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Parity Assisted Decoding for QAM in AWGN Channels

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Abstract* - In this paper, we propose and analyze a new Parity Assisted Decoding (PAD) technique to improve performance when hard decision decoding is used. When a symbol is received, a decision is made not solely by its Euclidean distances to the constellation points. Rather, the two parity bits are used to assist in making the decision. Unlike standard Error Correcting Codes (ECC), the proposed method operates on the received symbols at the detector level and before ECC. The method is demonstrated for a simple scheme involving the transmission of Quadrature Amplitude Modulation (QAM) symbol and two parity bits in separate channels. The parity bits and the information symbols can be sent in different channels (frequency division) or at different times on the same channel (time division). The parity bits are introduced to improve the "decision making" at the receiver and thus reduce the Bit Error Rate (BER). A technique for fine tuning the performance by changing the relative energies of the information bearing symbols and the parity bits is described. Using the parity bits for decision making compensates for the loss of soft decision decoders when high computational power is not feasible. A variation of the method in which the parity bits themselves are coded is shown. Simulation results show large gains in required Signal-to-Noise-Ratio (SNR) over uncoded system to achieve the same performance.

Index Terms - **Parity Assisted Decoding (PAD), Quadrature Amplitude Modulation (QAM), SNR, Error Correcting Codes (ECC)**.

I. INTRODUCTION

Hard decision decoding, where the output of the detector is quantized into one of the *M* possible values associated with the constellation diagram, is simple to implement and requires less hardware and/or computational resources as opposed to soft decision decoding. However, this simplicity comes at a price in terms of performance. Soft decision decoding, where the output of the detector is not quantized or quantized to a number of levels much more than *M* achieves a better Bit Error Rate (BER) for the same energy per bit as the hard decision one. It is not unusual to save as much as 3 dB per bit when soft decision decoding is used to achieve the same BER performance as when hard decision decoding is used. In traditional QAM, each *k* information bit is associated with a point in an *M*-point constellation diagram where $M = 2^k$. A waveform corresponding to the coordinates of the constellation point is transmitted to convey the information. In hard-decision decoding, the output of the demodulator is quantized into a small number, usually *M*, of discrete levels. This kind of decision-making highly simplifies the receiver structure. This simplification comes at a cost in terms of performance. In AWGN channels and when the symbols occur with equal probabilities, the receiver makes a decision as to which symbol was transmitted according to the Euclidean distances of the received symbol to all constellation points. Soft-decision decoding, on the other hand, does not quantize the output of the demodulator. The demodulator output is sent as is to the decoder for further processing. It is also possible to quantize the output of the demodulator to a large number of levels, much larger than *M*, and still achieves performance close to that of pure un-quantized output. In BPSK where $M = 2$, for example, the performance when 3-bit quantization, i.e., 8 levels, is used is similar to that of no quantization. When hard decision is used instead of soft decision, there is a loss of about 2-3 dB per bit to achieve the same level of performance in terms of BER. To compensate for the loss associated with hard decision, we have developed a new PAD scheme which involves sending two parity bits in addition to the QAM symbol. The parity bits are sent using QPSK in a different channel. The parity bits can also be sent at alternating times with the information symbols on the same channel. The two parity bits are simply logical functions of the bits of the word corresponding to the symbol to be transmitted. The proposed technique is particularly attractive for systems where high computational power is not available. Standard Error Correcting Codes (ECC) correct errors after the received symbols have been detected. The received word is compared, through certain algorithms, to all possible code words. The codeword closest to the received one, in Hamming distances for hard decisions and Euclidean distances for soft decisions, is chosen as the most probable transmitted codeword. The PAD proposed in this paper, on the other hand, operates on the received symbols during detection and before they are passed to ECC decoders. It uses the two parity bits for decision making at the matched filter or correlator bank level before they are further processed by the ECC. It should be emphasized that the parity bits are used in decision making at the receiver of a communication system rather than using them for error correction, though, ultimately, it leads to reducing the error rate. The proposed method can be used on its own in many communication applications. We also report preliminary results whereby the proposed PAD is combined with Turbo codes to assist in mapping binary bits into QAM symbols. This coding technique, along with semi-soft technique described in [1] is used to map Turbo codes into high dimension modulation. The parity bits are chosen to be as different as possible for the neighboring constellation points and the symbols they represent. Unlike

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standard ECC, the proposed method improves the system performance by assisting the receiver in decoding the received signals. Simulation results show gains of the system compared with uncoded systems with little computational complexity.

II. THE PAD ENCODER

The operation of the PAD can be summarized as follows. At the transmitter, two parity bits are generated for each symbol to be transmitted. Only two parity bits are generated regardless of the number of points in the constellation. This enables the transmission of the parity bits using QPSK which inherently has a low BER performance for a fixed SNR compared to other modulation techniques. It is also possible to generate three parity bits instead of two and sending them using a different modulation technique, like PSK-8 for example. However, PSK-8 has a high BER compared to that of QPSK of the same SNR. Moreover, PSK-8 is slightly more complex than QPSK. The two parity bits are logic functions of the bits representing the symbol to be transmitted. If the bits of information symbol are b_0, b_1, b_{k-1}, then the two parity bits are represented as

$$p_0 = f_0\left(b_0,\, b_1,\, \cdots,\, b_{k-1}\right),$$
$$p_1 = f_1\left(b_0,\, b_1,\, \cdots,\, b_{k-1}\right), \qquad (1)$$

where f_0 and f_1 are logic functions. The two functions, f_0 and f_1 can be implemented either as combinational logic functions using, for example, sum of products or product of sums structures. They can also be implemented using look up tables (LUT). Since a symbol error is most likely to occur when a symbol is received (due to additive noise) at a neighboring constellation point decision region, the functions are chosen to be as different as possible for neighboring points. Fig. 1 shows a simulation run of 10000 transmissions of a fixed symbol [the one at coordinates (1, -3)] and the number of occurrences of the symbol being received at neighboring symbols' decision regions for some fixed SNR. A count of a similar simulation run is shown in Fig. 2 for a relatively moderate SNR. The mathematical expression for the expected number of points to fall in any region is simply the area (volume) under a two-dimensional Gaussian distribution function centered at the coordinates of the central point and with a covariance matrix diag$\{N_0/2,\, N_0/2\}$ multiplied by the number of simulation runs, where N_0 is the one-sided noise power spectral density.

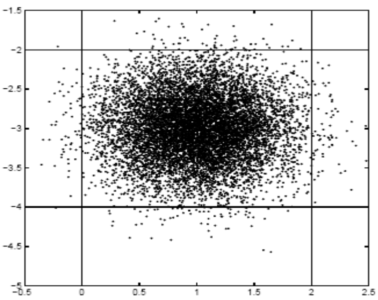

Fig 1: 10,000 received symbols after transmission of the symbol at (1, -3) in a QAM-64.

2	110	2
131	9512	138
0	103	2

Fig 2: Simulation run of the number of symbols that are received in neighboring regions when 10000 symbols of the center point are transmitted.

The first function, f_0, can be chosen to be checkered like in a chessboard among the symbols on the constellation diagram. Fig. 3 shows a typical function truth table for QAM-16.

1	0	1	0
0	1	0	1
1	0	1	0
0	1	0	1

Fig 3: The first parity bit function f_0 truth table for a QAM-16. The function is simply checkered bits like in a chess-board.

This can be extended to constellations with a larger number of points. This arrangement ensures that most errors, which as shown in Fig. 1 occur at the four neighboring regions, can be detected and corrected. Errors occurring at diagonal neighbors or those falling in the four closest neighbors but further than half the decision region's width will not be corrected by f_0. The function f_1 augments the first function to overcome the above shortcomings. The second function, f_1, can be chosen as shown in Fig. 4. This function ensures that diagonal neighboring regions have different parity. However, the function does not solve the problem when a received symbol arrives at a close neighbor but further than half the region's width. It should be noted that such errors occur very infrequently except for very low SNR's.

0	1	0	1
0	1	0	1
0	1	0	1
0	1	0	1

Fig 4: The second parity bit function f_1 truth table for a QAM-16. This and in conjunction with f_0 constitute the parity bits of PAD.

We note that f_0 is more effective in correcting errors than f_1. However, combining both functions increases the effectiveness of the PAD scheme. One might also map the two parity bits so as to protect the bit representing f_0 more than the bit representing f_1. However, this increases the complexity of the proposed PAD method even further. While mapping the bits to the constellation diagram can be done in a multitude of ways and yielding the same symbol error rate, it is customary

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to use Gray mapping. In Gray mapping neighboring symbols differ in a single bit. Therefore, symbol errors will most likely result is a single bit error. For a constellation of M points, the BER, P_e is approximated by:

$$P_e \approx \frac{P_M}{\log_2 M}, \quad (2)$$

Where P_M is the symbol error rate.

III. DECODING USING PAD

Unlike traditional QAM, the decoder of the proposed PAD scheme uses both the Euclidean distance of the received symbol as well as the parity bits to make the decision. The distances of the received symbol to N closest constellation points are calculated and sorted in ascending order. Then a decision is made for the first point whose parity matches the parity bits. If the parity of none of the N points matches the received parity bits, then a decision is made to the one which matches the parity bit corresponding to f_0, which as shown above is more effective in making correct decisions as compared to f_1. If that is also not met, then the first point—the one with shortest Euclidean distance—is selected. For a practical system and when the number of constellation points is small or medium, N can be less than 9. When the received symbol is close to the perimeter constellation points or "outside" the constellation diagram, then N can be made even smaller. We note here that the proposed scheme is not restricted to QAM as it can be used in conjunction with other modulation techniques like PSK and PAM. It can also be used in multi-dimensional modulation techniques like lattices.

IV. VARYING RELATIVE ENERGIES

In a typical M−point QAM modulation, each k bit of information where $k = \log_2 M$ is mapped to a point in the QAM constellation diagram. Associated with each point is a waveform whose amplitude and phase are determined by the coordinates of the point in the diagram.

Let the average energy per symbol (i.e. waveform) be E_s. Then, the average bit energy is

$$E_b = \frac{E_s}{k}. \quad (4)$$

In the proposed system, the available symbol energy is divided among the k information bits and the two parity bits. The division, however, need not be equal. Since the parity bits are used to make decisions on the information symbols, they need to have a lower probability of error than those of the information symbols. That necessitates assigning the parity bits a higher portion of the available energy. However, assigning them a very large portion will lead to low error rates for the parity but will leave a small amount of energy available to the information symbol. A possible solution to this problem is to divide the energy in such a way that the probability of error of the parity bits, P_p, is a small fraction of the probability of error of the information symbols. Thus, the average waveform energy is divided among the k information bits and the two parity bits like

$$E_s = k\widetilde{E}_b + 2E_p, \quad (5)$$

where $\widetilde{E}_b$ the energy per bit of the information symbol and E_p is the allotted energy to each parity bit. Further, let the ratio of the energy allocated to each parity bit relative to each "bit" in the QAM symbol be α. Thus,

$$E_p = \alpha\widetilde{E}_b, \quad (6)$$

for some $\alpha > 0$. From (4)-(6) we can write

$$kE_b = \widetilde{E}_b(k + 2\alpha). \quad (7)$$

The probability of error for the parity bits, P_p, assuming Gray coding is given by [2]

$$P_p = Q\left(\sqrt{\frac{2E_p}{N_0}}\right), \quad (8)$$

where N_0 is the one-sided noise power density. Similarly, the probability of symbol error for the QAM symbol PQAM is [2]

$$P_{QAM} = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k\widetilde{E}_b}{(M-1)N_0}}\right). \quad (9)$$

The parity bits are used to make a decision for the QAM symbol. Hence, it is imperative that the probability of error for the parity bits P_p to be much lower than the expected error for the QAM symbol. We let P_p be a small fraction of PQAM. Let the two probabilities be related by

$$P_p = \beta P_{QAM}, \quad (10)$$

where $0 < \beta << 1$. It follows from (4)-(10) that

$$Q\left(\sqrt{\frac{2E_p}{N_0}}\right) = 4\beta\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k\widetilde{E}_b}{(M-1)N_0}}\right). \quad (11)$$

Since $\widetilde{E}_b$ is related to E_b, via,

$$\widetilde{E}_b = \frac{k}{k + 2\alpha}E_b, \quad (12)$$

and from (6), we can write (11) as

$$Q\left(\sqrt{\frac{2\alpha k E_b}{(k+2\alpha)N_0}}\right) = 4\beta\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k^2 E_b}{(k+2\alpha)(M-1)N_0}}\right). \quad (13)$$

For a given SNR, E_b/N_0, this equation can be solved numerically for α and hence $\widetilde{E}_b$. The parameter β is an adjustable design parameter. A value of 0.1 implies that the parity bits' errors occur approximately a tenth of the times less frequently than those of the QAM symbol. Currently, there is no algorithm to obtain the optimal value of β. The choice is made mainly from numerical simulations.

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IV. NUMERICAL RESULTS

Since the parity bits control the decision making for the QAM symbols, they need to have as low an error rate as possible. From (8) we see that the error rate of the parity bits can be decreased by increasing the portion of energy assigned to the parity bits, E_p, because the Q function is a monotonically decreasing function. However, increasing E_p may not be desirable as that would reduce the energy available to the QAM symbol. Another way to reduce the parity error probability P_p and hence improve the overall performance is by coding the parity bits using either convolutional or block codes. Such coding, however, will necessitate expanding the required bandwidth even further. One can, however, use Trellis-Coded Modulation (TCM) for the parity bits [3]. By adding a little bit complexity at the receiver, it is possible to gain as much as 3-6 dB for the parity bits to achieve the same error performance without bandwidth expansion. We note, however, that when M is large, then the overall performance improves only slightly as the 'saved' parity energy is distributed among a large $k = log_2M$ bits of the information symbols. Fig. 5 demonstrates the performance of PAD combined with QAM-16. The figure shows a gain of more than three dB at BER of 10^{-5}. This gain comes at a very modest computational cost compared to traditional coding techniques. Fig. 6 demonstrates similar gains when QAM-64 is used for modulation. While PAD can be used as a stand alone ECC for systems with low computational power, it can also be used in conjunction with Turbo codes where hard decision are made at the receiver at the symbol's level. PAD then improves the performance of the hard decision before passing the output to a MAP decoder for further processing. Fig. 7 shows the performance of hard decision Turbo codes embedded on QAM-16.

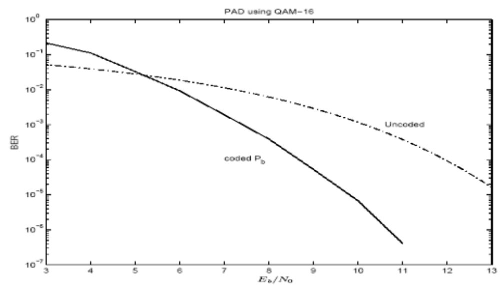

Fig 5: The performance of PAD on QAM-16 in terms of bit error probability P_b.

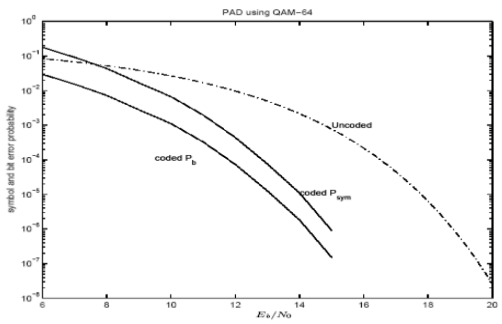

Fig 6: The performance of PAD on QAM-64 in terms of bit error probability P_b and symbol error rate P_{sym}.

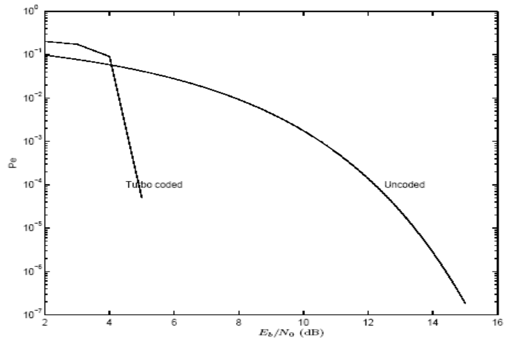

Fig 7: The performance of symbol hard decision with Turbo codes on QAM-16.

V. CONCLUSION

In standard ECC, parity bits are used to correct errors after decisions based on either the soft or quantized (hard) information has been made by the decoder. The parity bits and the information bits are used to detect/correct errors. A new technique proposed in this paper involves sending the parity bits in a different channel and using them to improve the decision making of the decoder. The available energy is distributed unevenly amongst the parity and the information symbols. Then, the parity bits are used at the receiver for assisting in decision making, thus improving the communication reliability. The technique, which does not require complex computations, performs well considering its simplicity. It shows a gain of about 3 dB at BER of 10^{-5} for QAM over AWGN. It costs about a doubling of the bandwidth if the same bandwidth is used for both the information symbols and the parity bits. The bandwidth expansion is only $(k+2)/k$ when QAM-2^k is used if all bits, information and parity, occupy the same bandwidth.

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Amer Al Habsi, he got his PhD from University of Arkansas at Little Rock, Arkansas, USA, 200, MS, from Northeastern University, Boston, Massachusetts, USA, 1998, and BS, Biomedical Engineering, Case Western Reserve University, Cleveland, OH, USA, 1994. His research Interest, Signal Processing, Communication Theory, Channel Coding. His related Experience, Supervisor, Petroleum Development Oman (PDO), Fahud, 1993-1994. He is one of the Professional Membership in IEEE Communication. Society.

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Dr.Hussain Al-Rizzo received his B.Sc. in Electronics and Communications (1979) (High Honors), Postgraduate Diploma in Electronics and Communications (1981) (High Honors) and M.Sc. in Microwave Communication Systems (1983) (High Honors) from the University of Mosul, Mosul, Iraq. From May 1983 to October 1987 he was working with the Electromagnetic Wave Propagation Department, Space and Astronomy Research Center, Scientific Research Council, Baghdad, Iraq. On December, 1987, he joined the Radiating Systems Research Laboratory, Electrical and Computer Engineering Department, University of New Brunswick, Fredericton, NB, Canada where he obtained his Ph.D. (1992) in Computational Electromagnetics, Wireless Communications, and the Global Positioning System. For his various academic achievements he won the nomination by the University of New Brunswick as the best doctoral Graduate in science and engineering. Since 2000, he joined the Systems Engineering Department, University Arkansas at Little Rock where he is currently a tenured Professor. He has published over 40 peer-reviewed journal papers, 70 conference presentations, and several patents. His research areas include implantable antennas and wireless systems, smart antennas, WLAN deployment and load balancing, electromagnetic wave scattering by complex objects, design, modeling and testing of high-power microwave applicators, design and analysis of micro strip antennas for mobile radio systems, precipitation effects on terrestrial and satellite frequency re-use communication systems, field operation of NAVSTAR GPS receivers, data processing, and accuracy assessment, effects of the ionosphere, troposphere and multipath on code and carrier-beat phase GPS observations and the development of novel hybrid Cartesian/cylindrical FD-TD models for passive microwave components.

Dr. Akl has over 20 years of industry and academic experience. He is currently a Tenured Associate Professor at the University of North Texas and a Senior Member of IEEE. He has designed, implemented, and optimized both hardware and software aspects of several wireless communication systems for CDMA, WiFi, and sensor networks. Dr. Akl has broad expertise in wireless communication, Bluetooth, CDMA/WCDMA network optimization, GSM, LTE, VoIP, computer architecture, and computer Networks. He is a very active researcher and is well published and cited. He has given depositions, trial testimony, and has prepared expert reports on claim construction, invalidity, infringement, and non-infringement. He has handled both ITC and district court cases. Dr. Akl was the 2008 recipient of the IEEE Professionalism Award and winner of the 2010 Tech Titan of the Future Award.

Maytham M. Hammood received his B.Sc. in Computer Science (2002) and M.Sc. in Computer Science (2005) from University of Technology, Baghdad, Iraq. From June 2005 to October 2008, he enrolled at Tikrit University, Tikrit, Iraq, as an instructor in the computer science department. . On august, 2009, he joined the Department of Applied Science, Computer Science, University of Arkansas at Little Rock pursuing his Ph.D as Full time graduate student. His current research areas include Data Security in Wireless Sensor Network.