Solution of Game Problems Using New Approach

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Abstract - In this paper, a new approach to the solution of game problems is suggested which is based on the iterative procedure. The proposed new technique is computationally more efficient and easier as compared to traditional simplex method.

Key Words - Alternative Approach, game problem, optimum solution, simplex method.

I. INTRODUCTION

Many practical problems require decision-making competitive situation where there are two or more opposing parties with conflicting interests and where the action of one depends upon the opponent, for example, candidates for an election, advertising and marketing campaigns by competing business firms, countries involved in military battles, etc, have their conflicting interests. On a competitive situation the courses of action (alternatives) for each competitor may be either finite or infinite. A competitive situation will be called a “Game” if it has the following properties:

- There are a finite number of competitors (participants) called players.
- Each player has a finite number of strategies (alternatives) available to him.
- A play of the game takes place when each player employs his strategy.
- Every game results in an outcome e.g., loss or gain or a draw, usually called pay off, to some players.

Dantzig’s [1] suggestion is to choose that entering vector corresponding to which \( z_j - c_j \) is most negative. Khobragade’s [4] suggestion is to choose that entering vector corresponding to which \( \frac{(z_j - c_j)\theta_j}{c_j} \) is most negative. It is shown that if we choose the vector \( y_j \) such that

\[
\frac{(z_j - c_j)}{c_j} \sum y_{ij}, \quad (c_j > 0, \quad y_{ij} \geq 0)
\]

is most negative, then the iterations required are fewer in some problems. This has been illustrated by giving the solution of a problem. We also show that either the iterations required are same or less but iterations required are never more than that of the simplex method.

II. GENERAL SOLUTION OF M X N RECTANGULAR GAMES

In a rectangular game with an \( m \times n \) pay off matrix, if there does not exist any saddle point and also it is not possible to reduce the size of the game to \( m \times 2 \) or \( 2 \times n \) pay off matrix by using the concept of dominance, the following methods are generally used to solve the game [3].

a) Linear Programming Method, and
b) Interactive method for approximate solution.

Linear Programming Method [3]

To illustrate the connection between a game problem and a liner programming problem, let us consider an \( m \times n \) pay off matrix, if

\[
S_m = \begin{bmatrix} A_1, & \cdots & A_m \\ P_1, & \cdots & P_m \end{bmatrix} \quad \text{and} \quad \sum_{i=1}^{m} p_i = \sum_{j=1}^{n} q_j = 1
\]

be the mixed strategies for the two players respectively.

Then, the expected gain \( g_j \) (\( j = 1, \ldots, n \)) of player A against B’s pure strategies will be

\[
g_1 = a_{11} p_1 + a_{12} p_2 + \cdots + a_{1m} p_m
\]

\[
g_2 = a_{21} p_1 + a_{22} p_2 + \cdots + a_{2n} p_m
\]

and the expected losses \( l_i \) (\( i = 1, \ldots, m \)) of player B against A’s pure strategies will be

\[
l_1 = a_{11} q_1 + a_{12} q_2 + \cdots + a_{1n} q_n
\]

\[
l_2 = a_{21} q_1 + a_{22} q_2 + \cdots + a_{2n} q_n
\]

The objective of player A is to select \( p_i \) (\( i=1,2,\ldots, m \)) such that he can maximize his minimum expected gains; and
the player B desires to select $q_j$ ($j = 1, 2, \ldots, n$) that will minimize his expected losses. Thus, if we let
\begin{equation*}
u = \min_j \sum_{i=1}^{M} a_{ij} p_i, \quad (j = 1, 2, \ldots, n)
\end{equation*}
And
\begin{equation*}
v = \max_i \sum_{j=1}^{M} a_{ij} q_j, \quad (i = 1, 2, \ldots, m);
\end{equation*}
The problem of two players could be written as:

**Player A**

Maximize $u = \min \frac{1}{u} = \sum_{i=1}^{m} \frac{p_i}{u}$

subject to the constraints:

\begin{equation*}
\sum_{i=1}^{m} a_{ij} p_i \geq u, \quad \sum_{j=1}^{n} p_i = 1, \quad p_i \geq 0 \quad (i = 1, 2, \ldots, m)
\end{equation*}

**Player B**

Minimize $v = \max \frac{1}{v} = \sum_{j=1}^{n} \frac{q_j}{v}$

subject to the constraints:

\begin{equation*}
\sum_{j=1}^{n} a_{ij} q_j \leq v, \quad \sum_{j=1}^{n} q_j = 1, \quad q_j \geq 0 \quad (j = 1, 2, \ldots, n).
\end{equation*}

Assuming that $u > 0$ and $v > 0$, we introduce new variables defined by,

$$p'_i = \frac{p_i}{u} \quad \text{and} \quad q'_j = \frac{q_j}{v} \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n).$$

Then, the pair of linear programming problems can be re-written as:

**Player A**

Minimize $p_0 = p'_1 + p'_2 + \cdots + p'_m$

subject to the constraint:

$$a_{ij} p'_1 + a_{ij} p'_2 + \cdots + a_{ij} p'_m \geq 1 \quad p'_i \geq 0 \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$$

**Player B**

Maximize $q_0 = q'_1 + q'_2 + \cdots + q'_n$

subject to constraints:

$$a_{ij} q'_1 + a_{ij} q'_2 + \cdots + a_{ij} q'_n \leq 1 \quad q'_j \geq 0 \quad (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$$

Remarks: 1

It is easy to note that the L P P’ s of the two players represent a primal – dual pair. Therefore, by fundamental theorem of duality one can read off the optimal solution of one player, just from the optimum simplex table of the opponent. That is, we need to solve just one players L.P.P by simplex method.

Liner programming technique requires all variables to be non-negative and therefore, to obtain a non-negative value $v$ of the game; the data to the problem, i.e. $a_{ij}$ in the payoff table should all be non – negative. If there are some negative elements in the pay off table, a constant to every element in the pay off table must be added so as to make the smallest element zero; the solution to this new game will give an optimal mixed strategy for the original game. The value of the original game then equals the value of the new game minus the constant.

**III. STATEMENT OF THE PROBLEM**

Solve the following game by liner programming technique:

Player B

$$\begin{bmatrix} 1 & -1 & 3 \end{bmatrix}$$

Player A

$$\begin{bmatrix} 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}$$

**III. SOLUTION OF THE PROBLEM**

Since some of the entries in the pay off matrix are negative, we add a suitable constant to such of the entries to ensure them all positive. Thus, adding a constant $c = 4$ to each element, we get the following revised pay off matrix:

Player B

$$\begin{bmatrix} 5 & 3 & 7 \\ 10 & 6 & 2 \end{bmatrix}$$

Player A

$$\begin{bmatrix} A_1 & A_2 & A_3 \\ p_1 & p_2 & p_3 \end{bmatrix}, \quad \text{S_B} = \begin{bmatrix} B_1 & B_2 & B_3 \\ q_1 & q_2 & q_3 \end{bmatrix},$$

where $p_1 + p_2 + p_3 = 1$ and $q_1 + q_2 + q_3 = 1$.

The linear programming formulation for the two players problems are:

For Player A

Maximize $v = \min \frac{1}{v} = x_1 + x_2 + x_3$

Subject to the constraints:

$$5x_1 + 7x_2 + 10x_3 \geq 1,$$

$$3x_1 + 9x_2 + 6x_3 \geq 1,$$

$$7x_1 + x_2 + 2x_3 \geq ;$$

$$x_j \geq 0 \quad (j = 1, 2, 3)$$

For Player B
Minimize \( v = \text{Maximize} \frac{1}{v} = y_1 + y_2 + y_3 \)

Subject to the constraints;
\[
5y_1 + 3y_2 + 7y_3 \leq 1 \quad \text{and} \quad y_j \geq 0 \quad (j = 1, 2, 3)
\]

Where \( x_j = p/u \ (j = 1, 2, 3) \) and \( y_j = q_j / v \), \((j=1,2,3)\);
\( u \) = minimum expected gain to A and 
\( v \) = minimum expected loss to B.

Let us now solve the problem for player B, by introducing slack variables \( x_4 \geq 0, x_3 \geq 0 \) and \( x_6 \geq 0 \); 
We can write problem as follows:
Max \( Z = x_4 + x_3 + x_6 \)
Subject to the constraints
\[
5x_4 + 3x_2 + 7x_3 + x_4 = 1 \quad \text{and} \quad 7x_4 + 9x_2 + x_3 + x_5 = 1 \quad \text{and} \quad 10x_4 + 6x_2 + 2x_3 + x_6 = 1
\]

**Initial Iteration:**

<table>
<thead>
<tr>
<th>Basis</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_B</td>
<td>y_4</td>
<td>y_1</td>
<td>y_2</td>
<td>y_3</td>
<td>y_4</td>
<td>y_5</td>
<td>y_6</td>
</tr>
<tr>
<td>y_4</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y_5</td>
<td>0</td>
<td>7</td>
<td>9</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>y_6</td>
<td>0</td>
<td>10</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Here we are applying our method;

Choose most – ve of \( \left( \frac{Z_j - C_j}{C_j} \right) \sum y_j \) .

Most – ve is \( \left( \frac{-1}{12}, \frac{-1}{18}, \frac{-1}{10} \right) = \left( \frac{-1}{10} \right) \).

Here the entering vector is \( y_3 \)

**2nd interaction:**

<table>
<thead>
<tr>
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<th>1</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_B</td>
<td>y_3</td>
<td>y_1</td>
<td>y_2</td>
<td>y_3</td>
<td>y_4</td>
<td>y_5</td>
<td>y_6</td>
</tr>
<tr>
<td>y_3</td>
<td>1</td>
<td>5/7</td>
<td>3/7</td>
<td>1</td>
<td>1/7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y_5</td>
<td>0</td>
<td>44/7</td>
<td>60/7</td>
<td>0</td>
<td>-1/7</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>y_6</td>
<td>0</td>
<td>60/8</td>
<td>36/7</td>
<td>0</td>
<td>2/7</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**3rd Iteration:**

Choose most – ve of \( \left( \frac{Z_j - C_j}{C_j} \right) \sum y_j \) .

i.e. most – ve of \( \left( \frac{-1}{203}, \frac{-4}{99} \right) \) is \( \frac{-4}{99} \)

Since all \( Z_j - C_j \geq 0 \), an optimum solution has been reached.

The expected value or game, therefore is obtained as
\( \nu = v - 4 = 5 - 4 = 1 \)

Optimum strategies for player B are,
\( q_1^0 = 0, q_2^0 = 0, q_3^0 = \frac{1}{10} x 5 = \frac{1}{2} \) and \( q_4^0 = \frac{1}{10} x 5 = \frac{1}{2} \)

Making use of duality, the optimum strategies for player A are obtained as,
\( p_1^0 = \frac{2}{15} x 5 = \frac{2}{3} \) and \( p_2^0 = \frac{1}{15} x 5 = \frac{1}{3} \) and \( p_3^0 = 0 \)

Hence, optimum solution to the given problem is

\[
S_A = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix} \quad \text{and} \quad S_B = \begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix}
\]

**IV. STATEMENT OF THE PROBLEM - II**

Solve the following 3 x 3 game:

Player B

\[
\begin{bmatrix}
3 & -2 & 4 \\
1 & 4 & 2 \\
2 & 2 & 6
\end{bmatrix}
\]

**V. SOLUTION OF THE PROBLEM**

Since some of the entries in the play off matrix are negative, we add a suitable constant to such of the entries to ensure them all positive.
Thus, adding a constant C= 4 to each element, we get the following revised pay off matrix:

Player B

\[
\begin{bmatrix}
6 & 1 & 7 \\
2 & 7 & 5 \\
5 & 5 & 9
\end{bmatrix}
\]

Player A

Let the strategies of the two players be,

\[
S_A = \begin{bmatrix}
A_1 & A_2 & A_3 \\
P_2 & P_2 & P_3
\end{bmatrix}, \quad S_B = \begin{bmatrix}
B_1 & B_2 & B_3 \\
q_1 & q_2 & q_3
\end{bmatrix}
\]

Where,

\[p_1 + p_2 + p_3 = 1 \quad \text{and} \quad q_1 + q_2 + q_3 = 1.\]

The linear programming formulation for the two players problems are:

For player A

Maximise \( \frac{1}{v} = x_1 + x_2 + x_3 \)

Subject to the constraints:

\[
\begin{align*}
6x_1 + 2x_2 + 5x_3 & \geq 1 \\
x_1 + 7x_2 + 5x_3 & \geq 1 \\
7x_1 + 5x_2 + 9x_3 & \geq 1 \\
\text{and } x_j & \geq 0 \quad (j = 1,2,3)
\end{align*}
\]

For player B

Minimise \( \frac{1}{v} = y_1 + y_2 + y_3 \)

Subject to the constraints:

\[
\begin{align*}
6y_1 + 2y_2 + 7y_3 & \leq 1 \\
2y_1 + 7y_2 + 5y_3 & \leq 1 \\
5y_1 + 5y_2 + 9y_3 & \leq 1 \\
\text{and } y_j & \geq 0 \quad (j = 1,2,3)
\end{align*}
\]

where \(x_j = \frac{p_j}{u} (j=1,2,3)\) and \(y_j = \frac{q_j}{v} (j = 1,2,3)\)

\[u = \text{minimum expected gain to A and } v = \text{minimum expected loss to B} \]

Let us now solve the problem for player B.

By introducing slack variables \(y_4, y_5, y_6, \geq 0\)

we can write problem as follows:

Max \( z = x_1 + x_2 + x_3 \)

Subject to the constraints:

\[
\begin{align*}
6x_1 + 2x_2 + 7x_3 + x_4 & = 1 \\
2x_1 + 7x_2 + 5x_3 + x_4 & = 1 \\
5x_1 + 5x_2 + 9x_3 + x_4 & = 1 \\
x_j & \geq 0
\end{align*}
\]

Initial iteration:

<table>
<thead>
<tr>
<th>(C_B)</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>Ratio (1)</th>
<th>Ratio (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_4)</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(y_5)</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(y_6)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(Z_j-C_j)</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Choose Most - ve of \(- \frac{1}{13}, -\frac{1}{13}, -\frac{1}{21}\)

1st Iteration [5]:

<table>
<thead>
<tr>
<th>Basis</th>
<th>(C_B)</th>
<th>(y_1)</th>
<th>(y_2)</th>
<th>(y_3)</th>
<th>(y_4)</th>
<th>(y_5)</th>
<th>(y_6)</th>
<th>(X_B)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_4)</td>
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<td>40/7</td>
<td>0</td>
<td>44/7</td>
<td>1</td>
<td>-1/7</td>
<td>0</td>
<td>6/7</td>
<td>0.15</td>
</tr>
<tr>
<td>(y_5)</td>
<td>1</td>
<td>2/7</td>
<td>1</td>
<td>5/7</td>
<td>0</td>
<td>1/7</td>
<td>0</td>
<td>1/7</td>
<td>0.5</td>
</tr>
<tr>
<td>(y_6)</td>
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<td>25/7</td>
<td>0</td>
<td>38/7</td>
<td>0</td>
<td>-5/7</td>
<td>1</td>
<td>2/7</td>
<td>0.08</td>
</tr>
<tr>
<td>(Z_j-C_j)</td>
<td>-5/7</td>
<td>0</td>
<td>-2/7</td>
<td>0</td>
<td>1/7</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Most negative of \((-5, -\frac{2}{67}, -\frac{2}{87}) = -\frac{5}{67}\)

2nd Iteration:

<table>
<thead>
<tr>
<th>Basis</th>
<th>(C_B)</th>
<th>(y_1)</th>
<th>(y_2)</th>
<th>(y_3)</th>
<th>(y_4)</th>
<th>(y_5)</th>
<th>(y_6)</th>
<th>(X_B)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>0</td>
<td>-84/35</td>
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<td>1</td>
<td>-8/5</td>
<td>14/35</td>
</tr>
<tr>
<td>(y_5)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>7/25</td>
<td>0</td>
<td>1/5</td>
<td>-2/25</td>
<td>3/25</td>
</tr>
<tr>
<td>(y_6)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>38/25</td>
<td>0</td>
<td>-1/5</td>
<td>7/25</td>
<td>2/25</td>
</tr>
<tr>
<td>(Z_j-C_j)</td>
<td>0</td>
<td>0</td>
<td>4/5</td>
<td>0</td>
<td>0</td>
<td>1/5</td>
<td></td>
<td>1/5</td>
</tr>
</tbody>
</table>

Since all \(z_j - c_j \geq 0\), an optimum solution has been reached.

The expected value of game, therefore is obtained as \(v^0 = 5 - 3 = 2\)

Optimum strategies for player B are,

\[q_1^0 = \frac{2}{25} \times 5 = 2/5 \]
\[q_2^0 = \frac{3}{35} \times 5 = 3/5 \]
\[q_3^0 = 0 \]

Making use of duality, the optimum strategies for player A are obtained as,

\[p_1^0 = 0 \times 5 = 0, \]
\[p_2^0 = 0 \times 5 = 0, \]
\[p_3^0 = \frac{1}{5} \times 5 = 1 \]

Hence optimum solution to the given problem is
S\_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 0 & 0 & 1 \end{bmatrix}, \quad S\_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ 2 & 3 & 5 \\ 5 & 5 & 0 \end{bmatrix}
\qquad \text{and}
\nu = 2

VI. STATEMENT OF THE PROBLEM

In a town there are only two discount stores ABC and XYZ. Both stores run annual pre–diwali sales during the first week of October. Sales are advertised through local newspapers with the aid of an advertising firm. ABC stores constructed following pay off in units of Rs. 1,00,000. Find the optimum strategies for both stores and the value of the game.

Strategies of XYZ

\[ \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & -2 \\ -1 & 2 & 3 \end{bmatrix} \]

Strategies of ABC

\[ \begin{bmatrix} 4 & 1 & 4 \\ 2 & 6 & 1 \end{bmatrix} \]

VII. SOLUTION OF THE PROBLEM

Since some of the entries in the pay off matrix are negative, we add a suitable constant to such of the entries to ensure them all positive. Thus, adding a constant \( c = 3 \) to each element, we get the following revised pay off matrix:

\[ \begin{bmatrix} 4 & 1 & 4 \\ 2 & 6 & 1 \end{bmatrix} \]

Let the strategies of the two stores be,

\[ S\_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ p_1 & p_2 & p_3 \end{bmatrix}, \quad S\_B = \begin{bmatrix} B_1 & B_2 & B_3 \\ q_1 & q_2 & q_3 \end{bmatrix} \]

where, \( p_1 + p_2 + p_3 = 1 \) and \( q_1 + q_2 + q_3 = 1 \).

The linear programming formulation for the ‘two stores’– problems are:

For stores ABC

Maximize \( \nu = \text{Minimize} \ \frac{1}{\nu} = x_1 + x_2 + x_3 \)

Subject to the constraints

\[ \begin{align*}
4x_1 + 2x_2 + 2x_3 & \geq 1 \\
x_2 + 6x_2 + x_3 & \geq 1 \\
4x_1 + x_2 + 6x_3 & \geq 1 \\
x_j & \geq 0 \quad (j = 1, 2, 3)
\end{align*} \]

For store XYZ

Minimize \( \nu = \text{Maximize} \ \frac{1}{\nu} = y_1 + y_2 + y_3 \)

Subject to the constraints,

\[ \begin{align*}
4y_1 + y_2 + 4y_3 & \leq 1 \\
2y_1 + 6y_2 + y_3 & \leq 1 \\
2y_1 + y_2 + 6y_3 & \leq 1 \\
y_j & \geq 0 \quad (j = 1, 2, 3)
\end{align*} \]

Where \( x_i = p_i/u \) and \( y_j = q_j/v \)

\( u = \text{minimum expected gain to A} \)
\( v = \text{minimum expected loss to B} \)

Let us now solve the problem for store XYZ.

By introducing slack variable \( x_4 \geq 0, x_5 \geq 0, x_6 \geq 0 \)

we can write problem as follows:

Max \( z = x_1 + x_2 + x_3 \)

Subject to the constraint

\[ \begin{align*}
x_1 + x_2 + 4x_3 + x_4 & = 1 \\
2x_1 + 6x_2 + x_3 + x_5 & = 1 \\
2x_1 + x_2 + 6x_6 & = x_6 + 1
\end{align*} \]

Initial Iteration:

<table>
<thead>
<tr>
<th>Basis</th>
<th>C_B</th>
<th>y_1</th>
<th>y_2</th>
<th>y_3</th>
<th>y_4</th>
<th>y_5</th>
<th>y_6</th>
<th>X_B</th>
<th>Ratio (1)</th>
<th>Ratio (2)</th>
</tr>
</thead>
<tbody>
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Choose Most – ve of \( \left( -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{11} \right) = -\frac{1}{8} \)

1st Iteration:

<table>
<thead>
<tr>
<th>Basis</th>
<th>C_B</th>
<th>Y_1</th>
<th>Y_2</th>
<th>Y_3</th>
<th>Y_4</th>
<th>Y_5</th>
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<th>X_B</th>
<th>Ratio</th>
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<tr>
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Choose Most – ve of \( \left( -\frac{2}{17}, -\frac{5}{17} \right) = -\frac{2}{17} \)

II\textsuperscript{nd} Iteration:

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<th>Y_6</th>
<th>X_B</th>
<th>Ratio</th>
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<td>0.11</td>
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<th>X_B</th>
<th>Ratio</th>
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<td>-ve</td>
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</tbody>
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<table>
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<th>C_B</th>
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<th>Y_2</th>
<th>Y_3</th>
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<th>X_B</th>
<th>Ratio</th>
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<tr>
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</table>

III\textsuperscript{rd} Iteration:
Basis

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<th>$y_3$</th>
<th>$y_4$</th>
<th>$y_5$</th>
<th>$y_6$</th>
<th>$x_B$</th>
</tr>
</thead>
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<td>11/45</td>
<td>1/9</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3/18</td>
<td>6/45</td>
<td>1/30</td>
<td>1/3</td>
</tr>
</tbody>
</table>

Since all $z_j - c_j \geq 0$, an optimum solution has been reached.

The expected value or game, therefore is obtained as

$$v^0 = 3 - 3 = 0$$

Optimum strategies for store XYZ are

$$q_1^0 = \frac{1}{9} \times 3 = \frac{1}{3}$$
$$q_2^0 = \frac{1}{9} \times 3 = \frac{1}{3}$$
$$q_3^0 = \frac{1}{9} \times 3 = \frac{1}{3}$$

Making use of duality, the optimum strategies for store ABC are obtained as,

$$p_1^0 = \frac{3}{18} \times 3 = \frac{1}{2}$$
$$p_2^0 = \frac{6}{45} \times 3 = \frac{2}{5}$$
$$p_3^0 = \frac{1}{30} \times 3 = \frac{1}{10}$$

Hence the optimum solution to the given problem is

$$S_A = \begin{bmatrix} A_1 & A_2 & A_3 \\ 1/2 & 2/5 & 1/10 \end{bmatrix},$$
$$S_B = \begin{bmatrix} B_1 & B_2 & B_2 \\ 2/3 & 1/3 & 1/3 \end{bmatrix}$$
and $v = 0$.

VIII. CONCLUSION

Game problems are successfully solved using new method. It has been observed that the number of iterations are either less or remained the same when compared to the solution using simplex method. The number of iteration does not increase in any of the cases tried.

REFERENCES


AUTHOR BIOGRAPHY

Dr. N.W. Khobragade for being M.Sc in statistics and Maths he attained Ph.D. He has been teaching since 1986 for 27 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems and its application. Published more than180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and four students submitted their thesis in University for award of Ph.D Degree under their guidance.