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Influence of Kerr Effect on Tweezer Center Location in Nonlinear Medium

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Abstract* - The influence of the Kerr effect on the tweezer center location of linear dielectric Nan particle in nonlinear medium irradiated by the intense Gaussian beam is investigated. The expressions of the focal length of nonlinear lens, intensity distribution of modified Gaussian beam in nonlinear medium, longitudinal and transverse gradient forces acting on dielectric particle are derived. The distribution of the optical forces in the cylinder of nonlinear medium is simulated and the motion of the tweezer center is discussed for same cases of nonlinear refractive index coefficient.

Index Terms-**Kerr effect, nonlinear medium, optical tweezer, optical force, self-focusing.**

I. INTRODUCTION

The previous works [1, 2], the optical tweezer to trap the dielectric nanoparticle embedded in a Kerr medium is concerned. The distribution of the optical forces acting on the nanoparticle in the Kerr medium has been discussed [1]. In work [2], the self-focusing relating to Kerr effect affecting on the optical forces have been concerned, and the influence of the nonlinear coefficient on optical forces is investigated with the approximation of the plane-wave laser beam. Unfortunately, this approximation has not allowed evaluate the influence of thickness of the nonlinear medium on the optical forces and tweezer center location, which are two important qualities of the tweezer. Therefore, in this paper, the influence of the self-focusing of the intense Gaussian laser beam on the distribution of optical force acting on dielectric nanoparticle is investigated. This article is organized as follows: in Sec.2 we derive the expressions of the optical forces concerning self-focusing, which is arisen from the Kerr effect in the nonlinear medium irradiated by the intense Gaussian laser beam (spherical wave); in Sec.3 we present the simulated distribution of the intensity and optical forces in a cylinder of nonlinear medium and discussion about the motion of tweezer center location.

II. OPTICAL FORCES

As a example, we consider an optical tweezer to trap a dielectric nanoparticles in the cylinder of Kerr medium (Fig.1). A spherical wave of the laser beam described by Gaussian function irradiating the dielectric nanoparticle embedded in Kerr medium, and its intensity is given by [3]:

$$I(\rho,z)=I_0\times\left(\frac{W_0}{W(z)}\right)^2\times\exp\left[-\frac{\rho^2}{W^2(z)}\right] \quad (1)$$

where, $I_0=I(0,0)$ is the maximum intensity at the point $(0,0)$ relating to the total power $P=\pi I_0 W_0^2/2$, W_0, $W(z)=W_0\sqrt{1+(z/z_0)^2}$ are the radius of the beam at 0 (of beam waist) and z, respectively, $z_0=\pi W_0^2/\lambda$ is the Rayleigh range, λ is the wavelength of laser, $\rho=\sqrt{x^2+y^2}$ is the radial coordinate.

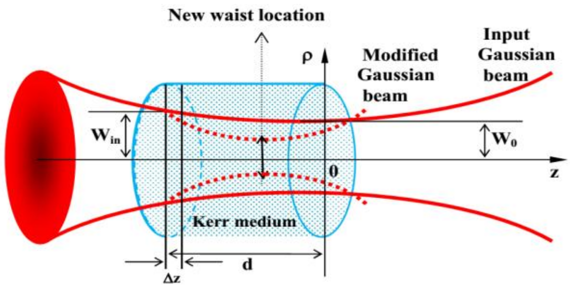

Fig.1 Sketch of optical tweezer with Kerr medium using Gaussian beam.

The Gaussian transverse intensity distribution (1) is incident upon the Kerr medium whose refractive index is altered to

$$n_m(\rho,z)=n_2+n_{nl}I(\rho,z) \quad (2)$$

where n_2 is the linear refractive index, and the nonlinear refractive index coefficient n_{nl} of the medium is assumed to be positive. As a result of this nonlinear response (if laser beam is intense enough), the refractive index of the medium is larger at the center of laser beam than at its periphery, with the result that the medium is in effect turned into a positive lens which potentially leads to self-focusing occurred if the power is greater than the critical power [4, 5]

$$P_{cr}=1.8962\frac{\lambda^2}{4\pi n_2 n_{nl}} \quad (3)$$

Assuming the radius of the beam at entrance face of the Kerr medium is $W_{in}=W(d)$, where d means the waist location (distance from beam waist to entrance face and $d<0$). From (1) and (2), we have

$$n_m(\rho,z)=n_2+n_{nl}I_0\left(\frac{W_0}{W(z)}\right)^2\exp\left[-\frac{\rho^2}{W^2(z)}\right] \quad (4)$$

As well known, the Kerr appears powerfuly in the cylinder limited by $\rho<W(z)$ and $\Delta z=d/m$ where m is a interger, so we can use the following approximation

$$\exp\left[-\frac{\rho^2}{W^2(z)}\right]\approx 1-\frac{\rho^2}{W^2(z)} \quad (5)$$

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Substituting (5) into (4), we have

$$n_m(\rho,z)=n_2+n_{nl}I_0\left(\frac{W_0}{W(z)}\right)^2\left(1-\frac{\rho^2}{W^2(z)}\right)=N_0-N_2\rho^2 \qquad (6)$$

where

$$N_1=n_2+n_{nl}I_0\left(\frac{W_0}{W(z)}\right)^2 \text{ and } N_2=n_{nl}I_0\left(\frac{W_0}{W(z)}\right)^2\frac{1}{W^2(z)} \qquad (7)$$

As shown in work [6] and from (6), the Kerr medium cylinder with thickness of Δz becomes the nonlinear lens with the focal length is given by

$$f_{nl,1}=\frac{1}{2N_2\Delta z}=\frac{W_{in}^4}{2n_{nl}I_0W_0^2\Delta z}=\frac{\left[1+(\Delta z/z_0)^2\right]^2}{2n_{nl}I_0\Delta z} \qquad (8)$$

Consequently, the Gaussian beam (1) will be modified to new one (Fig.1), whose intensity $I_{m,1}(\rho,z)$ given as [3]

$$I_{m,1}(\rho,z)=\frac{I_0}{1+\left[\frac{z+(z_1+\Delta z)}{z_{0,1}}\right]^2}\times\exp\left[-\frac{\rho^2}{W_{0,1}^2}\frac{1}{1+\left[\frac{z+(z_1+\Delta z)}{z_{0,1}}\right]^2}\right] \qquad (9)$$

with $W_{0,1}=M_1W_0$, $z_1=M_1^2(\Delta z-f_{nl,1})+f_{nl,1}$, $z_{0,1}=M_1^2z_0$, where, $W_{0,1}$, z_1, $z_{0,1}$ are the new waist radius, waist location from entrance face and Rayleigh range of modified beam, and $M_1=M_{r,1}/\sqrt{1+r_1^2}$, $r_1=z_0/(\Delta z-f_{nl,1})$, $M_{r,1}=\left|f_{nl,1}/\Delta z-f_{nl,1}\right|$, $z_0=\pi W_0^2/\lambda$ is the Rayleigh range of the input Gaussian beam. When the Kerr effect can be ignored, i.e. $f_{nl}=\infty$, then $M_1=1$ and $z_1=-\Delta z$, consequently, Eq.(9) coinsides with Eq. (1). In Eq. (9), the term $z_1+\Delta z$ gives us the change of waist location.

Eq. (9) is intensity of modified Gaussian beam propagating through the first Kerr medium cylinder with radius $W(z)$ and thickness Δz. Next, this beam with be focused by the second, third, …ith… mth nonlinear lens (second, third, …ith … mth Kerr medium cylinder), with focal length $f_{nl,i}$ given as:

$$f_{nl,i}=\frac{\left[1+(\Delta z/z_{0,(i-1)})^2\right]^2}{2n_{nl}I_0\Delta z} \qquad (10)$$

where, $z_{0,i}=M_i^2z_{0,(i-1)}$, $M_i=M_{r,i}/\sqrt{1+r_i^2}$, $r_i=z_{(i-1)}/(\Delta z-f_{nl,(i-1)})$, $M_{r,i}=\left|f_{nl,i}/\Delta z-f_{nl,i}\right|$. And then the intensity of modified Gaussian beam at exit face of ith Kerr medium cylinder will be given as:

$$I_{m,i}(\rho,z)=I_0\times\frac{1}{1+\left[\frac{z+(z_i+\Delta z)}{z_{0,i}}\right]^2}\times\exp\left[-\frac{\rho^2}{W_{0,i}^2}\frac{1}{1+\left[\frac{z+(z_i+\Delta z)}{z_{0,i}}\right]^2}\right] \qquad (11)$$

where

$$W_{0,i}=M_iW_{0,(i-1)},\quad z_i=M_i^2(\Delta z-f_{nl,i})+f_{nl,i},\quad z_{0,i}=M_i^2z_{0,(i-1)}. \qquad (12)$$

Using (11)and (12), the intensity distribution of modified Gaussian beam at plane $(\rho,-d+i\Delta z)$ and consequence, the optical force distribution can be simulated.

For simplicity, we assume that the radius (*a)* of the linear particle is much smaller than the wavelength of the laser (i.e., $a<<\lambda$), in this case we can treat the dielectric particle as a point dipole. We also assume that the refractive index of the dielectric particle is n_1 and $n_1>>n_2$, i.e., the necessary condition for trapping operation of the tweezer using Gaussian beam satisfied [7]. Using (2), the relative refractive index is given by

$$m_{n,i}(\rho,z)=\frac{n_1}{n_{m,i}(\rho,z)}=\frac{n_1}{n_2+n_{nl}I_{m,i}(\rho,z)} \qquad (13)$$

Using (13) and as shown in work [8, 9], we have the scattering cross sections (α_n) and polarizabilities (β_n) of particle in nonlinear medium as follows:

$$\alpha_{n,i}(\rho,z)=\left(\frac{128\pi^5a^6}{3\lambda^4}\right)\left[\frac{\left(m_{n,i}(\rho,z)^2-1\right)}{\left(m_{n,i}(\rho,z)^2+2\right)}\right]^2 \qquad (14)_1$$

and

$$\beta_{n,i}(\rho,z)=4\pi n_{m,i}^2(\rho,z)\varepsilon_0a^3\frac{\left(m_{n,i}(\rho,z)^2-1\right)}{\left(m_{n,i}(\rho,z)^2+2\right)} \qquad (14)_2$$

As shown in previous works [1, 2, 10, 11, 12], the transverse gradient force is reduced to:

$$\vec{F}_{grad,\rho}(\rho,z)=-\hat{\rho}\frac{2\beta_{n,i}(\rho,z)I_{m,i}(\rho,z)\rho}{cn_{m,i}(\rho,z)\varepsilon_0W_{0,i}} \qquad (15)$$

and the total longitudinal force is given by:

$$\vec{F}_{total,z}(\rho,z)=\vec{F}_{grad,z}(\rho,z)+\vec{F}_{scat}(\rho,z)=$$
$$-\vec{z}\frac{2\beta_{n,i}(\rho,z)I_{m,i}(\rho,z)}{n_{m,i}(\rho,z)\varepsilon_0ckW_{0,i}}\frac{1}{\left(1+\left(\frac{z+(z_i+\Delta z)}{z_{0,i}}\right)^2\right)^2} \qquad (16)$$
$$\times\frac{z+(z_i+\Delta z)}{z_{0,i}^2}+\vec{z}\frac{n_{m,i}(\rho,z)}{c}\alpha_{n,i}(\rho,z)I_{m,i}(\rho,z)$$

The general Exps. (15) and (16) describe the redistribution of transverse gradient force and longitudinal force in phase plane (ρ,z) depending on parameter collection

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($\{I_0, W_0, n_{nl}.d\}$) with chosen interger m.

III. REDISTRIBUTION OF INTENSITY AND OPTICAL FORCE AND MOTION OF WAIST LOCATION AND TWEEZER CENTER

Consider a Kerr medium with nonlinear refractive index coefficient of $n_{nl} = 1\times10^{-10} cm^2/W$ which is seem as the lowest one and linear refractive index of $n_2 = 1.332$ is irradiated by the laser beam of wavelength of $\lambda = 1.06\mu m$. Consequently, from (3) the critical power is

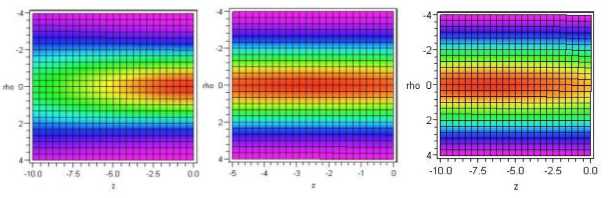
$$P_{cr} = 1.8962\frac{\lambda^2}{4\pi n_2 n_{nl}} = 0.127\times10^2 W,$$

and it decreases when the nonlinear refractive index coefficient increases. And considering the Gaussian beam with waist radius of $W_0 = 2\mu m$, the self-focusing effect occurs when the maximum intensity is greater than

$$I_0 > \frac{2P_{cr}}{\pi W_0^2} = \frac{2\times0.127\times10^2}{3.14\times4\times10^{-8}} \approx 2\times10^8 W/cm^2$$

In following numerical simulation we choose parameters of dielectric particle as: $a = 20nm$ is $n_1 = 1.592$.

The input Gaussian beam with $\lambda = 1,06\mu m$, $W_0 = 2\mu m$, $I_0 = 3.5\times10^8 W/cm^2$, whose waist location at exit face ($z = 0$) of medium of thickness of $d = 10\mu m$ (Fig.2a) is modified depending on the refractive index coefficient, i.e., in medium with $n_{nl} = 1\times10^{-10} cm^2/W$ and $n_{nl} = 2\times10^{-10} cm^2/W$, the waist radius decreases to modified one of $W_0^{mod} = 1.8\mu m$ and $W_0^{mod} = 1.4\mu m$ and its waist location moves a distance of $z^{mod} - d = 3.18\mu m$ and $z^{mod} - d = 6.18\mu m$ (Fig.2b and Fig.2c, respectively).



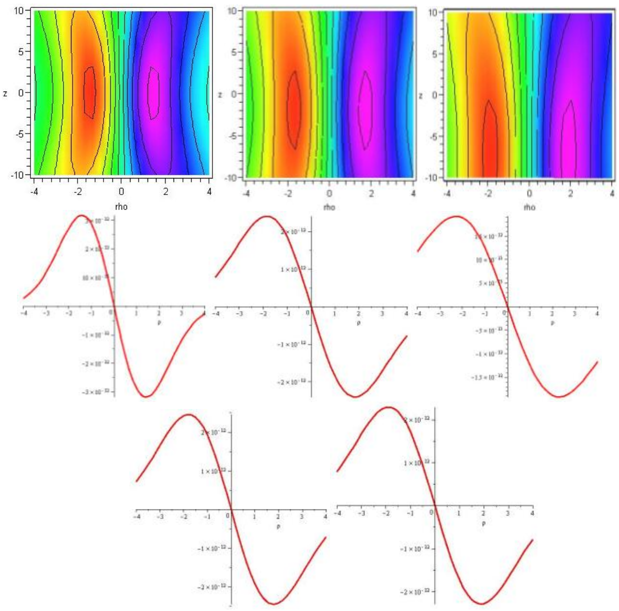
a b c

Fig.2. Intensity distribution in plane ρ-z ($\mu m - \mu m$).

a) Input Guassian beam with $\lambda = 1.06\mu m$ $W_0 = 2\mu m$, $I_0 = 3.5\times10^8 W/cm^2$, $d = -10\mu m$; b) Modified Gaussian beam with new radius of modified waist of $W_0^{mod} \approx 1.8\mu m$ and new distance from entrance face to new waist $z^{mod} \approx -6.82\mu m$ in medium with $n_{nl} = 1\times10^{-10} cm^2/W$; c) Modified Gaussian beam with with $W_0^{mod} = 1.4\mu m$, $z_0^{mod} = -3.82\mu m$ in medium with $n_{nl} = 2\times10^{-10} cm^2/W$.

The motion of waist location, consequently leads to move of the tweezer center and influence the force distribution. These questions will be investigated in detail as follows.

The transverse gradient force $F_{grad,\rho}$ and longitudinal force $F_{total,z}$ are calculated by expression (13) and (14) in the ranges: $\rho = (-4 \div 4)\mu m$ and $z = (-10 \div 0)\mu m$. The distributions of the transverse gradient force (Fig.3), longitudinal gradient force (Fig.4) and total longitudinal force (Fig.5) in the sphase plane (ρ, z) are simulated and shown as follows.



a) b) c)

Fig 3. Upper: Distribution of transverse gradient force (N) on the phase plane (ρ,z); Middle: (ρ,0) and downer: (ρ,z^{mod}-d).
a): $n_{nl} = 0$, b): $n_{nl} = 1\times10^{-10} cm^2/W$,
c): $n_{nl} = 2\times10^{-10} cm^2/W$.

From Fig.3, the distribution of the transverse gradient force in plane (ρ,z) is similar to that of intensity. Here is different that the absolute magnitude of maximum force in plane (ρ,0) decreases from $3.2\times10^{-12} N$ (Fig.3a) downto $2.0\times10^{-12} N$ (Fig.3c) with increasing of the nonlinear refractive index coefficient. Instead of that the diameter of trap region in specimen plane (z=0) increases from $3\mu m$ (Fig.3a) upto $4.5\mu m$ (Fig.3c). Meanwhile, that properties are almost not changed in the waist plane (ρ, z^{mod}-d). As principle, the trap center, where the all forces will be zero, locates in the waist plane as shown in Fig.3, i.e. it

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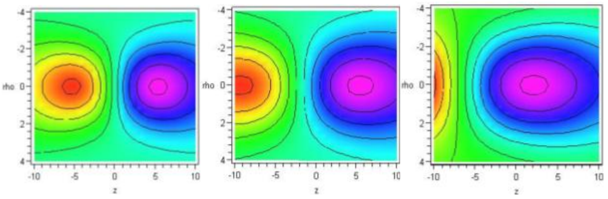
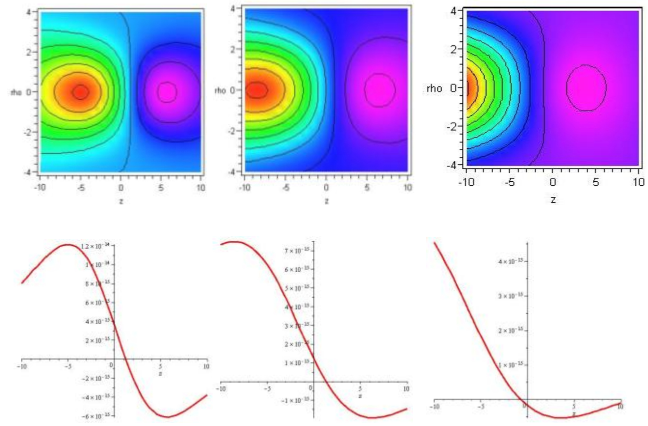
moves a distance of $z^{\text{mod}} - d = 0\mu m$ (Fig.4a), $z^{\text{mod}} - d = 3.18\mu m$ (Fig.4b) and $z^{\text{mod}} - d = 6.18\mu m$ (Fig.4c), respectively. Moreover, with increasing of refractive index coefficient the tweezer center backs more and more far from the exit face of medium.



a b c

Fig. 4. Distribution of the longitudinal gradient forces, $F_{grad,z}$ (N) in phase plane (ρ,z). a): $n_{nl} = 0$, b): $n_{nl} = 1\times10^{-10} cm^2/W$, c): $n_{nl} = 2\times10^{-10} cm^2/W$.

But, the considered tweezer uses one beam, so the particle is acted by the scatt force, $\vec{F}_{scat}(\rho, z)$ (see Eq.16). Since that, the tweezer center which is shown in Fig.**3** and Fig.4 moves forward as shown in Fig.5. Finally, the trap center is pulled to the exit face of medium and the total longitudinal force in specimen plane decreases.



a b c

Fig. 5. Distribution of the longitudinal forces, $F_{total,z}$ (N) in phase plane (ρ,z) (upper) and in the beam axis (downer).
a): $n_{nl} = 0$, b): $n_{nl} = 1\times10^{-10} cm^2/W$, c): $n_{nl} = 2\times10^{-10} cm^2/W$.

IV. CONCLUSION

The expression of intensity of the modified Gaussian beam and the optical forces acting on the linear nanoparticle embedded in Kerr medium are derived with approximation that the nonlinear lens appears on the cylinder of thickness Δz and radius $W_{in} = W(\Delta z)$. In the Rayleigh regime, expressions of optical forces are resulted. The cascade simulated results for single Gaussian beam tweezer show: i) the tweezer center is pulled behind the specimen plane if refractive index coefficient, n_{nl} is small; ii) with increasing of n_{nl} the tweezer center moves more and more far from infront of specimen plane and the magnitude of total longitudinal force decreases; iii) with increasing of n_{nl} the optical forces decrease, that means the stability of the particle decreases; iv) the tweezer center is always kept in the beam axis. In other words, the Kerr effect will affecting on the distribution of optical forces, especially, on longitudinal one and the motion of tweezer center in the beam axis. The attentions should be paid on problems when the medium surrounding the nanoparticle is sensitive to Kerr effect, mainly when the particle hangs in medium . Moreover, those above mentioned properties depend on other parameters as I_0, W_0 of input Gaussian beam and d of nonlinear medium, also. Those questions will be investigated in detail in the furture.

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