Rayleigh and Rician Fading Environment for Dual Hop Relayed Transmission

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Abstract- This paper presents the performance analysis of dual-hop relayed transmission over asymmetric Rayleigh and Rician fading environment. In many scenarios the commonly assumed symmetry in relay channels is unrealistic. Closed form expression for Outage Probability and Average Bit Error Probability are derived to study the performance of the system and also illustrate the positive impact of the Rician parameter on the system performance. Present the relation of Rayleigh–rician and rician–relay for both the hops and their ABEP, outage probability.

Keywords-Rayleigh, Rician fading, Amplify and forward, outage probability

I. INTRODUCTION

Recently wireless relaying is very popular because its enabling very effective and efficient information exchange. For many different cases the link experience different fading condition like for some time rician and for some other situation rayleigh. Wireless application in Adhoc network and military applications. Amplify and forward (AF) relaying has now become a well studied protocol under thermal noise-limited conditions, (see e.g. [1]–[3] and references therein). However, in practice, the interference present in relay networks can cause severe performance degradation [4]. Several recent works have studied the performance of AF and decode-and-forward (DF) relaying for interference limited (negligible thermal noise) relay(s) or destination(s), see e.g. [4]–[10]. In [4], the impact of interference on the performance of a dual-hop channel state information (CSI)-assisted AF relay network has been investigated. The authors of [5] have analyzed the performance gains of a half-duplex multi-user network where the relay-destination slot is reused, causing interference at the destination nodes. In [6], [7], the performance of different DF relaying systems in the presence of interference has been investigated. Assuming an interference-limited destination, in [8], the outage probability of a fixed gain AF relay system over Rayleigh fading channels has been investigated. In [9], the performance of a dual-hop CSI-assisted AF system with interference-limited relays has been studied. In [10], the outage probability of dual-hop CSI-assisted AF relaying with interference-limited relays and destinations has been derived. In cellular applications, the case of interference at the relay has practical significance since relays may operate at the cell edge providing extended coverage. Thus, due to the reuse of frequency bands it is likely that relays are affected by line-of-sight (LoS)/non LoS co-channel interference and thermal noise. Nonetheless, the joint effects of interference and thermal noise on the performance of dual-hop fixed gain relay networks has not been investigated in the existing literature. Motivated by the absence of such an analysis, in this paper, we study the outage probability and the average bit error rate (BER) of dual-hop fixed gain AF relaying with interference and thermal noise at the relay. In addition, the case where there exists interference at the destination is also considered, and analyzed separately. Assuming Rayleigh fading for the source-relay and relay-destination channels and Rician fading for the interfering channel, we derive new outage and BER expressions upon statistically characterizing the signal-to-interference plus noise ratio (SINR). The Rician fading assumption on the interference link is motivated by the fact that it is used to model wireless propagation comprising a LoS component and a scattered component. Therefore, the derived analytical expressions quantify the impact of interference on relay system performance for a large set of environments. The analysis of Rician channels is generally more difficult and as a special case, includes the commonly assumed Rayleigh fading. In this work, the outage probability and the ABEP of a dual-hop relay system in an asymmetric fading environment is investigated. We assume that the source-relay and relay destination links experience Rayleigh or Rician fading. First, using the cumulative distribution functions (cdfs) of the end-to-end signal-to-noise ratio (SNR), the outage probability is derived. Next, using the CDF, the ABEP of M-ary square QAM modulation is derived. Simulation results are also presented to verify the theoretical analysis. The rest of the paper is organized as follows: In Section II we outline the system and channel model. In Section III, we derive outage probability expressions for the asymmetric channel.

II. SYSTEM AND CHANNEL MODEL

Assume a dual-hop fixed gain AF relay system (cf. [3, Fig. 1]) operating in an asymmetric fading environment. The source, $S$ communicates with the destination, $D$ using a relay, $R$. Each transmission period is divided into two signaling intervals: In the first signaling interval, the received signal at $R$ is multiplied by a gain factor $G$ and in the second signaling interval, it is retransmitted to $D$. Assuming that $S$ transmits a signal with an average power normalized to unity, the instantaneous end-to-end SNR at destination, $^{\text{SNR}}_{\text{D}}$ [3].
\[ y_{eq} = \left( \frac{\alpha_1^2}{N_01} \right) \left( \frac{\alpha_2^2}{N_02} \right) \]  

(1)

where \( \alpha_1, \alpha_2 \) are the fading amplitudes of the wireless channels in the \( S \rightarrow R \) and \( R \rightarrow D \) links respectively, \( N_01 \) and \( N_02 \) are the power of the AWGN component at the input of the relay and the destination, and \( G \) is the relay gain. If, \( C = 1/G^2 N_01 \), then (1) simplifies to

\[ y_{eq} = \frac{\alpha_1 \alpha_2}{C + \gamma_i} \]  

(2)

In this work, we consider two cases for the fading distributions of the \( S \rightarrow R \) and \( R \rightarrow D \) links, namely:

- The \( S \rightarrow R \) link is subject to Rayleigh fading and the \( R \rightarrow D \) link is subject to Rician fading. In the following, this Rayleigh/Rician fading condition will be identified as scenario (a).
- The \( S \rightarrow R \) link is subject to Rician fading and the \( R \rightarrow D \) link is subject to Rayleigh fading. In the following, this Rician/Rayleigh fading condition will be identified as scenario (b).

If a link experiences Rayleigh fading, \( \gamma_i \), (with \( i = 1 \) or 2), is an exponentially distributed random variable (RV). That is, its probability density function (PDF) is given by

\[ f(\gamma) = \frac{1}{\Omega_i} e^{-\frac{\gamma}{\Omega_i}}, \quad \gamma \geq 0 \]  

(3)

where \( \gamma_i = \Omega_i/N_0i \) and \( \Omega_i \) is the average fading power of that link. If a link experiences Rician fading, the PDF of \( \alpha_i \) is given by

\[ f(\alpha) = \frac{2(K + 1)}{\Omega_i} \exp \left( -K \frac{(K + 1)\alpha^2}{\Omega_i} \right) \frac{\alpha^{K+1} I_0 \left( \frac{K(K + 1)}{\Omega_i} \alpha \right)}{I_0(K)}, \quad \alpha \geq 0 \]  

(4)

where \( K \) is the Rician \( K \)-factor defined as the ratio of the powers of the LoS component to the scattered components and \( I_0( \cdot ) \) is the zeroth order modified Bessel function of the first kind. \( \gamma_i \) is distributed according to a non-central-\( \chi^2 \) distribution given by

\[ P(\gamma | \gamma_i) \alpha^k e^{-\gamma/\gamma_i} \frac{(-1)^{k+1}}{2^k k!} I_0 \left( 2\gamma \sqrt{(k + 1)}/\gamma_i \right) \]  

(5)

Observe that when \( K = 0 \) the Rician distribution becomes the Rayleigh distribution. As \( K \rightarrow \infty \), the distribution approximates that of an AWGN (no fading) channel. Values of the \( K \)-factor in indoor/outdoor land mobile applications normally range from 0 - 12 dB [8].

### III. OUTAGE PROBABILITY

Outage probability is an important performance measure that is commonly used to characterize a wireless communication system. It is defined as the probability that the instantaneous end-to-end SNR, \( \gamma_{eq} \), falls below a threshold \( \gamma_{th} \). Therefore mathematically, the outage probability is given by [3]

\[ P_{out} = f(\gamma_{eq}) \frac{\gamma_{th}}{C + \gamma_i} \]  

where \( f(\gamma_{eq}) \) is the cdf of the end-to-end SNR. Next, we calculate the outage probability applicable to scenarios (a) and (b).

#### A. Scenario (a)

In the case of scenario (a) we can express \( P_{out} \) as, refer appendix also

\[ P_{out} = \int_{\gamma_{th}/\gamma_i}^{\infty} \frac{\gamma_{th}}{\gamma_i} p_{\gamma_i}(\gamma \gamma_i) d\gamma \]  

(6)

#### The cdf of \( \gamma_i \)

and using (5), (7) can be written as

\[ P_{out} = \int_{\gamma_{th}/\gamma_i}^{\infty} \frac{\gamma_{th}}{\gamma_i} \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)!} \right) \frac{\gamma_{th}}{\gamma_i} \]  

(7)

#### The integral required to compute the outage probability in (8) is

\[ I_k = \frac{1}{\gamma_i} \frac{\gamma_{th}}{\gamma_i} \]  

(8)

We are unaware of a closed-form analytical solution to this integral. Nevertheless, using the infinite-series representation of \( I_0(\cdot) \) [14, Eq. (8.447-1)] see appendix ,

\[ I_0(x) = \sum_{j=0}^{\infty} \frac{(-1)^j}{2^{2j} (j!)^2} x^{2j} \]  

(9)

we can rewrite \( I_k \) as

\[ I_k = \left( \sum_{j=0}^{\infty} \frac{k^2(k + 1)^j}{(j + 1)!} \right) \frac{\gamma_{th}}{\gamma_i} \]  

(10)

Observe that when \( \gamma_{th} = 0 \) the Rician distribution becomes the Rayleigh distribution. As \( K \rightarrow \infty \), the distribution approximates that of an AWGN (no fading) channel. Values of the \( K \)-factor in indoor/outdoor land mobile applications normally range from 0 - 12 dB [8].
The integral in (11) can be evaluated using [14, Eq. (3.471-9)]. We write \( I \) as

\[
I = \sum_{n=0}^{\infty} \frac{2^{\nu} \Gamma(\nu+1)}{(\nu+1)!} \Gamma(\nu+2) \frac{(k+1)^{\nu+1}}{y_2^{\nu+2}} \frac{\Gamma(\nu+1)^{\nu+1}}{y_2^{\nu+2}}
\]

(12)

for \( \gamma > 0 \) and \( K \nu(\cdot) \) is the \( \nu \)-th order modified Bessel function of the second kind. Finally, \( \text{Outage} \), for scenario (a) is given

\[
P_{\text{out}} = 1 - e^{-\frac{k}{y_1 T^2}} \sum_{n=0}^{\infty} \frac{2^{\nu} \Gamma(\nu+1)}{(\nu+1)!} K_\nu(\nu+1) \gamma_{th}/y_1^2
\]

(13)

Concerning the convergence of the infinite series in (13), the truncation error if \( T_1 \) terms are used is

\[
P_{\text{out}} = 1 - e^{-\frac{k}{y_1 T^2}} \sum_{n=0}^{T_1} \frac{2^{\nu} \Gamma(\nu+1)}{(\nu+1)!} K_\nu(\nu+1) \gamma_{th}/y_1^2
\]

(14)

For \( \nu > 0 \) and fixed \( x \), \( K \nu(x) \) can be asymptotically approximated as [15]

\[
K_\nu(x) \propto \left(\frac{x}{2}\right)^\nu \Gamma(\nu+1)
\]

(15)

Substituting (15) in (14) and after simplifications we get

\[
R_1 \propto e^{-\frac{k}{y_1 T^2}} \left(1 - \frac{1}{T_1 + 1}\right)^{1/k}
\]

(16)

Finally using [16, Eq. (4.1.7.10)]

\[
R_1 \propto e^{-\frac{k}{y_1 T^2}} (1 - (-T_1 + 1, k))/T_1
\]

(17)

Where \( -x^\alpha \) is the complementary incomplete gamma function [16, p. 792].

**B. Scenario (b)**

In the case of scenario (b) we can express \( \text{Outage} \) as

\[
P_{\text{out}} = \int_0^\infty \int_{y_2}^{\infty} \text{Pr}_1(y_2)dy_1
\]

(18)

and after some manipulations (18) can be reexpressed as using rician fading by some other ways, let \( U = y_1 - y_{th} \) in the above eq. then

\[
I_2 = -e^{-\frac{k}{y_1 T^2}} \int_0^\infty \frac{k^{\nu+1}}{y_2^{\nu+2}} \left(1 + \frac{k}{y_2}ight)^{\nu+1} dy_2
\]

(19)

Using the binomial expansion and the integral result of [14, Eq. (3.471-9)] \( I_2 \) can be solved. Therefore, \( \text{Outage} \) for scenario (b) is expressed as

\[
P_{\text{out}} = 1 - e^{-\frac{k}{y_1 T^2}} \sum_{n=0}^{\infty} \frac{2^{\nu} \Gamma(\nu+1)}{(\nu+1)!} \frac{\Gamma(\nu+1)^{\nu+1}}{y_1^2}
\]

(20)

Similar as scenario (a) with the help of binominal; distribution overall output power determined. For numerical verification, assume that both the \( S \rightarrow R \) and \( R \rightarrow D \) links experience Rayleigh fading, i.e., the scenario considered in [3]. Substituting \( K = 0 \) in (13) and (20) we get[3, Eq. (9)]. In all cases of practical significance, the infinite series representations involved in (20) can be truncated without sacrificing numerical accuracy. Suppose that we truncate (20) after \( T_2 \) terms. Therefore, the remainder, \( R_2 \) after applying (15) becomes a symptomatically

\[
R_2 = e^{-\frac{k}{y_1 T^2}} \sum_{n=0}^{T_2} \frac{1}{\Gamma(1+1)} y_{th} \gamma_{th}/y_1
\]

(21)

And simplifying further

\[
R_2 \propto e^{-\frac{k}{y_1 T^2}} \left(1 + \frac{1}{1+1}\right)^{\nu+1}
\]

(22)

We are unable to obtain a closed-form solution for the summation in (22). However note that, using the L’Hospital rule it can be shown that \( \lim x \rightarrow \infty K\nu(x)!2 = 0 \). Therefore, \( R_2 \) asymptotically converges to a finite number. Figs. 1 and 2 show the outage probability for scenarios (a) and (b) respectively. In all cases, we have employed the fixed gain \( G \) assumed in [4] under the so-called average power scaling, with the simulated results.

**IV. RESULTS**

The plots in Figs. 1 and 2 show similar trends, i.e., with an increasing \( K \)-factor, outage events decrease. However, in all cases, the outage probability of Rician /Rayleigh fading is lower than Rayleigh/Rician fading. Better channel conditions in the first hop (Rician fading) leads to an outage performance improvement because the end-to-end SNR probabilistically reaches higher values more often than in the first hop Rayleigh fading scenario.

![Fig.1 Outage probability in Rayleigh/Rician fading](image-url)
Fig. 2. Outage probability in Rician/Rayleigh fading (Outage probability Vs average SNR in db per hop)

We can use Matlab, maple, mathematical software’s for simulation purposes, in this particular paper we used Matlab software for simulating. For future purpose we can also simulate the ABEP ,average bit error probability and compare the results which will show that the rician fading gives better performance in source to relay link compare to Rayleigh fading and outage probability also less in rician. The system performance can be evaluated by other fading channels also.

V. CONCLUSION
In this paper, we have presented two infinite-series representations for the outage probability of a dual-hop communication system equipped with a single fixed-gain amplify and-forward relay in Rayleigh/Rician and Rician/Rayleigh fading environments respectively. These results demonstrate that the system exhibits an improved performance in a Rician/Rayleigh (source-relay/destination link) environment compared to a Rayleigh/Rician environment. Furthermore, both Rayleigh/Rician and Rician/Rayleigh asymmetric fading environments, and that means the rician fading channel in source- relay link is much efficient than the Rayleigh fading channels because it gives better channel conditions. Since it has been found in practice that different links of a relay system could experience line-of-sight/non line-of-sight fading conditions, this analysis is useful to the system design engineer for performance evaluation purposes.

VI. APPENDIX
In typical one defines an “outage” to occur if the signal-to-noise-plus-interference ratio drops below a certain minimum threshold value z. For instance for experiments and measurements revealed that a C/I of at least 9.5 dB is required. At lower C/I ratios, excessive burst errors occurs, synchronization acquisition becomes problematic, equalizer convergence is hampered, etc. We denote the threshold ratio az. The probability that the wanted signal power p0 sufficiently exceeds the joint interference plus noise power pt is

\[ P_e(p_0 > z, p_t) = \frac{1}{2} \int_{p_t}^{\infty} \left( \frac{-1}{z} \right) f_2(x) dx \]

Where we insert the appropriate pdf of received wanted signal power \( f_2(x) \) and interfering signal power \( f_3(x) \). These pdfs can for instance be derived from Rayleigh, rician distribution. The joint interference signal pt is the sum of the powers of each individual interfering signals. For independent fading and independently modulated signals, the pdf of the joint interference power is the convolution of the pdf of individual interference powers. In the special case of a Rayleigh-fading wanted signal, the integral over y can be solved analytically: Inserting the exponentially distribution of wanted signal power, we get

\[ P_e(p_0 > z, p_t) = \frac{1}{2} \int_{p_t}^{\infty} \left( \frac{-1}{z} \right) f_3(x) dx \]

An elegant mathematical framework has been developed by interpreting the result as a Laplace transform of the pdf of joint interference power: for a wanted signal subject to Rayleigh fading, this probability can be expressed in the form

\[ P_e(p_0 > z, p_t) = L \left\{ f_2(x) \right\} \]

where \( L(f,s) \) denotes the one-sided Laplace transform of the function f at the point s.

Rician channel
This approach can be applied to a Rician-fading wanted signal, using the series expansion

\[ L_0(\xi) = \sum_{n=0}^{\infty} \frac{1}{2^n n!} \xi^n x^n \]

for the modified Bessel function I0. This gives

\[ P_e(p_0 > z, p_t) = \sum_{n=0}^{\infty} \frac{1}{2^n n!} \exp\left\{ -z^2 \right\} \frac{1}{2} \int_{\frac{z}{\sqrt{2}}}^{\infty} \left( \frac{z^2}{x^2} \right)^n f_3(x) dx \]

Using the properties of the Laplace Transform, this can be written as

\[ P_e(p_0 > z, p_t) = \exp\left\{ -z^2 \right\} \int_{\frac{z}{\sqrt{2}}}^{\infty} \left( \frac{z^2}{x^2} \right)^n f_3(x) dx \]

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