

GCSR: A GPS Acquisition Technique using Compressive Sensing enhanced implementation

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Abstract— Current GPS receivers deploy large number of correlators involving huge samples and processing/integration to try to acquire weak signals. This is a drain on any Smartphone’s battery. Compressive sensing (CS) reduces the measurements/correlations required to acquire wireless signals if presented in a sparse form. CS allows sampling the signal at below Nyquist (sN) recommended rates saving processing time and power consumption. This paper implements a CS based GPS acquisition technique with an enhanced sampling process using a deterministic orthogonal waveform such as the Jacket matrix. This enables GCSR to sample the signal at the information band and reconstruct it without information loss. MATLAB simulation and analysis of this implementation shows that we achieve 20% better overall performance, which is much desirable for acquiring weak signals in harsh environment, in a simple frontend implementation.

Index Terms—GPS signal acquisition, compressive sensing, sub-Nyquist sampling, correlation

I. INTRODUCTION

Smart phones account for more than half the handsets sold nowadays. All handsets are GPS enabled to satisfy the ever increasing demand for localisation from many applications/services. Other GNSS technologies, such as Glonass & Galileo, are available and are rolling out into Smartphones gradually. Therefore, the hunt for a more energy efficient, faster to acquire and high-sensitivity GNSS multi-signal acquisition solutions is ongoing. Our GCSR focuses on the GPS signal acquisition since it is the most computational intensive process especially during cold-start and in harsh environments such as in urban areas and near indoors scenarios. GPS signal acquisition is a two dimensional process, where the acquisition algorithm has to search for the beginning of the C/A code (Code phase delay) in the received signal and the coarse value of its carrier frequency (Doppler shift). A conventional GPS receiver generates a replica C/A code for each GPS satellite, then search for its alignment with the incoming code and carrier frequency with ± 4 KHz range of Doppler frequencies. i.e. This process will determine the right code delay and Doppler shift when the generated codes and frequencies are perfectly aligned with the incoming signal. This matching/correlation process becomes very intensive when the signal’s power is degraded, depending on the environment where clear outdoors signal power is around -125dBm, and it can be around -160dBm in light indoors. If sampling is performed at the Nyquist recommended rates, then a weak signal will make

the receiver thrash for all the 20ms available to try to find the signal. CS is adopted so to either use the 20ms more efficiently by giving the Doppler/Code matching process a better chance to find the signal and/or do this matching at much less sampling and correlations without impacting the resolution/sensitivity of the acquired signals [1]. Our GCSR will further enhance this by moving the acquisition stage from the Analogue Front End (AFE) to the DSP domain. The main features of the GCSR implementation are:

- Lower computational complexity achieved by reducing the correlations of the problem leading to reduced measurements required to acquire the signals. Simple AFE implementation is achieved by transferring the complexity from the AFE domain to the DSP domain. This means a significant saving in process time and therefore battery energy.
- Sampling the received signal at the information band below the recommended double Nyquist sampling rate. i.e. The GPS signal is sampled at sN rate of 2.046 MHz. This means a further saving is achieved.
- CS does require that the signal is sparse. Fortunately, GPS, like any wireless RF signal, is relatively-sparse [2]. GCSR enhances the sparseness of the GPS received signal by using deterministic waveforms such the Jacket matrix. This has improved the sensing of weak GPS signals by 20%.

Fig. 1 shows a block diagram of our GCSR implementation as follows:

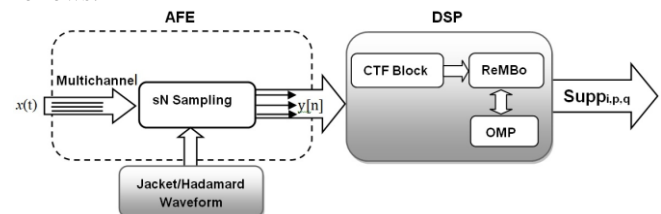


Fig. 1. Our GCSR acquisition block

1. The received analogue signal $x(t)$ is multiplied by a periodic waveform into pre-determined “m” number of channels to generate $y[n]$ baseband vectors representing unique values of the information band at that time. To enhance the sensing of $x(t)$, a number of periodic waveforms such as square, sawtooth (Ramp) and sinusoidal waveforms were tested. However, we concluded that a deterministic waveforms such as the Hadamard or the Jacket blocks offer better performance when we construct the signals. The Jacket block [3] is an extension of the Hadamard matrix and it is a centre

weighted matrix, where the inverse matrix can be obtained from closed form entity as shown below. i.e. either block inverse or element-wise inverse are possible.

$$[J]_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -c & c & -1 \\ 1 & c & -c & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}, \quad [J]_4^{-1} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1/c & 1/c & -1 \\ 1 & 1/c & -1/c & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Where $[J]_4$ a 4x4 Jacket matrix, and c is the non-zero arbitrary number that represents a weighted factor.

2. For “m”, the larger the number of channels selected, the more this will influence on the resolution of the sensing. Therefore, based on our experiments, we chose $m=480$ for normal GPS signal and $m=600$ for acquiring low sensitivity signals.
3. For the DSP part, the $y[n]$ vectors are now converted into a Frame using the Continuous-To-Finite (CTF) block [4]. The new block frame size is now “m x m”, i.e. converting the problem from an infinite measurement vector (IMV) (the length of the tested signal) to a multi measurement vector (MMV) (number of channels). This reduces the required processing from a typical acquisition search for a weak GPS signal (20ms signal length x 2 samples per chip x 1023 chips) to the number of channels chosen.
4. A greedy algorithm such as the Orthogonal Matching Pursuit (OMP) algorithm [5] or the Approximate Conjugate Direction Gradient Pursuit (ADGP) algorithm [6], is now applied to find the dictionary elements of the acquired signals. Each dictionary element comprises of three parameters as shown in Fig. 1; “I” the GPS satellite number, “p” the code phase delay and “q” the Doppler shift.

Also, when compared with other GPS acquisition techniques using CS or other serial-search (hardware correlation engine) or parallel-search (software FFT processing) methods, our GCSR implementation reduces the number of samples as well as reducing the problem from “N” number of samples to “M” number of sampling channels, and where $M \ll N$. Therefore, our acquisition is easier and faster than these methods. The rest of the paper has been organized as follow. Section II summarize the related work on sampling/acquiring signals based on CS. Section III derive the mathematical model for the GPS signals and its correlation properties. Section IV describes our GCSR. Section V explains the signal scenarios used to test the GCSR implementation and discuss the results. Section VI conclude this work and outlines future work.

II. RELATED WORK

The principle of CS is to permit the sampling of sparse signal below Nyquist rate and reconstruct the signal without information loss. When typically used for direct sequence spread spectrum (DSSS) IEEE 802.15.4 standard technology signal [7], sampling, using random demodulation (RD) has been successfully implemented in various CS frameworks.

The received signal is typically mixed with a square waveform, generated by an LFSR, at or greater than Nyquist rate, and then sampled below the Nyquist rate [8]. However, the RD sampling techniques has be performed in the AFE, requires synchronization with the signal during the reconstruction which necessitates the addition of more header bits to the spread signal. The Xampling technique improves on the RD technique by a) improving the sparse representation of the received signal by aliasing it into multichannel using a locally generated arbitrary periodic waveform. The mixing rate of these channels waveforms is below the Nyquist rate (equal to the sampling rate, which is the maximum information band of the received signals), b) this multi-channel Aliasing arrangement helps for direct signal reconstruction without the need for code synchronization [9]. Another CS-based approach for sampling and sparsing GPS signals uses “random Compressive Multichannel Samplers” (CMS) as shown in Fig. 2. The resulted multichannel sparse vectors are then converted to a frame/matrix by utilizing the CTF block. The dictionary elements are then found via applying the OMP algorithm to match this frame with the sensing matrix [1]. This method acquires GPS signals at 2.046MHz sampling rate. On the other hand, the requirement for constructing the “random compressive multichannel samplers” requires complex analogue hardware implementation. This method allows for accumulating the received power for acquiring 20ms length with the same number of channels at once. Our GCSR simplifies the hardware implementation of the CMS method by transfer the matching to the DSP domain. We also enhance the sensing process by utilizing the Jacket matrix for sparsing the received signal.

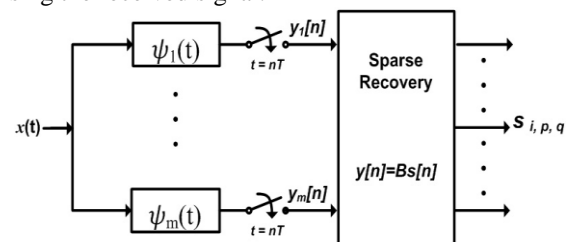


Fig. 2. CMS acquisition block

III. GPS L1-C/A CODE SIGNAL GENERATION

The GPS signal has been designed based on the DSSS technique; each satellite is assigned to a unique C/A code. This code is generated at exactly 1.023MHz with code length 1023 chips and it repeats every 1ms. The property of this code is nearly orthogonal, which means the correlation of any two individual codes results in “close to zero” value (cross-correlation), while correlation of same signal codes with a 1-chip phase delay will result “high” value (autocorrelation). The C/A code spreads the navigation messages produced at 50bps. Using BPSK modulation, the spread code is then modulated with an L1 carrier frequency centered at 1575.42MHz [10]. The received GPS L1-C/A signals are under the GPS-noise-ground-level of -111dBm.

Also, arriving GPS signals have different Doppler shifts and code phase delays dependent on the direction and position of the transmitting SV as well as the receiver environment that determines if the signals are direct or reflected (multipath). The received GPS signals are expressed as:

$$x(t) = \sum_{i=1}^I \sum_{z=0}^Z \sqrt{2P_{i,z}} C_{i,z}(t - \tau_{i,z}) D_{i,z}(t - \tau_{i,z}) e^{j(2\pi f_{d,i,z} t + \theta_{i,z})} + w_{i,z}(t)$$

Where $x(t)$ is the received signal, $\sqrt{2P}$ power of received signal, C represents the C/A code, D is the navigation message, τ is the code phase delay, f_d comprises the carrier frequency with Doppler shift, θ is the carrier phase, I is the number of satellites ($I=24$), Z is the number of multipath signals and $w(t)$ is the complex AWGN.

IV. OUR GCSR IMPLEMENTATION

Our GCSR implementation is as follows:

A1. Sampling rate: the GCSR has been designed to sample the GPS signal at low sampling rate 2.046MHz equal to the maximum information bandwidth. Fig. 3 shows the GCSR multichannel sampling block. $x(t)$ goes through “ m ” sampling channels simultaneously. Each of these channels consist of the periodic waveforms $\phi_m(t)$ multiplied by a copy of the received $x(t)$. The rate of the periodic waveform f_p is equal to the sampling rate f_s , ($f_p = f_s = 2.04\text{MHz}$). These mixed signals are a linear combination of replicas of $x(t)$ in any of the f_p and thus containing the full information of the $x(t)$ band. Therefore, the LPF in each channel will only allow baseband signals to pass to the ADC to produce a vector of samples $y_m[n]$.

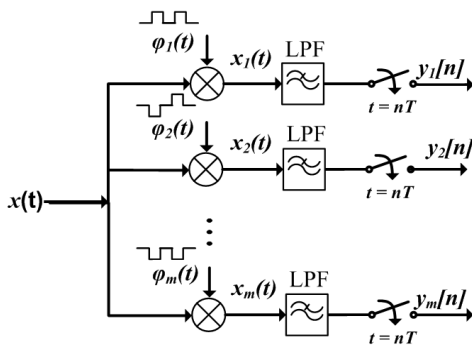


Fig. 3. GCSR multichannel sampling block

A2. Frame construction: the resultant sampled vectors $y_m[n]$ are then converted to a frame “ F ” by using CTF block as illustrated in equations (1) and (2) and shown in Fig. 4. The purpose of using the CTF block is to reduce the space of the problem from “ n ” dimension (number of samples) to “ m ” dimension (number of chosen channels), i.e., the CTF block converts the problem from IMV to MMV.

$$y[n] = [y_1[n], \dots, y_m[n]]^T \quad (1)$$

$$F = \sum_n y[n] y^T[n] \quad (2)$$

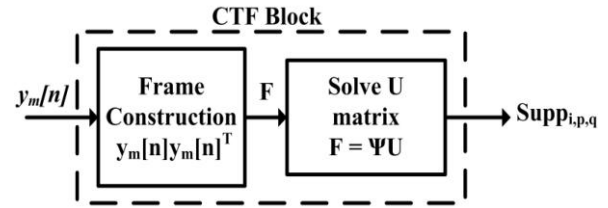


Fig. 4. Continuous-to-finite block

A3. Acquisition: for acquiring the GPS signal, the “ U ” matrix in “ $F = \Psi U$ ” block, as shown in Fig. 4, need to be solved. To achieve this, the measurement matrix Ψ illustrated in equation (3) is first constructed from multiplying the transformation matrix $\Phi(w)$ with the GPS dictionary matrix $\beta(w)$ illustrated in equation (4). Note that the dictionary matrix is built from a bank of correlators of I , P and Q , where the P is the “search step of code phase delay at half chip resolution” and the Q is the “Doppler frequency shift with 500Hz frequency search steps”.

$$\psi(\omega) = \Phi(\omega) \beta_{I,P,Q}(\omega) \quad (3)$$

$$[\beta]_{i,p,q} = C_i(t - pT_c) e^{j(2\pi(F_d + q)T_c)} \quad (4)$$

A4. Detecting the Dictionary elements: to relax the problem from MMV to single measurement vector (SMV) we use the “reduce and boost” algorithm. The OMP algorithm is now used to find the dictionary elements (support values) of the measurement matrix Ψ . After the completion of the support recovery of the sparse matrix U , the right code phase delay and Doppler frequency shift of acquired satellites ($\text{supp}_{i,p,q}$) are determined by calculating the maximum values of the column vectors of U matrix ($\text{supp}_{i,p,q} = \text{supp}(U)$).

V. EXPERIMENTAL RESULTS

To prove the GCSR, Seven scenarios have been chosen to illustrate the various environment conditions of the GPS signals. These scenarios are grouped according to the carrier to noise ratio (C/N) of the received signals as illustrated in table I. The first scenario represents an outdoors signal reception, where only LOS signals are received. The second scenario represents a single multipath signal with each of the LOS signals to understand the effect of the delay in the code phase and the signal power degradation of the multipath signal. The third scenario includes more multipath signals with the LOS signals. The last scenario represents an indoors environment, where only multipath signals are present.

TABLE I. GPS SIGNALS SCENARIOS

C/N dB-Hz	LOS & Multipath signals Scenarios	
	Number of received signals	Scenario name
50	5 LOS signals, no multipath signal	S1
45 to 40	5 LOS signals, each with one multipath signal	S2

C/N dB-Hz	LOS & Multipath signals Scenarios	
	Number of received signals	Scenario name
35	5 LOS signals, each with two multipath signals	SM
30 to 20	5 NLOS signals, each has three multipath signals	M

These scenarios are simulated using MATLAB. The front-end is designed to have 2 MHz bandwidth of the low-pass filter, 3 dB cascade noise figure and the nominal power of the received signal is set at -125 dBm. Considering the acquisition and the computational performance, we have compared our receiver with the CMS implementation. Choosing to simulate a 20ms length signal will add around 13dB gain to the C/N of the received signal at the sample rate of 2.046MHz. The acquisition rate can be improved by increasing the number of channels representing the rows in the sensing matrix. This will increase the reliability in the sampling signal, which results detecting weak signals. To illustrate this, two types of channels are used; 480-channels for comparison with CMS method, and 600 channels to demonstrate the performance of our GCSR, as shown in Fig. 5.

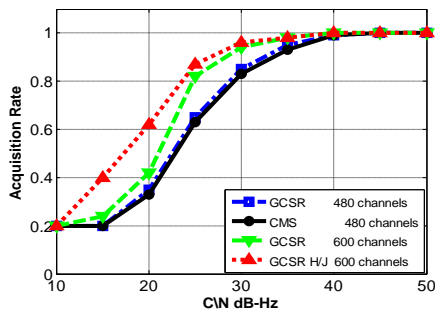


Fig. 5. Acquisition rate for different channels and waveforms

The deterministic orthogonal waveforms “Hadamard or Jacket matrices” used in GCSR to sample the received signal and to construct the measurement matrix has increased the acquisition rate by 20% more than when using a square waveform. i.e. we gain around 3dB-Hz clearly noticeable at high noise situations. This is achieved by the low coherence between our GPS dictionary matrix and the Jacket/Hadamard matrix, as a result of the perfect orthogonality of these matrices. To ensure performance’s stability of the GCSR selected waveforms, these seven different C/Ns’ scenarios have been run 100 times for each waveform (7 x 100 x 5) as shown in Fig. 6. The use of the Jacket/Hadamard matrix is more stable than using others periodic waveforms.

The GCSR has a simple implementation as the CMS method uses multichannel random samplers to sample the GPS signal; using the measurement matrix (complex hardware and pre-processing to construct the measurement matrix). GCSR achieves better signal matching. This is because finding the dictionary elements in the CMS method depends on the matching between the sensing matrix and the frame F, whereas, our GCSR method depend on matching between the measurement matrix and the frame F. This matching can produce more reliable correlation, since both of received signal and generated dictionary are multiplied by the

sensing matrix. To assure the quality of acquisition, GCSR has been also compared with a traditional GPS receiver as shown in Fig. 7. To do this, the resultant GCSR signals are fed into a GPS decoder to recover the actual acquired SV navigation message.

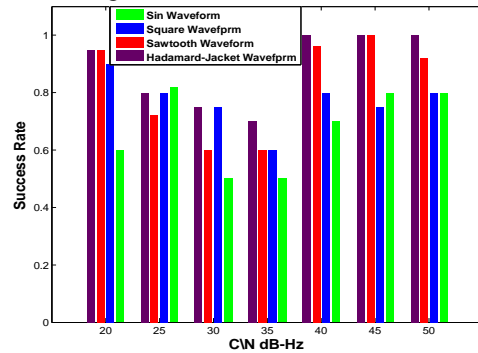


Fig. 6. Success rate with 100 runs for each of 5 waveforms and for each of 7 scenarios

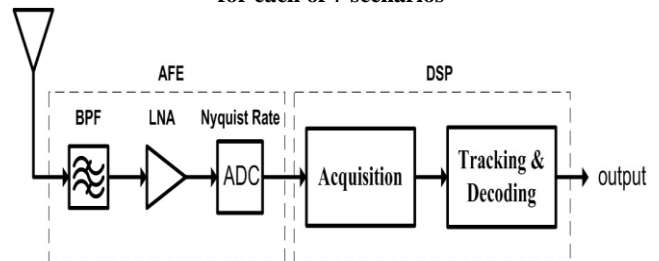


Fig. 7. Traditional GPS receiver

The bit error rate (BER) and the error vector magnitude (EVM) are used for evaluating and analyzing the effect of GCSR acquired signals decoded messages. The BER performance of both the GCSR receiver and the traditional GPS receiver, as a function of the normalize C/N, are shown in Fig. 8. The results illustrate that there is a small degradation of BER of about 1 dB. Actually, this small degradation is expected as an overhead of using the CS technique [11].

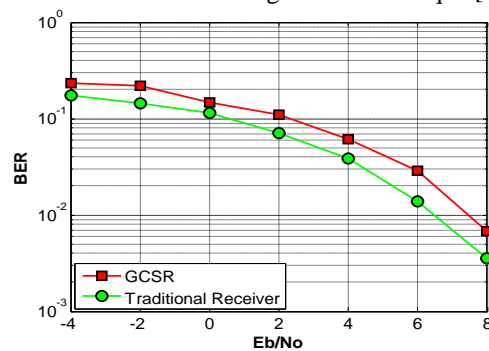


Fig. 8. Bit error rate versus energy per bit to noise power spectral density

The EVM analysis defines the difference between the estimated phase and amplitude values of the demodulated/decoded symbol with the values of the actual received symbol. This will show if the GCSR has preserved the distance between any pair of samples (phases and amplitudes) during compressing and reconstruction of the GPS signals. Fig. 9 shows the EVM values of the estimated phase and amplitude of the GCSR and a traditional GPS

receiver. There is a bit of distortion in estimating the phases & amplitudes in the GCSR, however, this distortion is acceptable for a solution based on the CS technique [11].

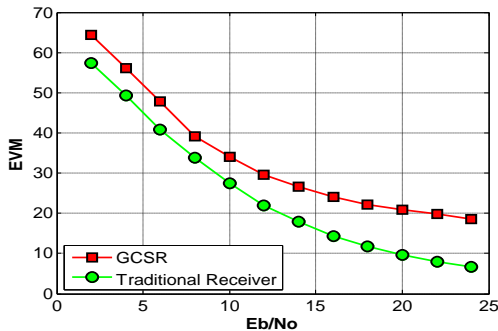


Fig. 9. Error vector magnitude curves (RMS)

A final prove of the GCSR implementation is to measure the stability of the phase locked loop (PLL) discriminator in decoder part. The basic work of the PLL is to recover/track the actual phase, amplitude and frequency of the received signal. The traditional receiver and our GCSR have almost the same steady-state values during running the simulation of tracking one-second of GPS data, as shown in Fig. 10.

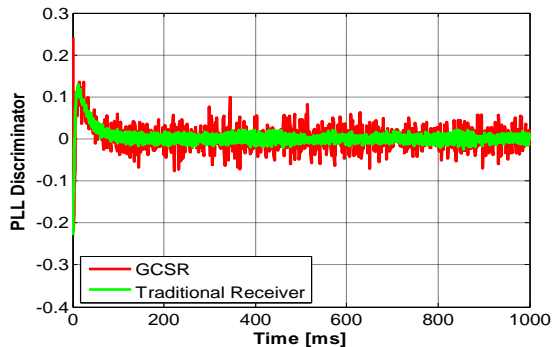


Fig. 10. The steady state of PLL discriminator

VI. CONCLUSION

The enhancements adopted in the GCSR implementation do simplify the frontend design as well as introduce better performance when compared with other CS-based or traditional GPS receiver solutions. The higher acquisition rate is achieved by using a deterministic waveform generator to sparse the received signal such as the Hadamard or the Jacket matrices. This is because these matrices have best orthogonality. As a result, the sensing process has been enhanced as well as improving the signal sampling. The GPS signals have been sampled below the Nyquist rate and equal to the information band. Acquiring GPS signals therefore can be accomplished with fewer correlations as the compressive sensing process is now transferred from matching the length of the whole signal samples number to matching whole rows/channels of the sensing matrix. Increasing the number of rows/channels will increase the acquisition rate proportionately. Reconstructing the signal based on our implementation is simpler than others because we have moved the measurement process to the DSP side while others process it in the Analog side. Our test scenarios and analysis

showed a slight phase distortion and amplitude degradation of the decoded signal; however the integrity of the received signal was maintained. Our future work will focus on the minimization of the dictionary matrix dimension to achieve faster acquisition.

VII. ACKNOWLEDGMENT

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Maher Al-Aboodi received B.Sc. degree in Mathematics and M.Sc. degree Pure Mathematics from the University of Basra, Iraq in 2002 and 2005 respectively. He is currently pursuing a DPhil degree in applied mathematics at the University of Buckingham, UK. His research focuses is on using Partial Differential Equations to devise a method and process to time-share received signals from various wireless technologies. His present work centers on using the same receiver chain (RF and digital) to handle a number of wireless signals necessary for cognitive-radio and software-defined-radio systems.