Extended Golden Code for Digital Transmission Channels through Low Voltage Three Phase Power Lines

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Abstract— In this paper the performance of an extended golden code for transmission channels through low voltage three-phase power lines is analyzed and determined. According to the results obtained, the bit error probability is reduced in the order of magnitude of at least \(1 \times 10^{-3}\) only when the space-time technique is used.

Index Terms— Golden code; transmission channels for low voltage three phase power line.

I. INTRODUCTION

This paper analyzes and determines the performance of an extended golden code for digital data transmission channel, set up with low voltage three-phase power lines.

A complex space-time code for four full diversity antennas is given [1]. A golden code for space-time of two by two is constructed [2]. The generator array for the code has a determinant that does not vanish. A basic introduction about the golden code is provided and we discuss how to use the code in a practical concatenation for a coding scheme [3]. A golden code decoder with low implementation complexity and maximum likelihood decoding is given [4]. A family of full diversity for rotated constellations based on algebraic number theory in \(Z^n\) is provided. [5]. The golden space-time combination with Trellis codification is provided for a Rayleigh fading channel and a two inputs/outputs MIMO channel [6]. A basic introduction about golden codes is provided and we discuss how to use the golden code for MIMO systems [7]. A new pre-processing to decode by using algebraic reduction is introduced, which exploits the multiplicative structure of the code. Also the simulation results for golden codes are shown [8].

All previous work is lead at wireless channels, which has a fast Rayleigh fading and Gaussian white noise. Furthermore, there are no references for low voltage three-phase power line channels.

References [9]-[10], the low voltage power line channel has five different types of noise: Background white noise, narrow band noise, impulsive periodic-asynchronous noise with the network frequency, impulsive periodic-synchronous noise with the network frequency and asynchronous impulsive noise.

Middleton in 1977 presented a model of impulsive noise, consisting of the sum of white noise plus impulse noise. According to the noise bandwidth, he classified it into three general classes: class A, class B and class C, [11]-[12].

We consider that the noise in the low voltage power lines consists of background noise and impulsive noise. As the background noise is stationary and behaves as additive, white Gaussian noise, is modeled as a Gaussian process, and the impulsive noise is modeled as a Poisson process. Therefore, the probability density function for noise in the low voltage power lines (1), references [13]-[14]:

\[
p_f(n) = (1 - \epsilon) N(0, \sigma^2) + \epsilon N(0, k\sigma^2) \quad (1)
\]

Where:

\(\epsilon\) : Number of occurrence of impulsive noise.

\(N(0, \sigma^2)\) : Gaussian distribution with zero mean value and variance \(\sigma^2\) representing the background noise.

\(N(0, k\sigma^2)\) : Impulsive noise component, with Gaussian distribution with a variance \(k\) times the background noise.

This paper proposes to extend the golden code of an array of two by two to another of four by four because it shows that this code has full diversity and codes gain in fast fading Rayleigh channels [2]. In Chile the medium voltage power line has three lines and a neutral.

The organization of this paper is as follows: In Section II the extended gold code is built and the performance of the transmission/reception system with the bit error rate (BER) parameters versus signal to noise ratio (SNR) is obtained. In Section III, the results obtained are shown in graphs and then these results are analyzed. In Section IV, work conclusions are given.

II. ANALYSIS OF THE PROPOSED TRANSMISSION/RECEPTION SYSTEM

The low voltage power line channel has the same features as a wireless channel with fast fading Rayleigh channel [14]. As the structure of the low voltage network and the propagation signal differs from the coupled lines; numerous reflections are caused by the attachment of plugs. Besides, the series connection of cables with different characteristic impedances and the propagation signal is not "line of sight" between transmitter and receiver. Furthermore, the echoes and the result of all this is a multipath propagation signal with a fast fading must be considered.
To apply the space-time technique it is necessary to have diversity, both in the transmitter and the receiver. For the MIMO channel (multiple input and multiple output), namely, using multiple transmit antennas and multiple receive antennas, where there should be no correlation between them. To achieve this, there should be at least a separation between the antennas half wavelength.

To emulate the MIMO system in low voltage power lines, the three-phase electrical system in Chile is considered, which has three 380 Volt lines plus the neutral, with a frequency of 50 Hz. The equivalent MIMO channel has four transmit antennas and four receiving antennas. The separation between three-phase cables allows inferring that no correlation exists between them.

The transmission system consists of the low voltage three-phase electrical systems and the following differences from the wireless channel should be considered: In the space-time wireless channel with n transmit antennas and m receiving antennas, the signal received in the j antenna (j = 1, 2, ... m), at any time t is the combination of the signal emitted by the n transmitting antennas, which is not the case for the three-phase system, because each phase is isolated from one another, thus isolating the path of the transmitted signal. This gives a natural decoupling between any of the three remaining signals corresponding to different transmitted symbols. The signal received is (2).

\[
\sum_{i=1}^{4} y_i(t) = \sum_{j=1}^{4} h_{i,j}(t) s_j(t) + n_j(t)
\]

Where:
- \(y_i(t)\): Signal received by the line i, at time t.
- \(h_{i,j}(t)\): Signal fading at time t, due to the path i.
- \(n_j(t)\): Low voltage power line noise at time t, due to the route i.

The extended golden code is discussed as follows: If n is not a quadratic integer, then \(Z(\sqrt{n})\) is an integral domain, therefore, \(Z(\sqrt{n})\) is a subset of the field \(Q(\sqrt{n})\), called quadratic domain in \(Q(\sqrt{n})\).

The quadratic norm \(N : Q(\sqrt{n}) \rightarrow Q\) is defined as (3):

\[
N(a + b\sqrt{n}) = (a + b\sqrt{n})(a - b\sqrt{n}) = a^2 - nb^2 \in \mathbb{Q}
\]

The field of quadratic extension \(Q(i)\) is considered, which is a set of (4):

\[
L = \left\{ x = a + b\sqrt{d} \mid a, b \in Q(i) \right\}
\]

Denoted as:

\[
L = \frac{Q(i, \sqrt{d})}{Q(i)}
\]

Where:
- \(d\): Positive and quadratic integer.
- \(Q(i) \cap Q(\sqrt{d}) = Q\)

The Galois group is given by (6):
\[ \alpha = 1 + i \theta = 1 + i \cdot 2.618 \]  
\[ \hat{\alpha} = 1 + i \hat{\theta} = 1 + i \cdot 1.618 \]

It has:
\[ \alpha \hat{\alpha} = 2 + i \]

It is determined the minimum determinant from (16) is:
\[ \det(X) = \frac{2 + i}{5} \left[ \left( s_1 + s_2 \theta \right) \left( s_1 + s_2 \hat{\theta} \right) - i \left( s_3 + s_4 \theta \right) \left( s_3 + s_4 \hat{\theta} \right) \right] \]

Manipulate (20) is obtained (21):
\[ \det(X) = \frac{1}{2 - i} \left[ \left( s_1^2 + s_1 s_2 - s_2^2 \right) - i \left( s_3^2 + s_3 s_4 - s_4^2 \right) \right] \]

\[ \left[ \left( s_1^2 + s_1 s_2 - s_2^2 \right) - i \left( s_3^2 + s_3 s_4 - s_4^2 \right) \right] \] is at least 1, therefore \( \delta_{\text{min}} \) is given (22):
\[ \delta_{\text{min}} = \text{Min}_{X \neq 0} \left[ \det(X) \right]^2 = \left( \frac{1}{2 - i} \right) \cdot \left( \frac{1}{2 + i} \right) = \frac{1}{5} \]

It is made the golden code length for a channel with low voltage three phase power lines, but in this case the neutral adddress three phase power lines, but in this case neutral addresses as another digital data transmission channel, leaving a array of four by four (24).

\[
\begin{bmatrix}
\alpha s_1 + \alpha_\theta & \alpha s_2 + \alpha s_\theta & \alpha s_3 + \alpha s_{\theta} & \alpha s_4 + \alpha s_{\theta} \\
\alpha s_1 + \alpha s_{\theta} & \alpha s_2 + \alpha s_\theta & \alpha s_3 + \alpha s_{\theta} & \alpha s_4 + \alpha s_{\theta} \\
\alpha s_1 + \alpha s_{\theta} & \alpha s_2 + \alpha s_\theta & \alpha s_3 + \alpha s_{\theta} & \alpha s_4 + \alpha s_{\theta} \\
\alpha s_1 + \alpha s_{\theta} & \alpha s_2 + \alpha s_\theta & \alpha s_3 + \alpha s_{\theta} & \alpha s_4 + \alpha s_{\theta} \\
\end{bmatrix}
\]

Where:
\[ s_1, s_2, ..., s_{15}, s_{16} \] Constellation symbols.
\[ \theta = 1.618 \]
\[ \hat{\theta} = -0.6118 \]
\[ \alpha = 1 + i \cdot 2.618 \]
\[ \hat{\alpha} = 1 + i \cdot 1.618 \]
\[ i = \sqrt{-1} \]

As the three-phase power lines are isolated from one another, signals are only received from the respective line and do not coupling signals from other lines. That is why it is applicable the golden code length of the two by two dimensional array to the four by four array. In this model of four three-phase power lines (three lines plus neutral), the received signal (low voltage electrical line R) is given by (24), (25), (26) and (27):
\[ y_{11} = h_1 \left( \alpha s_1 + \alpha s_{\theta} \right) \]  
\[ y_{12} = h_2 \left( \hat{\alpha} s_3 + i \hat{\alpha} s_{4\theta} \right) \]  
\[ y_{13} = h_3 \left( \alpha s_0 + \alpha s_{10\theta} \right) \]  
\[ y_{14} = h_4 \left( \hat{\alpha} s_{11} + i \hat{\alpha} s_{12\theta} \right) \]  

The low voltage electrical line \( S \) receives the signals (28), (29), and (31):
\[ y_{21} = h_{21} \left( \alpha s_3 + \alpha s_{\theta} \right) \]  
\[ y_{22} = h_{22} \left( \hat{\alpha} s_1 + i \hat{\alpha} s_{4\theta} \right) \]  
\[ y_{23} = h_{23} \left( \alpha s_1 + \alpha s_{2\theta} \right) \]  
\[ y_{24} = h_{24} \left( \hat{\alpha} s_{10} + i \hat{\alpha} s_{11\theta} \right) \]

The low voltage electrical line \( T \) receives the signals (32), (33), (34) and (35):
\[ y_{31} = h_{31} \left( \alpha s_5 + \alpha s_{6\theta} \right) \]  
\[ y_{32} = h_{32} \left( \hat{\alpha} s_7 + i \hat{\alpha} s_{8\theta} \right) \]  
\[ y_{33} = h_{33} \left( \alpha s_{13} + \alpha s_{14\theta} \right) \]  
\[ y_{34} = h_{34} \left( \hat{\alpha} s_{15} + i \hat{\alpha} s_{16\theta} \right) \]

A maximum likelihood detector is used to detect the transmitted symbols, which makes an exhaustive search of all possible values of the transmitted symbols and decides in favor of the set of symbols that minimizes the Euclidean distance. To decrease the probability of error of the detector instead of detecting simultaneously the sixteen symbols, the process is carried out in groups of four symbols at a time, as follows (40) until (43):
\[ D(s_1, s_2, s_3, s_4) = \]
\[ D(s_5, s_6, s_7, s_8) = \]
\[ D(s_9, s_{10}, s_{11}, s_{12}) = \]

\[
\begin{bmatrix}
\| y_{11} - h_1 (\alpha s_1 + \alpha s_{\theta} \right) \|^2 \\
+ \| y_{12} - h_2 (\hat{\alpha} s_1 + i \hat{\alpha} s_{4\theta} \right) \|^2 \\
+ \| y_{13} - h_3 (\alpha s_0 + \alpha s_{10\theta} \right) \|^2 \\
+ \| y_{14} - h_4 (\hat{\alpha} s_{11} + i \hat{\alpha} s_{12\theta} \right) \|^2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\| y_{31} - h_{31} (\alpha s_5 + \alpha s_{6\theta} \right) \|^2 \\
+ \| y_{32} - h_{32} (\hat{\alpha} s_7 + i \hat{\alpha} s_{8\theta} \right) \|^2 \\
+ \| y_{33} - h_{33} (\alpha s_{13} + \alpha s_{14\theta} \right) \|^2 \\
+ \| y_{34} - h_{34} (\hat{\alpha} s_{15} + i \hat{\alpha} s_{16\theta} \right) \|^2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\| y_{91} - h_{41} (\alpha s_9 + \alpha s_{10\theta} \right) \|^2 \\
+ \| y_{92} - h_{42} (\hat{\alpha} s_9 + \hat{\alpha} s_{10\theta} \right) \|^2 \\
+ \| y_{93} - h_{43} (\alpha s_{11} + \alpha s_{12\theta} \right) \|^2 \\
+ \| y_{94} - h_{44} (\hat{\alpha} s_{11} + i \hat{\alpha} s_{12\theta} \right) \|^2 \\
\end{bmatrix}
\]
The two dimensional constellation used is given \[14\], which has a very good performance in noisy channel class A, because it has an optimal rotation angle.

The amplitude of impulse noise and background noise is modeled by a Gaussian distribution and is defined as index \[A = 2 \Gamma\]

The Middleton class A probability density is write as (44) and (45):

\[
P_M(n) = \sum_{m=0}^{2^n} A^m \frac{1}{m! 2 \pi \sigma_m^2} e^{-\frac{|H|^2}{2 \sigma_m^2}}
\]

And \(\sigma_m^2\) as (45):

\[
\sigma_m^2 = \sigma^2 A \frac{m + \Gamma}{1 + \Gamma}
\]

According to fig. 1, data to be transmitted \(b(t)\) is obtained. These bits are converted into symbols of the circular, rectangular, and hexagonal of four and sixteen symbols constellation used in the transmission. Then, the golden code encoder converts the symbols into code words \(c_i(t)\). The transmission channel adds noise and fast fading to the transmitted signal \(y_i(t)\). At the receiver, there is the fading coefficient estimator \(h_f(t)\) and the combiner which builds golden codewords locally \(r(t)\). Besides, the maximum likelihood symbol estimator determines the transmitted symbols and the constellation becomes demapper of symbols to bits.

### III RESULTS AND DISCUSSION

#### A Results

In the low voltage three-phase power line channel, it is assumed that route gains \(h_i\) are modeled as independent samples of complex Gaussian random variables with a variance 0.5 per real dimension. The transmission channel is assumed quasi-static; the routes gains are constant throughout the length of the frame \(p\) (number of rows of the transmission array) and vary from one frame to another. The class A noise samples are independent-identically-distributed (iid) complex random variables.

The parameters of the noise in the low voltage power line are: Impulsive index value \(A = 0.01\) and the ratio of impulse noise \(\Gamma = 0.001\). With these values the noise is highly impulsive.

Fig. 2 shows the contrast of the performance of extended golden code compared to the optimal space-time technique with four symbols constellations for three-phase power line channel and BER parameter versus SNR parameter.

Fig. 3 shows the comparison of the performance of extended golden code compared to the optimal space-time technique with sixteen symbols constellations for three-phase power line channel and BER parameter versus SNR parameter.

In the low voltage three-phase power line channel, it is assumed that route gains \(h_i\) are modeled as independent samples of complex Gaussian random variables with a variance 0.5 per real dimension. The transmission channel is assumed quasi-static; the routes gains are constant throughout the length of the frame \(p\) (number of rows of the transmission channel adds noise and fast fading to the transmitted signal \(y_i(t)\). At the receiver, there is the fading coefficient estimator \(h_f(t)\) and the combiner which builds golden codewords locally \(r(t)\). Besides, the maximum likelihood symbol estimator determines the transmitted symbols and the constellation becomes demapper of symbols to bits.
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The parameters of the noise in the low voltage power line are: Impulsive index value \( A = 0.01 \) and the ratio of impulse noise \( \gamma = 0.001 \). With these values the noise is highly impulsive.

Fig. 2 shows the contrast of the performance of extended golden code compared to the optimal space-time technique with four symbols constellations for three-phase power line channel and BER parameter versus SNR parameter.

\[
\begin{align*}
\text{BER} & \quad 0.00001 & \quad 0.0001 & \quad 0.001 & \quad 0.01 & \quad 0.1 & \quad 1 \\
\text{SNR (dB)} & \quad 0 & \quad 5 & \quad 10 & \quad 15 & \quad 20 & \quad 25 & \quad 30
\end{align*}
\]

Fig. 3 shows the comparison of the performance of extended golden code compared to the optimal space-time technique with sixteen symbols constellations for three-phase power line channel and BER parameter versus SNR parameter.

\[
\begin{align*}
\text{BER} & \quad 0.0000001 & \quad 0.000001 & \quad 0.00001 & \quad 0.0001 & \quad 0.001 & \quad 0.01 & \quad 0.1 & \quad 1 \\
\text{SNR (dB)} & \quad 0 & \quad 5 & \quad 10 & \quad 15 & \quad 20 & \quad 25 & \quad 30
\end{align*}
\]

Fig. 2 shows the comparison between the performance of extended golden code and the optimal space-time technique with four symbols constellations for three phase power line channel with BER parameter versus SNR parameter.

Fig. 3 shows the comparison of the performance of gold code extended compared to the optimal space-time technique with sixteen symbols constellations for three phase power line channel, with the BER versus SNR parameter.

**B Discussion**

From fig. 3 and 4, we infer that the extended code has better performance than its optimal space-time counterpart, for four and sixteen symbols constellations. Besides, it is proved that the performance is independent of the constellation form; this is because the extended code performance of the three constellations (rectangular, circular and hexagonal) does not get much difference together.

In the case of four symbols constellations, the extended code error decreases the probability by at least in an order of magnitude of \(1 \times 10^{-3}\) for a SNR of 15 dB, compared to the best optimal space-time technical performance. The same applies to the sixteen symbols constellation.

**IV. CONCLUSION**

According to the results obtained, we can infer that the extension of the golden code greatly improves the reliability of the proposed system for a low voltage three-phase power line channel.

The BER is reduced from \(10^{-3}\) to \(10^{-3}\) to SNR of 15 dB. A gain of 7 dB is obtained with extended golden code. Besides, the transmission speed (sixteen symbols) increases compared to optimal space-time (four symbols).

Future work will be focused on improving the symbols detector.

**REFERENCES**


AUTHOR BIOGRAPHY

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