Optimization of Plane and Space Trusses Using Genetic Algorithms

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Abstract—weight optimization of trusses is so important due to economic and sustainability considerations. Geometry, topology and sizing optimization is extensively found in literature. Applications found in literature uses the traditional design variables containing node coordinates, elements connectivity and member cross sections. This paper presents an approach based on the genetic algorithm for optimum design of plane and space trusses subjected to specified set of constraints. The proposed approach defined innovative design variables in terms of node coordinates and displacements. Such limited design variables lead to the reduction of genotype length resulting in less execution time. Topology and cross sections are estimated after using strength criteria. The proposed approach was applied on benchmark problems repeated in literature, the proposed approach resulted in more optimized results with less mathematical effort.

Index Terms—Optimization; plane truss; Space truss; Genetic algorithms

I. INTRODUCTION

Unequivocally the material cost is one of the foremost driving factors in the construction of a buildings; it can be minimized by reducing the weight or volume of the structural system. In addition, such reduction in the used material serves as a tool for sustainable design as step for green buildings. All of the methods used for decreasing the weight intend to reach an optimum design having a set of design variables under specific design constraints. It is essential to comprehend the characteristics of the problem while going for an appropriate optimization method for structural design. The focal attribute of structural design optimization is that the solution sought is the global optimal solution [1] and the design variables are discrete and must be chosen from a pre-determined set which is suitable for engineering design problems. Genetic Algorithms is a part of evolutionary computational technique, it is a global search method which has been preferred by various researchers over other classical optimization techniques for specific applications. [2] Furthermore, it can be used in wide ranges of optimization problems. In contrast to traditional optimization methods which begin from single point solution, GA starts with population of solutions within a search space. Moreover, this technique works with a coding of a parameter set as opposed to traditional optimization methods which work with the parameters themselves Each individual combination in population has a fitness value determined by a fitness function. Afterwards, according to used crossover and mutation values the selection process is applied which imitate natural evolution to produce new candidate solutions. At the end of the process, the newly created generation replaces the previous generation and evolution is repeated until obtaining appropriate solution to the problem while ensuring certain design criteria are satisfied or reaching the pre-determined maximum number of generations. The optimization of truss structures can be classified into three categories depending on which component of the structure is used as a design variable: 1) Sizing, 2) Shape and 3) Topology optimization as shown in Table 1. In sizing optimization cross-sectional area of the members are the design variables and the coordinates of the nodes and the connectivity of various members are fixed. However, in truss design problems, cross sections are considered discrete variables such that member cross-sectional areas are specific predefined values. In Shape optimization the design variables are the nodal coordinates, and in topological optimization the number of nodes and the connectivity between nodes are the design variables while nodal coordinates are assumed known. However, the most efficient design will be obtained by considering all three categories simultaneously. Generally, multilevel optimization methods are used in which topological optimization first performed keeping the shape and cross sectional sizes fixed. When an optimal topology is found, shape and/or sizing optimization is performed on the topology found in the previous step. But this technique may not lead to the most optimal solution as all the three problems are not mutually independent. As a result, traditional methods of optimization have not been suitable for such problem and the use of other techniques such as GAs is gaining popularity in the field of structural optimization. Although it becomes difficult to optimize in complex structures where variable interactions increase. Classical optimization methods can produce sub-optimal results because of these interactions [28] In this paper, new approach is introduced for simultaneous shape, topology and size optimization of plane trusses and sizing optimization for space trusses. Nodal coordinates and deflections are used as design variables instead of instead of using topology and cross sectional area. Topology and cross sections are determined after to cope with the used design variables satisfy strength conditions. This method avoids conventional disadvantages of traditional methods of conflict use of optimization categories and the assumption of constant A/L and keeps it variable.
II. THE PROPOSED APPROACH

As described before, the core idea of our proposed approach is to carry out topology and shape optimization using nodes deflections and coordinates as a design variable. Design variables of the truss in this approach are as follows:

- Coordinates of each node \((x_i, y_i)\) for plane truss and \((x_i, y_i, z_i)\) for space truss, where \(i = 1\) to \(N\), \(N\) is the number of truss nodes). The range of nodal coordinates is defined by the truss proposed geometric limits.
- Deflections of each node \((u_i, v_i, w_i)\) for plane truss and \((u_i, v_i, w_i)\) for space truss, where \(i = 1\) to \(N\), \(N\) is the number of truss nodes). The range of nodal deflections is defined by the code limits of allowable deflection.

For each chromosome in the generation at which the above mentioned design variables are included, the following procedure is applied. At first, from the nodal coordinates of the solution, member original lengths \((L)\) can be calculated assuming that all nodes are connected and that is represented in topology matrix. Topology matrix describes the distribution of members in truss between nodes defined by:

\[
\begin{bmatrix}
1 & D_1 & D_2 & D_3 & D_4 & D_5 & D_6 \\
2 & D_1 & D_2 & D_3 & D_4 & D_5 & D_6
\end{bmatrix}
\]

**Topology matrix**

Where each row represents member and first column represent member number from 1 to \(m\) where \(m\) is the number of truss members and columns No. 2, 3, 4 represent degree of freedom of start node of member and columns No. 5, 6, 7 represent degree of freedom of end node of same member. Using the other group of design variables which represent nodal deflections, the deformed length of each truss member can be calculated. The change in length \((\Delta L)\) can be derived using the relation

\[
\Delta L = \Delta x \cos \theta_x + \Delta y \cos \theta_y + \Delta z \cos \theta_z
\]

Where \(\Delta x, \Delta y, \Delta z\) are the net displacements of the member and defined as:

\[
\begin{align*}
\cos \theta_x &= \frac{x_i - x_j}{\Delta L} \\
\cos \theta_y &= \frac{y_i - y_j}{\Delta L} \\
\cos \theta_z &= \frac{z_i - z_j}{\Delta L}
\end{align*}
\]

\[
\begin{align*}
\Delta x &= u_j - u_i \\
\Delta y &= v_j - v_i \\
\Delta z &= w_j - w_i
\end{align*}
\]

Where \(x_i, y_i, z_i, u_i, v_i, w_i\) are the nodal coordinates and displacements of member joints, respectively. The strain of each member is then compared to the allowable strain of used material \((\varepsilon_{all})\) as:

\[
\frac{\Delta L}{L_o} \leq \varepsilon_{all}
\]

Excluding the members not complying with the condition at Eq. 4, the topology matrix is developed. This avoid us the complications resulting from adding topology as design variable which result in huge design solution chromosome containing contradicting connectivity. At this stage, the shape and connectivity of the truss are defined and the next step is to estimate the member cross sections. Stiffness analysis of the developed truss is then carried out assuming constant area to length of members to get the member forces \(F_i\) for each member \(i\). Now the cross section of each member can be derived using the strength and deflection criteria as:

\[
A_i = \frac{M_{max}}{\sigma_{all}} \left( \frac{I_i}{g_{def}}, \frac{E_{def}}{E_{def}} \right)
\]

Where \(F_i\) is the force obtained in member \(i\) and \(\sigma_{all}, E\) are the allowable stresses and modulus of elasticity of truss member material respectively. While the truss member forces obtained from the analysis are not corresponding to global deflections estimated in the design variables, the cross section obtained by Eq. 5 can be used as just suggested cross section. Such suggested sections are optimal or near optimal final cross sections especially when the assumed deflections in the GA is near that resulting from analysis so that helps GA to reach optimal solution more easily and quickly. The estimated area is then used to select the member section from discrete available cross sections. Lastly we will calculate the forces, stresses and deflections corresponding to these selected suggested cross sections to check if stresses or deflection exceed the allowable limits. The master flow chart of the proposed procedure is shown in Fig. 2.

III. PROPOSED VERSUS TRADITIONAL APPROACHES

The traditional approach of combining shape, topology and sizing design variables in the Genetic algorithm genotype has several drawbacks as reported in literature [27]. Our proposal constitutes a trial to possess powerful merits to overcome such
drawbacks by altering and reducing the design variables and obtaining the remaining design variables using engineering design roles. For the traditional approach encountered in literature, individual chromosomes combination in population may produces optimum topology and shape but due to random selection of cross sections this combination produces section(s) smaller than minimum needed cross section(s) and that cause exceeding constrain(s) limit(s) such stress or deflection so due to this selected section(s) which lead to unfit structure due to the penalty resulting from design violations usually expresses as [26].

\[ F(x) = 1/f(x) (1000v + 1) \]  

(6)

Where \( F(x) \) is fitness value, \( f(x) \) weight of truss and \( v \) is the count of the number of constraints violated by a given solution including check of stability and constructability. Also sometimes individual chromosomes combination produces a good topology and shape but due to random selection this combination produce section(s) larger than minimum needed cross section(s), so the outcome truss is overly weighted despite if we decrease this cross section(s) slightly until reaching the minimum safe cross section(s) the fitness will be improved. In addition, the chromosome length needed for representing sizing variable depend on the number of discrete available cross sections. The chromosome length reserved for member cross section is estimated as:

\[ \text{No. of Bits} = \log_2 \left( \frac{x_{\text{max}} - x_{\text{min}}}{\varepsilon} \right) \]  

(7)

Where \( x_{\text{max}}, x_{\text{min}} \) are the upper and lower bounds of the variable respectively and \( \varepsilon \) is the desired precision.

So if the available cross sections are few, the solution will not be accurate enough and if the available sections are more the chromosome length will be long and it means more complexity and more time consuming. So, it is clear that using sizing as variable doesn’t produce the optimum fitness for most of individual chromosomes combination especially in case of using infinite search and that is unpractical method [26]. In addition, Being no relation between topology as variable and other variables i.e. selecting topology is not depend on sizing or shape of truss that makes it very complicated to make topology sizing and shape optimization in same time and the number of possible solutions reaches extreme levels, which means very big populations and long calculation time and also stuck problems may occurs [27]. On the other hands, the proposed approach proved to give better solution than using sizing as variable to avoid the abovementioned drawbacks and make topology optimization by simple condition without need any chromosomes and also avoiding the disadvantages of traditional method. It also lead to the reduction of the chromosome length because as we mentioned before the chromosome length depend on the limits of variable and the limits of deflection which are lower than the limits of using all available of topology and cross sections. For some cases if we can easily predict the direction of deflection for each point so the range of deflection variable can be reduced by 50%.

Fig.2: Master Flow Chart of the Optimization Algorithm
The node deflection variable is also associated with nodes not members and as we know the number of truss node usually less than the number of its members so that also reduces the chromosome length needed to represent that variable. This procedure is willing to get the maximum fitness for each proposed shape and Topology because we get here the minimum safe sections depending on engineering design rules which can reduce the probability of getting unfit trusses exceed constrains limits.

IV. RESULTS

To verify our proposed approach and investigate its stability and efficiency regarding its results and computational effort, two benchmark problems are considered. The 10 member plane truss problem and the 25 member space truss problems are selected to represent plane and space trusses. The results of our proposed approach are compared with results of optimization of these examples found in previous work. Summary definition of problems and comparison of results are included in this section.

A. 10-Bar plane Truss

The first problem is shape, sizing and topology optimization for 10 bar plane truss. This ten-bar truss is often used as a benchmark problem in structural optimization. This structure is frequently found in literature related to plane truss optimization. The truss has two vertical supports with a distance ‘a’ of 9.144 meters (360 inches) and two loads ‘F’ of 445.374 kN (100 kips) at 9.144 and 18.288 meters from the lower support as shown in Figure 3. Weight is minimized by GA with parameters as follows: population size 600, maximum generation 200, 0.9 crossover and 0.05 mutation probability as justified. Aluminum is used, with Modulus of elasticity $E = 68.95$ GPa ($10^3$ ksi), density $\rho = 2,768$ kg/m$^3$ (0.1 lb/in$^3$) and element stresses are limited to 172.37 MPa (25 ksi) in both tension and compression while buckling is ignored. The displacements are limited to 50.8 mm (2 in) both horizontally and vertically as per code requirements. All available cross sections are used here, as mentioned before the large number of available cross section doesn’t affect time consumed or GA efficiency and not need much populations / generations number as member sections are not used as design variables. Shown in Figure 4, is the convergence history of the 10 bars truss example. At the figure, the fitness value is plotted against the generation number to clarify the how the GA converges to the optimum solution. As investigated from the plot, the fitness value improvement is very limited during the first 30 generation, and then greatly improved till the 100th generation after which stability is observed while Deb and Gulati [12] reached stability around the 140th generation using 110 populations and optimizing size and topology only.

Table 2 lists the results encountered in literature for the 10 member benchmark problem compared with the results of the proposed approach. As illustrated from the results, our propose approach resulted in the most optimized value of fitness (weight) which is less than almost all results found in literature. This may be attributed to the consideration of limited optimization categories in literature in terms of shape and sizing or topology and sizing while our approach considers the three categories simultaneously. Only Deb and Gulati [12], resulted in better solution, but as indicated, their optimum solution contains two members over each other one member between nodes P6 & P4 and other member between...
nodes P6 & P2 which is considered unreal and undesirable overlap [27].

Figure 5 shows the shape of optimized truss while table 3. And 4 show the results of nodal coordinates and member sections. The maximum deflection investigated in the optimum solution is 50.763 for node P2 in Y direction which reaches 99.92% of the maximum permissible value which means that the selected cross sections are almost optimum.

Table 3: Coordinate. Of Variable Node P3

<table>
<thead>
<tr>
<th>Node</th>
<th>X  (m) (Inch)</th>
<th>Y  (m) (Inch)</th>
<th>Z  (m) (Inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.9525</td>
<td>-37.5</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.9525</td>
<td>37.5</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-0.9525</td>
<td>-37.5</td>
<td>0.9525</td>
</tr>
<tr>
<td>4</td>
<td>0.9525</td>
<td>37.5</td>
<td>0.9525</td>
</tr>
<tr>
<td>5</td>
<td>0.9525</td>
<td>37.5</td>
<td>-0.9525</td>
</tr>
<tr>
<td>6</td>
<td>-0.9525</td>
<td>-37.5</td>
<td>-0.9525</td>
</tr>
<tr>
<td>7</td>
<td>-2.54</td>
<td>-100</td>
<td>2.54</td>
</tr>
<tr>
<td>8</td>
<td>2.54</td>
<td>100</td>
<td>2.54</td>
</tr>
<tr>
<td>9</td>
<td>2.54</td>
<td>100</td>
<td>-2.54</td>
</tr>
<tr>
<td>10</td>
<td>-2.54</td>
<td>-100</td>
<td>-2.54</td>
</tr>
</tbody>
</table>

The other benchmark problem is sizing optimization for the 25 bar space truss shown in Figure 6. The coordinates of joints and the member groups for section selection and applied loads are shown in Tables 5, 6 and 7.

Aluminum is used with modulus of elasticity \( E = 68.95 \text{ GPa} \) \( (10^4 \text{ ksi}) \) and density, \( \rho = 2.768 \text{ kg/m}^3 \) \( (0.1 \text{ lb/in}^3) \) and element stresses are limited to 275.8 MPa \( (40 \text{ ksi}) \) in both tension and compression while buckling is ignored. The displacements are limited to 8.9 mm \( (0.35 \text{ in}) \) in all directions as per code requirements.

Table 6: Loading conditions for 25-bar space truss.

<table>
<thead>
<tr>
<th>Node</th>
<th>F_x (KN) (lb)</th>
<th>F_y (KN) (lb)</th>
<th>F_z (KN) (lbs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.45374</td>
<td>-44.5374</td>
<td>-44.5374</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-44.5374</td>
<td>-44.5374</td>
</tr>
<tr>
<td>3</td>
<td>2.22687</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>2.672244</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The set of areas available for this truss is \( S = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6 \} \text{ (in}^2) \).

Weight is minimized by GA with parameters as follows: population size 400, maximum generation 100, 0.9 crossover and 0.05 mutation probability as justified. Shown in Figure 7, is the convergence history of the 25 bars truss example. At the figure, the fitness value is plotted against the generation number to clarify the how the GA converges to the optimum solution. As investigated from the plot, the fitness value is greatly improved from first generation till the 60th generation after which stability is observed that is because of making optimization for sizing only.
V. CONCLUSIONS

When dealing with simultaneous size, shape and topology optimization, the number of possible solutions reaches great levels, which need long chromosome, big populations and long calculation time. So to overcome these problems we should work on reducing the chromosome length without affecting the GA efficiency. An approach is proposed based on using nodal deflections as design variable instead of the member sections in addition to the nodal coordinates. This will reduce the length of genotype as nodes are always less than members in truss and as the range of nodal displacement is less than the range of available steel sections for truss members. In addition, according to loads and configurations the direction of deflection can be expected which reduces the deflection variables to 50% which can improve the calculations. By using simple condition it also allows removing and keeping on members i.e. making topology optimization without using chromosome for it and also reduce Complexity and stuck problems of making Topology optimization by traditional method because this condition make topology optimization depend on other variables (coordinates of nodes and deflections of nodes). The proposed procedure was applied to two of the classical truss problems and the results were compared to the results of previous work found in literature. The presented results not only produce better optimum weight than previous results but also reduced the calculation time and effort by using a few numbers of chromosomes and ability to choose sections from huge numbers of a pre-determined set without any increasing the chromosome length.

REFERENCES

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### Table 2: Results of previous works with same conditions

<table>
<thead>
<tr>
<th>Search</th>
<th>Method</th>
<th>Optimization category</th>
<th>Weight (lbs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kripakaran, Gupta and Baugh Jr. (2007) [6].</td>
<td>Hybrid search method.</td>
<td>√</td>
<td>5073.03</td>
</tr>
<tr>
<td>Li, Huang and Liu (2006) [5]</td>
<td>Particle swarm</td>
<td>√</td>
<td>5060.9</td>
</tr>
<tr>
<td>Rajan (1995) [9].</td>
<td>Genetic algorithm</td>
<td>√</td>
<td>4962.1</td>
</tr>
<tr>
<td>Su Ruiyi, Gui Liangjin, Fan Zijie (2009) [8].</td>
<td>Genetic algorithm</td>
<td>√</td>
<td>4962.07</td>
</tr>
<tr>
<td>Wenyan (2005) [10].</td>
<td>Genetic algorithm</td>
<td>√</td>
<td>4921.25</td>
</tr>
<tr>
<td>Deb and Gulati (2001) [3].</td>
<td>Genetic algorithm</td>
<td>√</td>
<td>4899.15</td>
</tr>
<tr>
<td>Deb and Gulati (2001) [12].</td>
<td>Genetic algorithm</td>
<td>√</td>
<td>4731.65</td>
</tr>
<tr>
<td>This study</td>
<td>Genetic algorithm</td>
<td>√</td>
<td>4762.1</td>
</tr>
</tbody>
</table>

### Table 4: Optimization results for 10-bar plan truss

<table>
<thead>
<tr>
<th>Element No.</th>
<th>Dimensions mm</th>
<th>Area in²</th>
<th>Stress KSI</th>
<th>% Stress of allowable</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>200x200x5</td>
<td>6.04</td>
<td>23.42</td>
<td>93.7%</td>
</tr>
<tr>
<td>M2</td>
<td>400x400x10</td>
<td>24.1</td>
<td>-8.398</td>
<td>33.6%</td>
</tr>
<tr>
<td>M3</td>
<td>260x260x12.5</td>
<td>19.1</td>
<td>-5.782</td>
<td>23.1%</td>
</tr>
<tr>
<td>M4</td>
<td>180x180x12</td>
<td>12.5</td>
<td>-8.236</td>
<td>32.9%</td>
</tr>
<tr>
<td>M5</td>
<td>350x350x10</td>
<td>21.0</td>
<td>6.812</td>
<td>27.2%</td>
</tr>
<tr>
<td>M6</td>
<td>400x400x12</td>
<td>28.8</td>
<td>7.124</td>
<td>28.5%</td>
</tr>
</tbody>
</table>