$ is not very satisfactory. We rewrite the above form as:

$$W(k) = \gamma_E E(k)k\left[2\int_0^k k'^2 E(k',t)dk'\right]^{1/2}, (\alpha_E = \sqrt{2}\gamma_E) \quad (8)$$

which is proposed by Ellison after historically first approximation for the energy transfer function

$$W(k) = \gamma_0\int_k^\infty E(k')dk' . \left[2\int_0^k k'^2 E(k',t)dk'\right]^{1/2} \quad (9)$$

One may choose $\gamma_E = \sqrt{\frac{2}{3}}P^{-3/2}$ in order to obtain Kolmogorov's spectrum

$$E(k) = P\varepsilon^{2/3}k^{-5/3} \quad (10)$$

as a solution of the spectral equation w(k)= ε in the inertial range. Here P is the dimensionless constant. Moreover, in the inertial subrange, where (10)is valid, we have [6] :

$$\sigma_w(k) = \left[2\int_0^k k'^2 E(k')dk'\right]^{1/2} = \sqrt{\frac{3\alpha}{2}}(\varepsilon k^2)^{1/3} \quad (11)$$

where $\sigma_w(k)$ is the mean square of the vorticity. A substitution of (11) into (8) leads to the approximation for F(k) suggested by Pao [10,11]:

$$W(k) = P^{-1}\varepsilon^{1/3}k^{5/3}E(k) \quad (12)$$

III. MODELLING OF TRANFER,PRODUCTION AND BUOYANCY TERMS

We concentrate our attention on the smallscale motion on the atmosphere, which areapproximately in local energetic equilibrium.For the equilibrium region

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$k \gg L_0^{-1}$,the equation for the energy balance is reducible(cf.Vinnichenco et al. [1]) to

$$\varepsilon = 2\nu\int_0^k k'^2\, E(k')dk' - \frac{d\bar{u}}{dz}\int_k^\infty \tau(k')dk' + w(k) + \beta\int_k^\infty H(k')dk' \quad (13)$$

The first term on the right hand side of eqn.(13) describes the dissipation of turbulent energy. The second term describes the production of turbulent energy due to mean velocity shear. The third term represents the inertial transfer of turbulent kinetic energy, while the last term describes the contribution of the buoyancy force towards turbulent energy. For W(k),we take the Modified Obukhov form proposed by Ellison [9], which is based on Onsger's [12] spectral jump concept,

as $W(k) = \gamma_E E(k)k\left[2\int_0^k k'^2\, E(k')dk'\right]^{1/2}$.

Expressionfor W(k) as given by (12) may be interpreted as:

$$W(k) = \nu_T.(Vorticity)^2 \quad (14)$$

Since the vorticity is approximated by $\varepsilon^{1/3}k^{2/3}$ [13], the coefficient of eddy viscosity (or turbulent kinematic viscosity) ν_T takes the form:

$$\nu_T = P^{-1}\varepsilon^{-1/3}k^{1/3}E(k) \quad (15)$$

As we are considering of week interaction between the main and turbulent motions,the second term on the right-hand side of (13) can be modelled for sufficiently largewave number as

$$\frac{d\bar{u}}{dz}\int_k^\infty \tau(k')dk' = -\nu_T\left(\frac{d\bar{u}}{dz}\right)^2 \quad (16)$$

where ν_T has the expression given by (15). For the same reason we may model heat flux or buoyancy term as

$$\beta\int_k^\infty H(k')dk' = -\nu_T{*}\left(\beta\frac{d\bar{T}}{dz}\right) \quad (17)$$

Where $\nu_T{*} = \alpha\ \nu_T$ (cf. Vinnicheno et. al.[1]and Lumley and Panofsky [14])and $\beta\ \frac{d\bar{T}}{dz}$ has the dimension of the square of the vorticity. Substituting (12),(16) and (17) in equation (13), we obtain

$$\varepsilon = 2\nu\int_0^k k'^2\, E(k')dk' + P^{-1}\varepsilon^{1/3}k^{5/3}E(k) + bP^{-1}\varepsilon^{-1/3}k^{1/3}E(k) \quad (18)$$

Where $b\left[=\left(\frac{d\bar{u}}{dz}\right)^2 - \alpha\beta\frac{d\bar{T}}{dz}\right]$.It is to be noticed that 'b' can be written as

$$b = \alpha N^2\left(\frac{1}{R_f} - 1\right),$$

WhereN is the Brunt Väisälä frequency and Rfis the flux Richardson number.

IV. EXACT SOLUTION FOR THE TURBULENT ENERGY SPECTRUM

We may rewrite equation (18) in the form

$$\varepsilon = 2\nu\int_0^k k'^2\, E(k')dk' + P^{-1}\varepsilon^{1/3}[k^{5/3}E(k)] + bP^{-1}\varepsilon^{-1/3}k^{-4/3}[k^{5/3}\ E(k)] \quad (19)$$

and find its exact solution. Differentiating (19) with respect to k and simplifying weobtain

$$\frac{d[k^{5/3}E(k)]}{k^{5/3}E(k)} = \frac{\frac{4}{3}b\varepsilon^{-1/3}k^{-7/3} - 2\nu Pk^{1/3}}{\varepsilon^{1/3} + b\varepsilon^{-1/3}k^{-4/3}} \quad (20)$$

.Integration of equation (20) yields

$$k^{5/3}E(k) = Q.\exp\int(\frac{4}{3}b\varepsilon^{-1/3}k^{-7/3} - 2\nu Pk^{1/3}).(\varepsilon^{1/3} + b\varepsilon^{-1/3}k^{-4/3})^{-1}dk \quad (21)$$

WhereQis the constant of integration. We now introduce a wave number kb, given by

$$k_b = b^{3/4}\varepsilon^{-1/2} \quad (22)$$

In view of (22), equation (21) can be written as

$$k^{5/3}E(k) = Q.\exp\int(\frac{4}{3}b\varepsilon^{-1/3}k^{-7/3} - 2\nu Pk^{1/3}).\varepsilon^{-1/3}[1+\left(\frac{k_b}{k}\right)^{4/3}]^{-1}dk$$

$$= Q\left[e^{\frac{-3}{2}P\nu\varepsilon^{-1/3}k^{4/3}}.k^{4/3}bk_b^{-4/3}\varepsilon^{-2/3}(k^{4/3} + k_b^{4/3})^{(\frac{3}{2}k_b^{4/3}P\nu\varepsilon^{-1/3} - bk_b^{-4/3}\varepsilon^{-2/3})}\right] \quad (23)$$

Above equation gets simplified to

$$E(k) = Qk^{-1/3}e^{\frac{-3}{2}P\nu\varepsilon^{-1/3}k^{4/3}}\left(k^{4/3} + b\varepsilon^{-2/3}\right)^{\frac{3P\gamma b - 2\varepsilon}{2\varepsilon}}$$

(24)

with the help of wave number kb, as defined by the relation (22). To determine the constant Q we choose Kolmogoroffwave number k_d as such

$$k_b \le k_d = \varepsilon^{1/4}\nu^{-3/4} \qquad (25)$$

which implies the condition

$$\left(\frac{b\nu}{\varepsilon}\right)^{3/4} \le 1 \qquad (26)$$

Following Peskin and Baw [15], constant Q is now determined as

$$Q = E(k_d)k_d^{1/3}e^{\frac{3}{2}P\nu\varepsilon^{-1/3}k_d^{4/3}}\left(k_d^{4/3}+b\varepsilon^{-2/3}\right)^{1-\frac{3Pb\nu}{2\varepsilon}} \quad (27)$$

Substituting the value of Q in (24) we get after some simplification,

$$\frac{E(k)}{E(k_d)} = \left(\frac{k}{k_d}\right)^{-1/3} e^{\frac{3P[1-\left(\frac{k}{k_d}\right)^{4/3}]}{2}} \left[\frac{\frac{b\nu}{\varepsilon}+\left(\frac{k}{k_d}\right)^{4/3}}{1+\frac{b\nu}{\varepsilon}}\right]^{\frac{3Pb\nu}{2\varepsilon}-1} \quad (28)$$

Scaling the wave number k and energy spectrum E(k) as $\tilde{k} = \frac{k}{k_d}$ and $\tilde{E} = \frac{E(k)}{E(k_d)}$, equation (28) can be written in the form $\tilde{E} = \tilde{k}^{-1/3}e^{\frac{3P}{2}(1-\tilde{k}^{4/3})}\left[\frac{\frac{b\nu}{\varepsilon}+\tilde{k}^{4/3}}{1+\frac{b\nu}{\varepsilon}}\right]^{\frac{3Pb\nu}{2\varepsilon}-1}$. When b = 0 we have $\tilde{E} = \tilde{k}^{-5/3}e^{\frac{3P}{2}(1-\tilde{k}^{4/3})}$,the model is reduced to its simple form, as obtained for the non-stratified turbulencephenomena[1].

4.1 **<u>Case:</u>** $k_b \le k$

Now under the assumption $k_b \le k$, eqn. (23) reduces to

$$k^{5/3}E(k) = Q.\exp\int\left(\frac{4}{3}b\varepsilon^{-1/3}k^{-7/3} - 2\nu Pk^{1/3}\right)\varepsilon^{-1/3}\left[1-\left(\frac{k_b}{k}\right)^{4/3}\right]^{-1}dk \quad (30)$$

[Neglecting the higher powers than $\left(\frac{k_b}{k}\right)^{4/3}$ in the expansion]. Putting the value of k_b in above equation it can be readily obtained

$$E(k) = Qk^{\frac{-5}{3}+2P\frac{b\nu}{\varepsilon}}\exp(-b\varepsilon^{-2/3}k^{-4/3})\exp(\frac{-3}{2}P\nu\varepsilon^{-1/3}k^{4/3})\exp(\frac{1}{2}b^2\varepsilon^{-4/3}k^{-8/3}) \quad (31)$$

In a similar fashion, following Peskin and Baw's prescription, constant Q can bedeterminedas

$$Q = E(k_b)k_b^{\frac{5}{3}-2P\frac{b\nu}{\varepsilon}}\exp(b\varepsilon^{-2/3}k_d^{-4/3})\exp(\frac{3}{2}P\nu\varepsilon^{-1/3}k_d^{4/3})\exp(-\frac{1}{2}b^2\varepsilon^{-4/3}k_d^{-8/3}) \quad (32)$$

Now using the above form of Q and scaling the wave number and energy spectrumfunction, eqn.(30) can be reformulated as

$$\tilde{E} = \tilde{k}^{\frac{-5}{3}+2P\frac{b\nu}{\varepsilon}}\exp[\frac{b\nu}{\varepsilon}(1-\tilde{k}^{-4/3})]\exp[\frac{3}{2}P(1-\tilde{k}^{4/3})]\exp[\frac{1}{2}\left(\frac{b\nu}{\varepsilon}\right)^2(\tilde{k}^{-8/3}-1)] \quad (33)$$

V. DISCUSSION OF THE RESULTS AND CONCLUDING REMARKS

In this paper we have expressed the energy transfer spectrum (section 3)as a product of eddy viscosity coefficient times the (Vorticity)^2. Monin and Yaglom (1975) showed in their analysis that the mean Reynolds stresses of the micro component may approximately be proportional to eddy viscosity coefficient kE(k).This expression according to them was dueto Ellison (1962) and the mean square of the vorticity may be calculated from the results

$$\left[2\int_0^k k'^2 E(k',t)dk'\right]^{1/2} = \frac{\sqrt{3\alpha}}{2}(\varepsilon k^2)^{1/3} \quad [13, 16]$$

which leads to the Pao's formula for the energy transfer spectrum. The solution for the turbulence energy spectrumin the wave number range $k \ge k_b$ where in the effect of stratification is no more and we obtained Heisenberg k^{-7} power law of the isotropic case. The energy spectrum function as described by (29) has been plotted in (Fig.1). and it is clear that from this graph that it represents the distribution of decay of energy spectra over the range $k_b \le k \le k_d$ and $k \ge k_d$. The energy spectrum function as predicted by equation (33) is plotted for a value of the non-dimensional parameter, satisfying the condition (26). For comparison with the case of homogeneous and isotropic turbulence, we also plot the energy spectrum from equation for b = 0. The value of the constant P is chosen to be 1.70. It is observable (Fig.2) that for $\tilde{k}$ <1, which implies the inertial subrange kb < k <kd as defined with condition (26) satisfied, the spectral density of turbulent energy of weakly-stratified shear flow is less than that of isotropic turbulent flow. It may be noticed that the effect of the non-dimensional parameter $\frac{b\nu}{\varepsilon}$ on the shape of the spectrum is significant in kb < k

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<kdand this corresponds to the case of stably stratified atmosphere. That is some of the turbulent energy besides usual energy transfer is converted into potential energy.

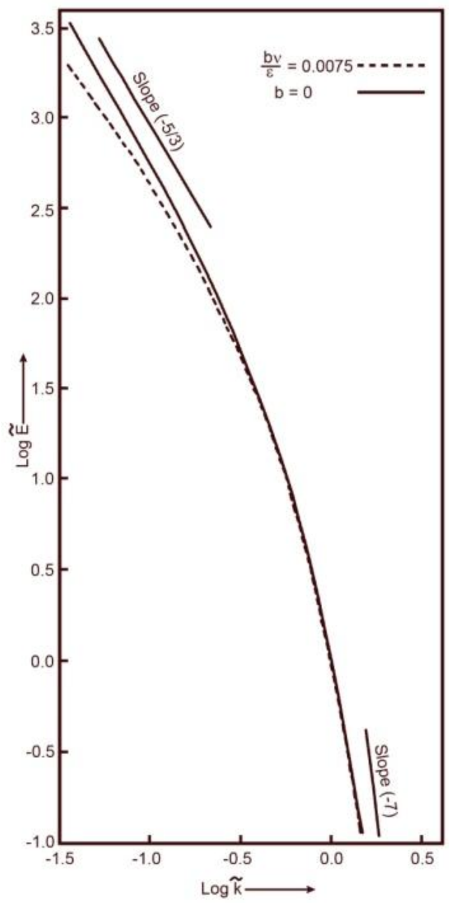

Fig 1: Plot of log $\tilde{E}$ as a function of log$\tilde{k}$ in the wave number range

The two curves ($\frac{bv}{\varepsilon}$ =.0075 and b=0) intersect at $\tilde{k}$ =1 and cross over for $\tilde{k}$ >1. In the dissipation range $\tilde{k}$ >1, the present model suggeststhat the spectral density of turbulent energy of weakly-stratified shear flow althoughslightly exceeds that of the isotropic turbulent flow but seems to be significant (Fig.2). More precisely,we recover the solution for the turbulence energy spectrumin the wave number range $k \ll k_b$,where in the effect of stratification is no more and we obtained Heisenberg k^{-7} power law of the isotropic case [4].We showed that byplotting $\log \tilde{E}$ vs $\log \tilde{k}$ for the range of wave number $\tilde{k} >> \frac{k_b}{k_d}$ that at low wave number spectral density does not exceed 5/3, whereas at intermediate wave numbers, $\tilde{E}$ exhibits approximately a $\tilde{k}^{-5/3}$ falls off. At high wave numbers, $\tilde{E}$ exhibits a $\tilde{k}^{-7}$ behaviour and at higher wave numbers it decays more faster. It may be concluded onthe above analysis that the physical energy transfer hypothesis of Obukhov as modified by Ellison is fully capable of explaining the energy transfer process in homogeneous turbulence. It is important to note that Chakra borty and Mazumder (1986)[18] solved the stably stratified turbulent flow by

$$b\left[=\left(\frac{d\bar{u}}{dz}\right)^2-\alpha\beta\frac{d\bar{T}}{dz}\right]$$

as a small non zero parameter. He determined kinetic energy spectra for the case $\frac{k}{k_d}$ <<1 and $\frac{k}{k_d}$ >>1, where kdis the Kolmogoroffwave number. We have discussed that energy is fed in this type of turbulence into energy containing range of the spectrum by mean velocity shear and extracted by the buoyancy forces of stably stratification. Gargett et al.(1984) argued that for Small scale of turbulence local isotropy may be achieved despite the presence of macroscopic properties of the mean flow, including its mean stability N. The influence of the buoyancy forces is not necessarily restricted to wave numbers of order L^{-1} ,whereL isa length comparable with the scale of flow as a whole. Gargett et al. (1984) [19] also studied the one-dimensional spectrum of the stably stratified turbulent flow in relation to the value of $I=\frac{k_d}{k_b}$ successfully where $k_d=\varepsilon^{1/4}\nu^{-3/4}$ and $k_b=N^{3/2}\varepsilon^{-1/2}$ two respective wave numbers depicting the stably stratified turbulent flows. The results with I = 630 case was demonstrated well. To sum up, the present analysis clearly suggests that the energy transfer function due to modified Obukhov form may be employed successfully for prediction of the energy density spectrum in the case of stably stratified turbulent flow.

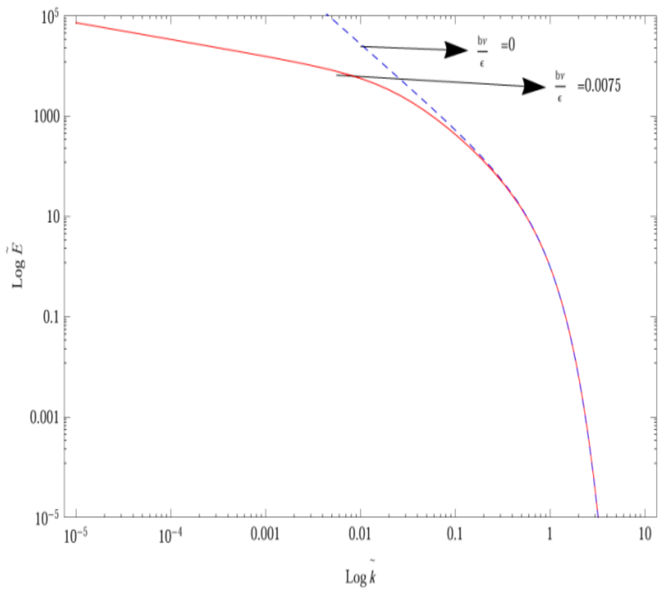

Fig 2: Non Dimensional energy spectra

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