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Operational Calculus on Fourier-Finite Mellin Transform

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Abstract*— The Fourier Transform and the Finite Mellin Transform has found various applications separately. Their usage is different from each other, but combining these two transforms may help in solving different problems which could not be solved by these two transforms separately. So combining and extending these two transforms is important. This newly formed Fourier-Finite Mellin Transform may be used for image recognition and processing, movement detection and derivation of densities for algebraic combinations of random variables and many more. In this paper some operators on the space $FM^{\beta}_{f,b,c,\alpha}$ is described, differential operator F-Type and also some results on differential operator are proved.

Index Terms—**Differential operator, Finite Mellin transform, Fourier-finite Mellin transform, Fourier transform.**

I. INTRODUCTION

Fourier transform (FT) is named in the honor of Joseph Fourier (1768-1830). In the theory of Integral transform, Fourier analysis is one of the most frequently used tools in signal processing and many other scientific fields [1,5,6]. Besides the Fourier transform (FT), time-frequency representation of signals, such as Wigner Distribution (WD), Short Time Fourier Transform (STFT), Wavelet Transform (WT) are also widely used in speech processing, image processing or quantum physics [3]. For electrical networks the Fourier transform is applied to functions describing a current or voltage as function of time. Fourier transform is also used to analyze and process a sequence of measurements or data, originating for example from an audio signal or a digitized photo. [3,4] The past implementations of the Mellin transform based on the FFT have required exponential sampling, interpolation, and the computation of a correction term all of which introduce error into the transform. A modified Mellin transform for digital implementation is developed and applied to range radar profiles of naval vessels [4]. The Mellin transformation is a basic tool for analyzing the behavior of many important functions in Mathematics and mathematical physics. Mellin transformation is also used in statistics. It is also used in time-frequency analysis.[7,8]. In the late 1970's, Casasent and Psaltis [1976,1997] contributed substantially to the implementation of a digital form of the Fourier-Mellin transform in application using physical lenses. The Fourier-Mellin transform was implemented on a digital computer and applied towards the recognition and differentiation of images of plant leaves regardless of translation, rotation or scale. Translated, rotated and scaled leaf images from seven species of plants were compared. It also used for movement detection [2]. Recently, the Fourier-Mellin transform has seen a revival with the advent of watermarking. It also useful in agriculture. Robbins and Huang described an implementation for the application of the Fourier-Mellin transform to correct various optical distortions, including noise, in lenses. In this way there are various application of Fourier- Mellin Transform. In the present paper, the Fourier- finite Mellin transform is discussed. Operators on the space $FM^{\beta}_{f,b,c,\alpha}$ is defined in the section II. Some theorems on Differential operator are proved in section III, some results on differential operator are proved in section IV, and lastly conclusions are given in section V. Notations and terminologies as per [9], [10], [11].

II. OPERATOR

If $\phi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ and ξ is any fixed real number then $\phi(t+\xi,x) \in FM^{\beta}_{f,b,c,\alpha}$, $t+\xi > 0$
And $\phi(t+\xi,x) \in F^{\nu} M^{\beta}_{f,b,c,\alpha}$, $t+\xi < 0$.

Proof: Consider, $\gamma_{b,c,k,q,l}\, \phi(t+\xi,x)$

$$= \sup_{I} \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t+\xi,x) \right|$$

$$= \sup_{I} \left| (t'-\xi)^k \lambda_{b,c}(x) x^{q+1} D_{t'}^l D_x^q \phi(t',x) \right| ,$$

Where $t' = t+\xi \quad \therefore\ t = t'-\xi$

$$\leq C A^k k^{k\alpha} B^l l^{l\beta}$$

Thus, $\phi(t+\xi,x) \in FM^{\beta}_{f,b,c,\alpha}$, for $t+\xi > 0$

Similarly it can be show that $\phi(t+\xi,x) \in F^{\eta} M^{\beta}_{f,b,c,\alpha}$, for $t+\xi < 0$

A. Proposition

The translation (shifting) operator $\xi :\ \phi(t,x) \to \phi(t+\xi,x)$ is a topological automorphism on $FM^{\beta}_{f,b,c,\alpha}$, for $t+\xi > 0$, and it is a topological

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isomorphism from $FM^{\beta}_{f,b,c,\alpha}$ onto $F^{\eta} M^{\beta}_{f,b,c,\alpha}$, for $t + \xi < 0$.

B. Proposition

If $\phi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ and $p > 0$, strictly positive number then $\phi(pt, x) \in FM^{\beta}_{f,b,c,\alpha}$.

Proof: Consider, $\gamma_{b,c,k,q,l}\, \phi(pt, x)$

$$= \sup_I \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(pt, x) \right|$$

$$= \sup_I \left| \left(\frac{T}{p}\right)^k \lambda_{b,c}(x) x^{q+1} D_T^l D_x^q \phi(T, x) \right|,$$

Where $pt = T \quad \therefore \quad t = \frac{T}{p}$

$$= M \sup_I \left| T^k \lambda_{b,c}(x) x^{q+1} D_T^l D_x^q \phi(T, x) \right|,$$

Where M is a constant depending on p.

$$\leq M\, C\, A^k k^{k\alpha} B^l l^{l\beta} \quad \leq \quad C' A^k k^{k\alpha} B^l l^{l\beta},$$

Where $C' = MC$

Thus, $\phi(pt, x) \in FM^{\beta}_{f,b,c,\alpha}$, for $p > 0$.

C. Proposition

If $p > 0$ and $\phi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ then the scaling operator $R : FM^{\beta}_{f,b,c,\alpha} \to FM^{\beta}_{f,b,c,\alpha}$ defined by $R\phi = \psi$, where $\psi(t,x) = \phi(pt, x)$ is a topological automorphism.

D. Proposition

If $\phi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ and η is any fixed real number then, $\phi(t, x+\eta) \in FM^{\beta}_{f,b,c,\alpha}$, $x + \eta > 0$ and $\phi(t, x+\eta) \in F^{\nu} M^{\beta}_{f,b,c,\alpha}$, $x + \eta < 0$

Proof:-Consider, $\rho_{b,c,k,q,l}\, \phi(t, x+\eta)$

$$= \sup_I \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t, x+\eta) \right|$$

$$= \sup_I \left| t^k \lambda_{b,c}(x' - \eta)(x' - \eta)^{q+1} D_t^l D_{x'}^q \phi(t, x') \right|,$$

Where, $x' = x + \eta \quad \therefore \quad x = x' - \eta$

$$\leq C A^k k^{k\alpha} B^l l^{l\beta}.$$

Thus, $\phi(t, x+\eta) \in FM^{\beta}_{f,b,c,\alpha}$, for $x + \eta > 0$

Similarly it can be shown that $\phi(t, x+\eta) \in F^{\nu} M^{\beta}_{f,b,c,\alpha}$, for $x + \eta < 0$.

E. Proposition

If $\phi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ and $p > 0$, strictly positive number then $\phi(t, px) \in FM^{\beta}_{f,b,c,\alpha}$.

Proof:-Consider,

$$\gamma_{b,c,k,q,l}\, \phi(t, px) = \sup_I \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t, px) \right|$$

$$= \sup_I \left| t^k \lambda_{b,c}\left(\frac{X}{p}\right) \left(\frac{X}{p}\right)^{q+1} D_t^l D_X^q \phi(t, X) \right|,$$

Where $px = X \quad \therefore \quad x = \frac{X}{p}$

$$= M \sup_I \left| t^k \lambda_{b,c}\left(\frac{X}{p}\right) X^{q+1} D_t^l D_X^q \phi(t, X) \right|$$

Where M is a constant depending on p.

$$\leq M\, C\, A^k k^{k\alpha} B^l l^{l\beta} \quad \leq \quad C' A^k k^{k\alpha} B^l l^{l\beta},$$

Where $C' = MC$

Thus, $\phi(t, px) \in FM^{\beta}_{f,b,c,\alpha}$, for $p > 0$

III. DIFFERENTIAL OPERATOR F-TYPE

A. Theorem

The operator $\phi(t, x) \to D_t \phi(t, x)$ is defined on the space $FM^{\beta}_{f,b,c,\alpha}$ and transforms this space into itself.

Proof: Let $\phi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$.

If $D_t \phi(t, x) = \psi(t, x)$, then we have

$$\gamma_{b,c,k,q,l}(\psi) = \sup_I \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \psi(t, x) \right|$$

$$= \sup_I \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q D_t \phi(t, x) \right|$$

$$\leq C A^k k^{k\alpha} B^{(l+1)} (l+1)^{(l+1)\beta}$$

$\therefore \quad \psi(t, x) \in FM^{\beta}_{f,b,c,\alpha}$ i.e. $D_t \phi(t, x) \in FM^{\beta}_{f,b,c,\alpha}$

Or $\gamma_{b,c,k,q,l}(D_t \phi(t, x)) = \gamma_{b,c,k,q,l+1}(\phi(t, x))$, $k, q, l \in N_0$.

B. Theorem

The operator $\phi(t, x) \to D_x \phi(t, x)$ is defined on the space $FM_{f,b,c,\gamma}$ and transform this space into itself.

Proof: Let $\phi(t,x) \in FM_{f,b,c,\gamma}$. If $D_x \phi(t, x) = \psi(t, x)$

We have,

$$\rho_{b,c,k,q,l}(\psi) = \sup_I \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \psi(t, x) \right|$$

$$= \sup_I \left| t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q D_x \phi(t, x) \right|$$

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$$= \sup_I \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^{q+1} \phi(t,x)\right|$$

$$\leq C_{lk} A^{q+1} (q+1)^{(q+1)\gamma}, \quad k,q,l = 0,1,2,\dots\dots\dots$$

Therefore, $\psi(t,x) \in FM_{f,b,c,\gamma}$ i.e. $D_x \phi(t,x) \in FM_{f,b,c,\gamma}$.

i.e. $\rho_{b,c,k,q,l}\left(D_x \phi(t,x)\right) = \gamma_{b,c,k,(q+1),l}(\phi(t,x))$

C. Proposition

The differential operator of F-type $F: \phi(t,x) \to D_t \phi(t,x)$ is a topological automorphism on $FM^{\beta}_{f,b,c,\alpha}$.

D. Proposition

The differential operator of M_f-type $M_f: \phi(t,x) \to D_x \phi(t,x)$ is a topological auto Orphism on $FM^{\beta}_{f,b,c,\alpha}$.

IV. RESULTS ON DIFFERENTIAL OPERATOR

A. Theorem

For $m = (m_1, m_2)$ where $m_1, m_2 = 0,1,2,\dots\dots\dots\dots\dots$ if $\phi(t,x) \in FL^{\beta}_{f,b,c,\alpha}$, then $\psi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ where $\psi(t,x) = D^m \phi(t,x)$. Further the mapping $\Delta = D^m \phi : \phi \to \psi$ is one-one, linear and continuous.

Proof: For $\psi \in FM^{\beta}_{f,b,c,\alpha}$,

$$\gamma_{b,c,k,q,l}\,\psi(t,x) = \sup_I \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \psi(t,x)\right|$$

$$= \sup_I \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q D^m \phi(t,x)\right|$$

$$= \sup_I \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^{l+1} D_x^{q+1} \phi(t,x)\right|$$

$$\leq C A^k k^{k\alpha} B^l l^{l\beta} \quad \text{------------} \quad (A1)$$

Thus $\psi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ if $\phi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ It is obviously linear.

It is injective for, if $D^m\phi = 0$ then $\phi = c$, c is a constant. If $c = 0$ then $\phi = 0$ and D is injective. But if $c \neq 0$ then, $\sup_I \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q c\right| = \sup_I \left|t^k \lambda_{b,c}(x) x^{q+1} c\right|$, for $l = q = 0$. As the right hand side is not bounded we conclude that $\phi \notin FM^{\beta}_{f,b,c,\alpha}$, which is a contradiction.

Hence 'c' must be zero and therefore $\phi = 0$. For continuity we observe from equation (A1) that,

$\gamma_{b,c,k,q,l}(D^m \phi) \leq M\,\gamma_{b,c,k,q,l}(\phi)$, where M is some constant. Thus the theorem is proved.

B. Theorem

For $\sigma \in R$ and $\phi(t,x) \in FM^{\beta}_{f,b-\sigma,c,\alpha}$,
$E(\phi) = \psi(t,x) = e^{-\sigma x}\phi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$.

Proof: Let $\phi(t,x) \in FM^{\beta}_{f,b-\sigma,c,\alpha}$

Consider, $\gamma_{b,c,k,q,l}\,\psi(t,x)$

$$= \sup_I \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q e^{-\sigma x} \phi(t,x)\right|$$

$$= \sup_I \left|\sum_{i=0}^{1} b_i e^{-\sigma x} t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x)\right|$$

Or **P**

$$= \sup_I \left|\sum_{i=0}^{1} b_i e^{-\sigma x} t^k x^{b+q+1} D_t^l D_x^q \phi(t,x)\right|$$

$$\leq C A^k k^{k\alpha} B^l l^{l\beta}$$

Thus $\psi(t,x) \in FM^{\beta}_{f,b,c,\alpha}$ if $\phi(t,x) \in FM^{\beta}_{f,b-\sigma,c,\alpha}$

In view of this proposition we have, the exponential multiplier operator, $E^h: FM^{\beta}_{f,b-\sigma,c,\alpha} \to FM^{\beta}_{f,b,c,\alpha}$ is a topological isomorphism.

V. CONCLUSION

This paper defines the operators on the space $FM^{\beta}_{f,b,c,\alpha}$ and proves some theorems on differential operator and also some results on differential operator which could be used for different applications mentioned earlier elsewhere.

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