Thermoelastic Analysis of a Solid Circular Cylinder

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I. INTRODUCTION

Nowacki [5] has determined study state thermal stress in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Wankhede [9] has determined the quasi – static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there are not many investigations on transient state. Roy Choudhuri [8] has succeeded in determining the quasi – static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In the recent work, some problems have solved by Noda [6] & Deshmukh [1]. Recently Nasser [3,4] proposed the concept of heat sources in generalized thermo elasticity and applied to a thick plate problem he has not however considered any thermo elastic problems with boundary conditions of radiation type in which sources are generated according to the linear function of the temperature which satisfies the time dependent heat conduction equation. Kamdi et al [10] solved transient thermo elastic problem for solid circular cylinder.

This paper is concerned with the transient thermo elastic problem in a circular solid cylinder in which sources are generated according to the linear function of temperature occupying the space $D = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq (x^2 + y^2)^{1/2} \leq a, \quad -h \leq z \leq h\}$ where $r = (x^2 + y^2)^{1/2}$ with radiation type boundary conditions by applying integral transform techniques. The results are obtained in terms of Bessel’s functions in the form of infinite series. Numerical results are carried out for a particular case of a circular solid cylinder made of Allumineum metal and the results depicted graphically.

Key Words: Transient Response, Solid Cylinder, Temperature Distribution, Thermal Stresses, Integral Transform.

II. STATEMENT OF THE PROBLEM

Consider a circular solid cylinder in which sources are generated according to the linear function of temperature occupying the space $D = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq (x^2 + y^2)^{1/2} \leq a, \quad -h \leq z \leq h\}$ where $r = (x^2 + y^2)^{1/2}$. The material is isotropic homogeneous and all properties are assumed to be constant. Heat conduction with internal heat source and the prescribed boundary conditions of the radiation type are considered. The equation for heat conduction [6] is

$$\frac{k}{r} \left( \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right) + \Theta(r, z, t, \theta) = \frac{\partial T}{\partial t} \quad \text{(1)}$$

where \( \Theta(r, z, t, \theta) \) is the source function and $k = \frac{\lambda}{\rho C}$.

\( \lambda \) being the thermal conductivity of the material, \( \rho \) is the density and \( C \) is the calorific capacity, assumed to be constant. We consider the under given functions as the superposition of the simpler function [10]:

$$\Theta(r, z, t, \theta) = \phi(r, z, t) + \psi(t) \theta(r, z, t) \quad \text{(2)}$$

and

$$T(r, z, t) = \theta(r, z, t) \exp \left[ -\frac{t}{\psi(\zeta)} d\zeta \right] \quad \text{(3)}$$

For the sake of brevity, we consider

$$\chi(r, z, t) = \frac{\delta(r - r_0) \nu(z) u(t)}{2 \pi r_0} , \quad 0 \leq r_0 \leq a$$

Where \( \nu(z) \), \( u(t) \) are arbitrary functions.

Substituting equations (2) and (3) in the heat conduction equation (1), one obtains

$$k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + \chi(r, z, t) = \frac{\partial T}{\partial t} \quad \text{(4)}$$

where $k$ is the thermal diffusivity of the material of the cylinder (which is assumed to be constant). Subject to the initial and boundary condition

$$M_i(T, 1, 0, 0) = 0, \quad \text{for all } 0 \leq r \leq a, -h \leq z \leq h \quad \text{(5)}$$
$M_x(T, 1, 0, a) = 0$, for all $-h \leq z \leq h$, $t > 0$ (6)

$M_z(T, 1, k_1, h) = \left(-\frac{Q_0}{\lambda}\right) \delta(r - r_0) u(t)$ (7)

$M_z(T, 1, k_2, -h) = 0$, for all $0 \leq r \leq a$, $t > 0$ (8)

The most general expression for these conditions can be given by

$M_v(f, k, \bar{k}, \bar{k}, s) = (\bar{k} f + \bar{\kappa} f)_{v=s}$

where the prime ($\lambda$) denotes differentiation with respect to $v$; $\delta(r - r_0)$, is the Dirac Delta function having

$0 \leq r_0 \leq a: \left(-\frac{Q_0}{\lambda}\right) u(t) \delta(r - r_0)$ is the additional sectional heat available on its surface at $z = h$; $\bar{k}$ and $\bar{\kappa}$ are radiation constants on the upper and lower surface of cylinder respectively. The Navier’s equations without the body forces for axisymmetric two dimensional thermo elastic problem can be expressed as [6]

$\nabla^2 u_r - \frac{u_r}{r^2} + \frac{1}{1 - 2\nu} \frac{\partial e}{\partial r} - \frac{2(1 + \nu)}{1 - 2\nu} \alpha_i \frac{\partial \theta}{\partial r} = 0$ (9)

$\nabla^2 u_z - \frac{1}{1 - 2\nu} \frac{\partial e}{\partial z} - \frac{2(1 + \nu)}{1 - 2\nu} \alpha_i \frac{\partial \theta}{\partial z} = 0$ (10)

where $u_r$ and $u_z$ are the displacement components in the radial and axial directions respectively and the dilation $e$ as [6]

$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}$ (11)

The displacement functions in the cylindrical coordinate system are represented by the Goodier’s thermo elastic displacement potential $\phi(r, z, t)$ and Love’s function $L$ as [6]

$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z}$ (12)

$u_z = \frac{\partial \phi}{\partial z} + 2(1 - \nu) \nabla^2 L - \frac{\partial^2 L}{\partial z^2}$ (13)

in which Goodier’s thermo elastic potential must satisfy the equation [6]

$\nabla^2 \phi = \left(1 + \nu\right) \alpha_i \theta$ (14)

and the Love’s function $L$ must satisfy the equation

$\nabla^2 \left(\nabla^2 L\right) = 0$ (15)

Where

$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$

The component of the stresses are represented by the use of the potential $\phi$ and Love’s function $L$ as [6]

$\sigma_{rr} = 2G \left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z} \left(\nu \nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)$ (16)

$\sigma_{r\theta} = 2G \left(\frac{1}{r} \frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z} \left(\nu \nabla^2 L - \frac{1}{r} \frac{\partial L}{\partial r}\right)$ (17)

$\sigma_{zz} = 2G \left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z} \left(2 - \nu\right) \nabla^2 L - \frac{\partial^2 L}{\partial z^2}$ (18)

$\sigma_{rz} = 2G \left(\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r} \left(1 - \nu\right) \nabla^2 L - \frac{\partial^2 L}{\partial r \partial z}\right)$ (19)

Where $G$ and $\nu$ are the shear modulus and Poisson’s ratio respectively. The boundary conditions on the traction free surface of a solid cylinder are

$\sigma_{rr} = \sigma_{rz} = 0$ at $r = a$

The equations (1) to (19) constitute the mathematical formulation of the problem under consideration.

**III. SOLUTION OF THE PROBLEM**

In order to solve fundamental differential equation (4) under the boundary conditions (6), we first introduce the method of Hankel transform of order $n$ over the variable $r$. Let $n$ be the parameter of transform, then the integral transform and its inversion theorem is written as [7]

$g^*(\xi_n, z, t) = \int_0^a g(r, z, t) r K_0(\xi_n r) \, dr$, (20)

$g(r, z, t) = \sum_{n=1}^\infty g^*(\xi_n, z, t) K_0(\xi_n r)$ (21)

Applying the transformation defined in equation (18) to the equations (4) (5) and (7) and using equation (6), one obtains

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Further applying the transform defined in equation (24) to the equation (19), (20), (23) and using equations (21), (22) one obtains

$$\begin{align*}
k &\left[ -\xi_n^2 \mathcal{T}^* (\xi_n, z, t) + \frac{\partial^2 T^* (\xi_n, z, t)}{\partial t^2} \right] + v(z) u(t) \
&= -v(z) P_m(z) u(t) K_0 (\xi_n, r_0) - \frac{d\mathcal{T}^* (\xi_n, m, t)}{dt} \end{align*}$$

(30)

$$M_f (\mathcal{T}^*, 1, 0, 0) = 0$$

(31)

Where $\mathcal{T}^*$ is the transformed function of $\mathcal{T}$ and $m$ is the transformed parameter. The symbol ($-$) means a function of the transformed domain and the nucleus is given by the orthogonal functions in the interval $-h \leq z \leq h$ as

$$P_m (z) = Q_m \cos (\mu_m z) - W_m \sin (\mu_m z)$$

In which

$$Q_m = \mu_m (k_1 + k_2) \cos (\mu_m h)$$

$$W_m = 2 \cos (\mu_m h) + (k_2 - k_1) \mu_m \sin (\mu_m h)$$

$$\lambda_m^2 = \int_{-h}^{h} P_m^2 (z) dz$$

$$= h [Q_m^2 + W_m^2 + \sin (2\mu_m h)] [Q_m^2 - W_m^2]$$

The eigen values $\mu_m$ are the positive roots of the characteristic equation

$$[k_1 a \cos (ah) + \sin (ah)] [\cos (ah) + k_2 a \sin (ah)] = [k_2 a \cos (ah) - \sin (ah)] [\cos (ah) - k_1 a \sin (ah)]$$

After performing calculations on the equation (25), the reduction is made to linear first order differential equation as

$$\frac{d\mathcal{T}^* (\xi_n, m, t)}{dt} + k \Lambda_{m,n} \mathcal{T}^* = \Omega (\xi_n, \mu_m)$$

(32)

where

$$\Lambda_{m,n} = \mu_m^2 + \xi_n^2$$

(33)

and

$$\Omega (\xi_n, \mu_m) = \left[ \frac{P_m (h)}{k_1} - \frac{P_m (-h)}{k_2} \right] \left[ \frac{Q_o}{\lambda} \right] u(t) r_0 K_0 (\xi_n, r_0)$$

$$+ \frac{v(z) P_m (z)}{2\pi} u(t) K_0 (\xi_n, r_0)$$

(34)

The transformed temperature solution of differential equation (27) is
\[
\bar{T}^* = \frac{\Pi(m, n)}{(k \Lambda_{m,n})} [u(t) - e^{-(k \Lambda_{m,n} t)}]
\]

(35)

where

\[
\Pi(m, n) = \frac{v(z) P_n(z)}{2\pi} K_0(\xi_n, r_0)
\]

(36)

Applying the inversion theorems of transformation rules defined in (18) and (24) one obtains

\[
T(r, z, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \Psi_{n,m} [u(t) - \exp(-k \Lambda_{m,n} t)]
\]

\[
\times P_m(z) K_0(\xi_n, r)
\]

(37)

where

\[
\Psi_{n,m} = \frac{\Omega(m, n)}{(k \Lambda_{m,n} t) \lambda_m}
\]

(38)

Taking into account of first equation (3), the temperature distribution is finally represented by

\[
\theta(r, z, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \Psi_{n,m} [u(t) - \exp(-k \Lambda_{m,n} t)]
\]

\[
\times P_m(z) K_0(\xi_n, r) \exp \left[ \int_{0}^{t} \psi(\zeta) \ d\zeta \right]
\]

(39)

The equation (38) represents the temperature at any instant and at all points of a circular cylinder when there are radiation type boundary conditions.

IV. THERMOELASTIC DISPLACEMENT

Referring to the fundamental equation (1) and its solution (34) for the heat conduction problem, the solution for the displacement functions are represented by Goodier’s thermo elastic potential \( \phi \) governed by the equation (11) as

\[
\phi(r, z, t) = \left( \frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\Psi_{n,m}}{\Lambda_{m,n}}
\]

\[
\times [u(t) - \exp(-k \Lambda_{m,n} t)]
\]

\[
\times P_m(z) K_0(\xi_n, r) \exp \left[ \int_{0}^{t} \psi(\zeta) \ d\zeta \right]
\]

(40)

Similarly the solutions for Love’s function \( L \) are assumed so as to satisfy the govern condition of equation (12) as

\[
L(r, z, t) = \left( \frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\Psi_{n,m}}{\Lambda_{m,n}}
\]

\[
\times [u(t) - \exp(-k \Lambda_{m,n} t)]
\]

\[
\times [A_n J_0(\xi_n r) + C_n J_1(\xi_n r)]
\]

\[
\times [\cos h(\xi_n z)] \exp \left[ \int_{0}^{t} \psi(\zeta) \ d\zeta \right]
\]

(41)

Using (35) and (36) in (9) and (10) one obtains

\[
u_r = \left( \frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\Psi_{n,m}}{\Lambda_{m,n}}
\]

\[
\times [u(t) - \exp(-k \Lambda_{m,n} t)]
\]

\[
\times \left[ \xi_n \sin h(\xi_n z) \right] \left[ A_n (-\xi_n J_1(\xi_n r)) + C_n J_0(\xi_n r) J_1(\xi_n r) \right]
\]

\[
\times \left[ \sqrt{\frac{2}{a}} \right] \frac{J_1(\xi_n a) P_m(z)}{J_0(\xi_n a)}
\]

\[
\times \exp \left[ \int_{0}^{t} \psi(\zeta) \ d\zeta \right]
\]

(42)

\[
u_z = \left( \frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\Psi_{n,m}}{\Lambda_{m,n}}
\]

\[
\times [u(t) - \exp(-k \Lambda_{m,n} t)]
\]

\[
[\mu_m [Q_m \sin (\mu_m z) + W_m \cos (\mu_m z)] K_0(\xi_n, r)
\]

\[
+ [A_n \xi_n^2 J_0(\xi_n r)] [\cos h(\xi_n z)]
\]

\[
- C_n \xi_n^2 [4(1-v) J_0(\xi_n r) - (\xi_n r) J_1(\xi_n r)]
\]

\[
\times [\cos h(\xi_n z)] \exp \left[ \int_{0}^{t} \psi(\zeta) \ d\zeta \right]
\]

(43)

Then the stress components can be evaluated by substituting the values of thermo elastic displacement potential \( \phi \) from equation (35) and Love’s function \( L \) from equation (36) in equations (13) (14) (15) and (16), one obtain

\[
\sigma_{rr} = -2G \left( \frac{1+v}{1-v} \right) \alpha_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\Psi_{n,m}}{\Lambda_{m,n}}
\]

\[
\times [u(t) - \exp(-k \Lambda_{m,n} t)]
\]
\[
\begin{align*}
\times [\{ & \mu_n^2 + 2\xi_n^2 \} J_0(\xi_n r) - \xi_n^2 J_1(\xi_n r)] \left[ \frac{\sqrt{2}}{a \xi_r J_1(\xi_r a)} \right] P_m(z) \\
- A_n \xi_n^2 \left[ J_1(\xi_n r) - J_0(\xi_n r) \right] \left[ \xi_n \sin h(\xi_n z) \right] \\
+ C_n \xi_n^2 [(2v - 1) J_0(\xi_n r)] \left[ \xi_n \sin h(\xi_n z) \right] \\
+ (\xi_n r) J_1(\xi_n r)] \left[ \xi_n \sin h(\xi_n z) \right] \times \exp \left[ \int_0^t \psi(\zeta) \, d\zeta \right] \quad (44)
\end{align*}
\]

\[
\sigma_{\theta\theta} = -2G \left( \frac{1 + v}{1 - v} \right) \alpha_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\Psi_{n,m}}{\Lambda_{m,n}} \\
\times [u(t) - \exp (-k \Lambda_{m,n} t)]
\]

\[
\begin{align*}
\Lambda_{m,n} J_0(\xi_n r) + \xi_n J_1(\xi_n r) r \left[ \frac{\sqrt{2}}{a \xi_n J_1(\xi_n a)} \right] P_m(z) \\
+ \left[ A_n \xi_n J_1(\xi_n a) \right] r \left[ \sin h(\xi_n z) \right] \\
+ C_n \xi_n [(2v - 1) J_0(\xi_n r)] \left[ \sin h(\xi_n z) \right] \\
\times \exp \left[ \int_0^t \psi(\zeta) \, d\zeta \right] \quad (45)
\end{align*}
\]

\[
\sigma_{zz} = -2G \left( \frac{1 + v}{1 - v} \right) \alpha_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\Psi_{n,m}}{\Lambda_{m,n}} \\
\times [u(t) - \exp (-k \Lambda_{m,n} t)]
\]

\[
\begin{align*}
\Lambda_{m,n} J_0(\xi_n r) + \xi_n J_1(\xi_n r) r \left[ \frac{\sqrt{2}}{a \xi_n J_1(\xi_n a)} \right] P_m(z) \\
- \left[ 2\mu_m^2 - \xi_n^2 \right] P_m(z) \left( K_0(\xi_n r) \right) \\
- \left[ A_n \xi_n^2 J_0(\xi_n r) \right] \left[ \sin h(\xi_n z) \right]
\end{align*}
\]

\[
\begin{align*}
- C_n \xi_n^2 [(2v - 2) J_0(\xi_n r) - (\xi_n r) J_1(\xi_n r)] \left[ \sin h(\xi_n z) \right] \\
\times \exp \left[ \int_0^t \psi(\zeta) \, d\zeta \right] \quad (46)
\end{align*}
\]

\[
\sigma_{rz} = -2G \left( \frac{1 + v}{1 - v} \right) \alpha_t \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\Psi_{n,m}}{\Lambda_{m,n}} \\
\times [u(t) - \exp (-k \Lambda_{m,n} t)]
\]

\[
\begin{align*}
\times [-\mu_m^2] \left[ Q_m \sin (\mu_m z) + W_m \cos (\mu_m z) \right] \\
\times A_n \xi_n \left[ J_1(\xi_n r) \right] \left[ \frac{\sqrt{2}}{a \xi_n J_1(\xi_n a)} \right] \left[ \cos h(\xi_n z) \right] \\
\times \left[ A_n \xi_n J_1(\xi_n r) \right] \left[ \cos h(\xi_n z) \right] \\
\times \left[ -C_n \xi_n J_0(\xi_n r) + 2(1 - v) \xi_n^2 J_1(\xi_n r) \right] \left[ \cos h(\xi_n z) \right]
\end{align*}
\]

V. DETERMINATION OF UNKNOWN ARBITRARY FUNCTIONS $A_n$ AND $C_n$

Applying boundary conditions (17) to the equations (39) and (42), one obtains

\[
\begin{align*}
A_n &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[ \xi_n^2 \left( B_0 + C_0 \right) \right] \left[ \cos h(\xi_n z) \right] \\
&- D_n E_0 \left( \hat{B} + \hat{C} \right) \left[ \cos h(\xi_n z) \right] P_m(z) \\
&- C_n \xi_n [(2v - 1) J_0(\xi_n r)] \left[ \sin h(\xi_n z) \right] \sin h(\xi_n z) \\
&- \sqrt{2} a^{-1} \hat{D} \left( \xi_n \right) \left[ \cos h(\xi_n z) \right] \\
&- \sqrt{2} a^{-1} \hat{D} \left( \xi_n \right) \left[ \cos h(\xi_n z) \right]
\end{align*}
\]

Where

\[
\begin{align*}
A_0 &= \left[ J_1(\xi_n a) \right] \\
B_0 &= (2v - 1) J_0(\xi_n a) \\
C_0 &= (\xi_n a) J_1(\xi_n a) \\
D_0 &= \mu_m^2 + 2\xi_n^2 - \xi_n \\
E_0 &= \left[ J_0(\xi_n a) \right] \\
&\left[ J_1(\xi_n a) \right] \left[ \frac{\sqrt{2}}{a} \right]
\end{align*}
\]

\[
\hat{A} = J_1(\xi_n a) \\
\hat{B} = \xi_n J_0(\xi_n a) \\
\hat{C} = 2(1 - v) J_1(\xi_n a) \\
\hat{D} = \mu_m^2 [Q_m \sin (\mu_m z) + W_m \cos (\mu_m z)] \left[ \frac{\sqrt{2}}{a} \right]
\]
VI. SPECIAL CASE AND NUMERICAL RESULTS

Set $\psi(\zeta) = -\zeta$, $v(z) = \delta(z - z_0)$.

\[ u(t) = \exp(-\omega t) \quad (48) \]

Using (43) into equation (34) and (37) to (42) one obtain.

\[ \theta(r, z, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \psi_{n,m} \exp(-k \Lambda_{m,n} t) \]
\[ \times P_m(z) K_0(\xi_n r) \exp(-t^2/2) \quad (49) \]

\[ u_r = \left( \frac{1+v}{1-v} \right) \alpha_n \sum_{m=1}^{\infty} \psi_{n,m} \Lambda_{m,n} \]
\[ \times \exp(-k \Lambda_{m,n} t) \quad (50) \]

\[ u_z = \left( \frac{1+v}{1-v} \right) \alpha_n \sum_{m=1}^{\infty} \psi_{n,m} \Lambda_{m,n} \]
\[ \times \exp(-k \Lambda_{m,n} t) \quad (51) \]

\[ \sigma_{\theta\theta} = -2G \left( \frac{1+v}{1-v} \right) \alpha_n \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{n,m} \Lambda_{m,n} \]
\[ \times \exp(-k \Lambda_{m,n} t) \quad (52) \]

\[ \sigma_{zz} = -2G \left( \frac{1+v}{1-v} \right) \alpha_n \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{n,m} \Lambda_{m,n} \]
\[ \times \exp(-k \Lambda_{m,n} t) \quad (53) \]

\[ \sigma_{rz} = -2G \left( \frac{1+v}{1-v} \right) \alpha_n \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{n,m} \Lambda_{m,n} \]
\[ \times \exp(-k \Lambda_{m,n} t) \quad (54) \]

\[ \sigma_{rr} = -2G \left( \frac{1+v}{1-v} \right) \alpha_n \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{n,m} \Lambda_{m,n} \]
\[ \times \exp(-k \Lambda_{m,n} t) \quad (55) \]

VII. CONCLUSION

The temperature distributions, displacement and stress functions at the edge $z = h$ of a circular cylinder in which sources are generated according to the linear function of the temperature have been obtained where the cylinder is subjected to known heat source function $Q_0 \delta(t-t_0) \delta(r-r_0)$. As a particular case
mathematical model is constructed for $\psi(\zeta) = \zeta$ and performed numerical calculations. We develop the analysis for the temperature

- Modules of elasticity, $E$ (dynes/cm$^2$) $6.9 \times 10^{11}$
- Shear modulus, $G$ (dynes/cm$^2$) $2.7 \times 10^{11}$
- Poisson ratio $\nu$ 0.281
- Thermal expansion coefficient, $\alpha_t$ (cm/cm$^0\cdot$C) $25.5 \times 10^{-6}$
- Thermal diffusivity, $k$ (cm$^2$/sec) 0.86
- Thermal conductivity $\lambda$, (Cal/cm$^0\cdot$C/sec/cm$^2$) 0.48
- Outer radius, $a$ (cm) 5
- Height, $h$ (cm) 100

The results obtained in terms of Bessel’s function in the form of infinite series. The expressions are represented graphically. As the radius of the cylinder increases, the temperature distribution, displacement and stresses of the cylinder increases gradually.

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REFERENCES


Fig (1) : Graph of $r$ vs $T(r,z,t)$

Fig (2) : Graph of $r$ vs $\theta(r,z,t)$

Fig (3) : Graph of $r$ vs $u_r$

Fig (4) : Graph of $r$ vs $\sigma_{rr}$

Fig (5) : Graph of $r$ vs $\sigma_{zz}$

AUTHOR BIOGRAPHY

Dr. N.W. Khobragade for being M.Sc in statistics and Maths he attained Ph.D. He has been teaching since 1986 for 27 years at PGTD of Maths, RTM Nagpur University, Nagpur and successfully handled different capacities. At present he is working as Professor. Achieved excellent experiences in Research for 15 years in the area of Boundary value problems and its application. Published more than 180 research papers in reputed journals. Fourteen students awarded Ph.D Degree and four students submitted their thesis in University for award of Ph.D Degree under their guidance.