ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

Adjacent Vertex Distinguishing Edge Colouring of Cactus Graphs

Nasreen Khan, Madhumangal Pal
Department of Mathematics, Global Institute of Management and Technology, Krishnagar-741102, W.B., INDIA
Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore-721102, W.B., INDIA

Abstract*— Adjacent vertex distinguishing edge colouring or avd -colouring of a graph G is a mapping from its edge set to the set of nonnegative integers such that no pair of adjacent vertices meet the same set of colours. The avd -chromatic number, denoted by $\chi'_a(G)$, is the minimum number of colours needed in an avd -colouring of G. A cactus graph is a connected graph in which every block is either an edge or a cycle and in other words, no edge belongs to more than one cycle. Here is proved that for a cactus graph G, $\Delta \le \chi'_a(G) \le \Delta+3$, where Δ is the degree of G. An optimal algorithm is also presented to colour the edges using avd -edge colouring technique on cactus graphs in $O(n)$ time, where n is the total number of vertices of the cactus graph.

***Keywords*--adjacent vertex distinguishing edge chromatic number, adjacent vertex distinguishing edge colouring, analysis of algorithms, cactus graph, design of algorithms, graph colouring**

I. INTRODUCTION

Cactus graph is a connected graph in which every block is a cycle or an edge, in other words, no edge belongs to more than one cycle. Cactus graph have extensively studied and used as models for many real world problems. This graph is one of the most useful discrete mathematical structure for modelling problem arising in the real world. It has many applications in various fields like computer scheduling, radio communication system, etc. Cactus graph have studied from both theoretical and algorithmic points of view. This graph is a subclass of planar graph and superclass of tree. Let G be a simple graph with n vertices. For $d \ge 0$ write n_d for the number of vertices of G of degree d. Let $\chi'(G)$ be the minimum number of colours required in a proper edge-colouring of G. By Vizing's theorem we know that $\Delta \le \chi'(G) \le \Delta+1$. A proper edge colouring is said to be *vertex-distinguishing* if each pair of vertices is incident to a different set of colours. Suppose that $G=(V,E)$ is a graph and $\phi: E \to \{c_1, c_2, \ldots, c_k\}$ is a proper edge colouring of G. For any vertex $v \in V$, let $d_G(v)$ or simply $d(v)$ denote degree of v in G and $\Phi(v)=\{\phi(vw)/vw \in E\}$. If $uv \in E$ then u is called a neighbor of v and v, a neighbor of u. We say a colour c_i is incident with a vertex $u \in V(G)$, if there exists an edge (u,v) is coloured by c_i. A proper edge colouring ϕ is called an *adjacent vertex distinguishing edge colouring* or avd -*edge colouring* if $\Phi(u) \ne \Phi(v)$ for all $uv \in E$. The vertex distinguishing proper edge colouring will also be called as *strong edge colouring*. It is clear that every graph without isolated edges has an *avd-colouring*. A k - avd -*colouring* is an *avd-colouring* using at most k colours. The *avd-chromatic number* of G, denoted by $\chi'(G)$, is the minimum number of colours needed in an *avd-colouring* of G. The concept of vertex-distinguishing edge colouring has been considered in several papers [1, 4, 5, 6, 7, 11, 12]. Zhang *et al.* [15] presented the following conjecture.

Conjecture 1 *[15] If G be a simple connected graph with at least three vertices and $G \ne C_5$, then $\Delta \le \chi'_a(G) \le \Delta+2$.*

II. REVIEW OF PREVIOUS WORK

Several results are known for adjacent vertex distinguishing edge colouring of graphs, but to the best of our knowledge no result is known for cactus graph. In this section, the known results for general graphs and some related graphs of cactus graphs are presented. A $\chi'_a(G)$ is at least as large as the edge chromatic number of G, it is clear that $\chi'_a(G) \ge \Delta$. Balister *et al.* [3] proved Conjecture 1 for all graphs with $\Delta = 3$, and for all bipartite graphs. They also showed that the bound is tight. Only much weaker bounds are known for general graphs without any isolated edges. Akbari *et al.* [2] obtained the bound $\chi'_a(G) \le 3\Delta$ for all graphs without any isolated edges. For very large Δ, Hatami [10] proved that $\chi'_a(G) \le \Delta+300$ if $\Delta > 10^{20}$, and Ghandehari and Hatami [9] proved that $\chi'_a(G) \le \Delta+27\sqrt{\Delta ln \Delta}$ if $\Delta > 10^6$. Recently, Edwards *et al.* [8] proved that if G is a planar bipartite graph with $\Delta(G) \ge 12$, the $\chi'_a(G) \le \Delta+1$. In [13], Liu and Liu proved that for any connected 3-colorable Hamiltonian graph G, $\chi'_a(G) \le \Delta+3$. In [15], Zhang *et al.* find out the result for complete

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

graph K_p $(p \geq 3)$, $\chi'_a(K_p) = p$, if $p \equiv 1 \pmod 2$ and $p+1$, if $p \equiv 0 \pmod 2$. For a tree T with $|V(T)| \geq 3$, $\chi'_a(T) = \Delta(T)$, if any two vertices of maximum degree are not adjacent and $\Delta(T)+1$, if T has two vertices of maximum degree which are adjacent.

III. THE AVD-COLOURING OF INDUCED SUB-GRAPHS OF CACTUS GRAPHS

Let $G = (V, E)$ be a given graph and subset U of V the induced subgraph by U, denoted by $G[U]$, is the given graph $G' = (U, E')$, where $E' = \{(u,v) : u, v \in U$ and $(u,v) \in E\}$.

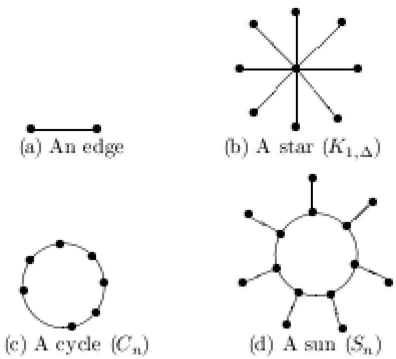

Fig. 1: Induce subgraphs of cactus graph.

The cactus graph have many interested subgraphs, those and their *avd-edge colouring* are illustrated below. An edge is denoted by P_2, so $\chi'_a(an\ edge) = 1$. The star graph $K_{1,\Delta}$ is a subgraph of $K_{n,m}$ therefore one can conclude the following result.

Lemma 1 *For any star graph* $K_{1,\Delta}$, $\chi'_a(K_{1,\Delta}) = \Delta$, *where* Δ *is the degree of the star graph.*

III. AVD-EDGE COLOURING OF CYCLES

A. *Avd-edge colouring of one cycle*

In [15], Zhang *et al.* have colour C_n by *avd-edge colouring* and they have obtained the following result. Here we have given a constructive prove of this result.

Lemma 2 *[15] For any cycle* C_n *of length* n,

$$\chi'_a(C_n) = \begin{cases} 3, & \text{if } n \equiv 0 \pmod 3; \\ 4, & \text{if } n \equiv 1 \pmod 3; \\ 5, & \text{if } n \equiv 2 \pmod 3. \end{cases}$$

Proof: We assume that C_n is a cycle of length n. Let v_i's and e_i's; $i = 0, 1, \ldots, n-1$ be the vertices and edges of C_n respectively. To colour the edges of the cycle by *avd-colouring* we classify the cycle into three groups, viz., C_{3k}, C_{3k+1} and C_{3k+2} respectively. Here $e_0 = (v_0, v_1)$, $e_{n-1} = (v_0, v_{n-1})$ and $e_i = (v_i, v_{i+1})$ for $i = 1, 2, \ldots, n-2$. Now we colour the edges of C_n as follows.

Case 1: If $n = 3k \equiv 0 \pmod 3$.

Then we colour the edges as

$$\varphi(e_i) = \begin{cases} 0, & \text{if } i \equiv 0 \pmod 3; \\ 1, & \text{if } i \equiv 1 \pmod 3; \\ 2, & \text{if } i \equiv 2 \pmod 3. \end{cases}$$

here $\Phi(v_i) = \begin{cases} \{2,0\}, & if\ i \equiv 0\ (mod\ 3); \\ \{0,1\}, & if\ i \equiv 1\ (mod\ 3); \\ \{1,2\}, & if\ i \equiv 2\ (mod\ 3). \end{cases}$

Case 2: If $n = 3k+1 \equiv 1 \pmod 3$.

Here we colour the first $3k$ edges of C_{3k+1} as

$$\varphi(e_i) = \begin{cases} 0, & \text{if } i \equiv 0 \pmod 3; \\ 1, & \text{if } i \equiv 1 \pmod 3; \\ 2, & \text{if } i \equiv 2 \pmod 3, \end{cases}$$

and remaining edge as $\phi(e_{3k}) = 3$. Here the colour set of the vertices v_i; for $i = 1, 2, \ldots, 3k-1$ are

$$\Phi(v_i) = \begin{cases} \{0,1\}, & if\ i \equiv 1\ (mod\ 3); \\ \{1,2\}, & if\ i \equiv 2\ (mod\ 3); \\ \{2,0\}, & if\ i \equiv 0\ (mod\ 3); \end{cases}$$

and

$$\Phi(v_i) = \begin{cases} \{2,3\}, & if\ i = 3k; \\ \{2,0\}, & if\ i = 0. \end{cases}$$

Case 3: If $n = 3k+2 \equiv 2 \pmod 3$.

We colour the edges e_i; $i = 0, 1, \ldots, 3k-7$ by using the same process given in case 1 of this lemma. Then we coluor the eight edges by *avd-colouring* as

$$\phi(e_i) = \begin{cases} 0, & if\ i = 3k-6, 3k-2; \\ 1, & if\ i = 3k-5, 3k-1; \\ 2, & if\ i = 3k-4, 3k; \\ 3, & if\ i = 3k-3, 3k+1. \end{cases}$$

And the colour sets for the vertices v_i; $i = 1, 2, \ldots, 3k-4$ are same as the colour sets of the vertices v_i; $i = 1, 2, \ldots, 3k-1$ of the above case. For remaining vertices

$$\Phi(v_i) = \begin{cases} \{3,0\}, & if\ i = 0, 3k-2; \\ \{2,3\}, & if\ i = 3k-3, 3k+1; \\ \{0,1\}, & if\ i = 3k-1; \\ \{1,2\}, & if\ i = 3k. \end{cases}$$

Case 4: If $n = 5$.

Then we colour the edges of C_5 as $\phi(e_0) = 0$, $\phi(e_1) = 1$, $\phi(e_2) = 2$, $\phi(e_3) = 3$ and $\phi(e_4) = 4$.

Thus from all above cases we see that minimum colours require to colour the edges of a cycle C_n are 3, when

$n \equiv 0, 4$ when $n \equiv 1, 2$ but $n \neq 5$ and 5 when n is equal to 5, respectively.

That is, $$\chi'_a(C_n) = \begin{cases} 3, & if\ n \equiv 0\ (mod\ 3); \\ 4, & if\ n \equiv 1,2\ (mod\ 3)\ and\ n \neq 5; \\ 5, & if\ n = 5. \end{cases}$$

B. Avd-edge colouring of two cycles

Lemma 3 *If a graph G contains two cycles of finite lengths and they are joined with a common cutvertex, then*

$$\chi'_a(G) = \begin{cases} 5, & \text{when two cycles are of length 5;} \\ 4, & \text{otherwise.} \end{cases}$$

Proof: Let us consider that the graph G contains two cycles of lengths n and m respectively, joined by a cutvertex v_0. Here degree of v_0 is 4. Again, let v_i, e_i; $i = 0,1,\ldots,n-1$ and v_0, v'_1, …, v'_{m-1} e'_j; $j = 0,1,\ldots,m-1$ be the vertices and edges of C_n and C_m respectively, where, $e_0 = (v_0, v_1), e_{n-1} = (v_0, v_{n-1})$ and $e_i = (v_i, v_{i+1})$; $i = 1,2,\ldots,n-2$, $e'_0 = (v_0, v'_1), e'_{m-1} = (v_0, v'_{m-1})$ and $e'_j = (v'_j, v'_{j+1})$; $j = 1,2,\ldots,m-2$.

Now we colour the edges of the graph by *avd-colouring* as follows.

Case 1: If $n = 3k \equiv 0$ (mod 3) and $m = 3k \equiv 0$ (mod 3).

First we colour the edges of C_n as per the rule of *avd-edge colouring* given in case 1 of previous lemma. Now, we colour the edges e'_j; $j = 3,4,\ldots,3k-2$ of C_m as

$$\varphi(e_i) = \begin{cases} 2, & \text{if } i \equiv 0\ (\text{mod } 3); \\ 1, & \text{if } i \equiv 1\ (\text{mod } 3); \\ 0, & \text{if } i \equiv 2\ (\text{mod } 3). \end{cases}$$

And the remaining edges as

$$\phi(e'_j) = \begin{cases} 1, & if\ j = 0; \\ 2, & if\ j = 1; \\ 0, & if\ j = 2; \\ 3, & if\ j = 3k-1. \end{cases}$$

Case 2: If $n = 3k \equiv 0$ (mod 3) and $m = 3k+1 \equiv 1$ (mod 3).

Here we colour the edges of C_n as same as given in case 1 of previous lemma. Now, we colour the edges e'_j; $j = 3,4,\ldots,3k-1$ of C_m as same process which has done in the above case for the edges e'_j; $j = 3,4,\ldots,3k-2$. Then the colouring procedure for the edges e'_j; $j = 0,1,3,3k$ are

$$\phi(e'_0) = 1,\ \phi(e'_1) = 2,\ \phi(e'_2) = 0, \text{ and } \phi(e'_{3k}) = 3.$$

Case 3: If $n = 3k \equiv 0$ (mod 3) and $m = 3k+2 \equiv 2$ (mod 3).

The colouring process of the edges of C_n are as same as given in case 1 of previous lemma. Now, we colour the edges e'_j; $j = 3,4,\ldots,3k$ of C_m as same process which has done in the above case for the edges e'_j ; $j = 3,4,\ldots,3k-1$. Then we colour remaining four edges as

$$\phi(e'_0) = 1,\ \phi(e'_1) = 2,\ \phi(e'_2) = 0, \text{ and } \phi(e'_{3k+1}) = 3.$$

Case 4: If $n = 3k+1 \equiv 1$ (mod 3) and $m = 3k+1 \equiv 1$ (mod 3).

We colour the edges of C_n as per rule given in case 2 of Lemma 2. Now, we colour the edges e'_j ; $j = 3,4,\ldots,3k-1$ of C_m as

$$\varphi(e'_i) = \begin{cases} 1, & \text{if } i \equiv 0\ (\text{mod } 3); \\ 3, & \text{if } i \equiv 1\ (\text{mod } 3); \\ 0, & \text{if } i \equiv 2\ (\text{mod } 3), \end{cases}$$

and for the last edge $\phi(e'_{3k}) = 2$.

Case 5: If $n = 3k+1 \equiv 1$ (mod 3) and $m = 3k+2 \equiv 2$ (mod 3).

Here the colouring process of the edges of C_n are same as given in case 2 of previous lemma. Then, we colour the first $3k+1$ edges of C_m using same process of the edges e'_j; $j = 0,1,\ldots,3k-1$ given in the case 4 of this lemma. And we colour remaining edge e'_{3k+1} as $\phi(e'_{3k+1}) = 2$.

Case 6: If $n = 3k+2 \equiv 2$ (mod 3) and $m = 3k+2 \equiv 2$ (mod 3).

The colouring process of the edges of C_n is as same as given in case 3 of Lemma 2. Now, we colour the edges of C_m by using the *avd-colouring* process given in case 5 of this lemma.

When n and m both are exactly equal to 5, then we colour the edges of the graph as

$$\phi(e_i) = \begin{cases} 0, & for\ i = 0; \\ 1, & for\ i = 1; \\ 2, & for\ i = 2; \\ 3, & for\ i = 3; \\ 4, & for\ i = 4; \end{cases} \quad \text{and} \quad \phi(e'_j) = \begin{cases} 1, & for\ j = 0; \\ 2, & for\ j = 1; \\ 0, & for\ j = 2; \\ 1, & for\ j = 3; \\ 3, & for\ j = 4. \end{cases}$$

So, from all above cases we see that 4 colours are required to colour the edges of the graph and 5 colours, when length of each cycle is equal to 5.

Thus, $$\chi'_a(G) = \begin{cases} 5 = \Delta + 1, & \text{when two cycles are of length 5;} \\ 4 = \Delta, & \text{otherwise.} \end{cases}$$

C. Avd-edge colouring of three cycles

By using the result of above lemma we can state the following result.

Lemma 4 *Let a graph G contains three cycles of finite lengths and they are joined with a common cutvertex v_0. If $\Delta(=6)$ be the degree of v_0, then $\chi'_a(G) = \Delta$.*

Proof: We assume that the graph G contains three cycles C_n, C_m and C_p respectively. They are joined by a common cutvertex v_0 with degree $\Delta = 6$. Let v_i , e_i ; $i = 0,1,\ldots,n-1$; v_0, v'_1, …, v'_{m-1} e'_j; $j = 0,1,\ldots,m-1$ and v_0, v''_1, …, v''_{p-1} e''_k; $r = 0,1,\ldots,p-1$ be the vertices, edges of C_n, C_m and C_p respectively, where

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

$e_0 = (v_0, v_1), e_{n-1} = (v_0, v_{n-1})$ and $e_i = (v_i, v_{i+1})$; $i = 1, 2, \ldots, n-2$, $e'_0 = (v_0, v'_1), e'_{m-1} = (v_0, v'_{m-1})$ and $e'_j = (v'_j, v'_{j+1})$; $j = 1, 2, \ldots, m-2$. $e''_0 = (v_0, v''_1), e''_{m-1} = (v_0, v''_{p-1})$ and $e''_r = (v''_r, v''_{r+1})$; $r = 1, 2, \ldots, p-2$.

First we colour two cycles according to the rule describe in Lemma 3. Now we colour the edges of C_p as follows.

Case 1: For $n = 3k$, $m = 3k$ and $p = 3k + i$, $i = 0, 1, 2$.

When $p = 3k$, then we colour edges e''_r ; $r = 1, 2, \ldots, 3k-2$ by

$$\phi(e''_r) = \begin{cases} 2, \text{ if } r \equiv 0 \text{ (mod 3)}; \\ 0, \text{ if } r \equiv 1 \text{ (mod 3)}; \\ 1, \text{ if } r \equiv 2 \text{ (mod 3)}. \end{cases}$$

And the remaining two edges by $\phi(e''_0) = 4$ and $\phi(e''_{3k-1}) = 5$.

When $p = 3k + 1$, then colouring scheme for the edges e''_r ; $r = 1, 2, \ldots, 3k-1$ are

$$\phi(e''_r) = \begin{cases} 2, \text{ if } r \equiv 0 \text{ (mod 3)}; \\ 0, \text{ if } r \equiv 1 \text{ (mod 3)}; \\ 1, \text{ if } r \equiv 2 \text{ (mod 3)}. \end{cases}$$

And for the edges e''_0 and e''_{3k} are $\phi(e''_0) = 4$ and $\phi(e''_{3k}) = 5$.

When $p = 3k + 2$, then we colour the edges e''_0 , e''_{3k+1} and e''_r ; $r = 1, 2, \ldots, 3k$ as

$$\phi(e''_r) = \begin{cases} 4, & \text{if } r = 0; \\ 5, & \text{if } r = 3k+1; \end{cases} \text{ and } \phi(e''_r) = \begin{cases} 2, \text{ if } r \equiv 0 \text{ (mod 3)}; \\ 0, \text{ if } r \equiv 1 \text{ (mod 3)}; \\ 1, \text{ if } r \equiv 2 \text{ (mod 3)}, \end{cases}$$

respectively.

Case 2: For $n = 3k + i$; $i = 0, 1, 2$, $m = 3k + j$ and $p = 3k + j$; $j = 1, 2$.

When $n = 3k + i$; $i = 0, 1$, $m = 3k + 1$ and $p = 3k + 1$, then we colour the edges of C_p according to the rule of case 1 (for $3k + 1$) of this lemma.

When $n = 3k + i$; $i = 0, 1, 2$, $m = 3k + j$; $i = 1, 2$ and $p = 3k + 2$, then the colouring procedure of the edges of C_p is same as given in case 1 (for $3k + 1$) of this lemma.

Case 3: For $n = 5$, $m = 5$ and $p = 5$.

Here we colour the edges e''_r ; $r = 0, 1, \ldots, 4$ as

$\phi(e''_0) = 3$, $\phi(e''_1) = 1$, $\phi(e''_2) = 0$, $\phi(e''_3) = 2$ and $\phi(e''_4) = 5$.

So, from the all above cases we see that six colours (which is equal to degree of cutvertex) are needed to colour the edges of the graph.

Thus, $\chi'_a(G) = 6 \ (= \Delta)$.

D. Avd-edge coloring of finite number of cycles

From lemma 3, lemma 4 we can conclude the general form of these lemmas which is given below.

Lemma 5 *If the graph G contains finite cycles of finite lengths, joined with a common cutvertex with degree Δ, then $\chi'_a(G) = \Delta$.*

Proof: From Lemmas 2, 3 and 4, we see that $\chi'_a(C_5) = \Delta + 3$, $\chi'_a(C_5^1 \bigcup_{v_0} C_5^2) = \Delta + 1$, where C_5^1 , C_5^2 are two different cycles of lengths 5 joined by v_0 and $\chi'_a(C_5^1 \bigcup_{v_0} C_5^2 \bigcup_{v_0} C_5^3) = \Delta$ respectively, where C_5^1 , C_5^2 and C_5^3 are three different cycles of lengths 5. When a graph G contains two or three cycles of finite lenghts other than 5 joined by a common cutvertex, then the value of χ'_a is Δ . So, if we prove for a graph which contains finite number of cycles of lengths 5, the value of χ'_a is equal to degree of the cutvertex, then general result will be proved automatically.

Let G contains k number of cycles of lengths 5 (shown in Fig. 2), joined by a common cutvertex v_0 with degree Δ . So, $\Delta = 2k$. Let v_0 , v_i ; v_0 , v'_i ; v_0 , v''_i ; …; v_0 , $v_i^{(k-1)}$; $i = 0, 1, \ldots, 4$ be the vertices of G . And e_i ; e'_i ; e''_i ; $e_i^{(k-1)}$; $i = 0, 1, \ldots, 4$ are the edges, where

$e_0 = (v_0, v_1), e_4 = (v_0, v_4)$ and $e_i = (v_i, v_{i+1})$; $i = 1, 2, 3$. $e_0^{(p)} = (v_0, v_1^{(p)}), e_4^{(p)} = (v_0, v_4^{(p)})$ and $e_i^{(p)} = (v_i^{(p)}, v_{i+1}^{(p)})$; $p = 1, 2, \ldots, k-1$, $i = 1, 2, 3$.

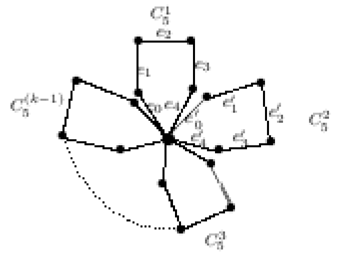

Fig. 2: A graph G contains k number of different C_5 's.

Now we colour the edges of first three cycles of lengths 5 by *avd-colouring* according to the process as given in case 5 of Lemma 4. Then we colour other edges as follows

$$\phi(e_i^j) = \begin{cases} 2j, \text{ if } i = 0; \\ 2j+1, \text{ if } i = 4 \text{ and } j = 3, 4, \ldots, k-1, \end{cases}$$ and

$$\phi(e_i^{(j)}) = \begin{cases} 0, \text{ if } i = 1; \\ 1, \text{ if } i = 2; \\ 2, \text{ if } i = 3 \text{ and } j = 3, 4, \ldots, k-1. \end{cases}$$

Here, $\phi(e_4^{(k-1)}) = 2(k-1) + 1 = 2k - 1$. So, $2k$ colours are needed to colour the edges of the graph G .

Thus, $\chi'_a(G) = \Delta$.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

IV. AVD-EDGE COLOURING OF OTHER SUBGRAPHS OF CACTUS GRAPH

Lemma 6 *Let G be a graph contains finite number of cycles of finite lengths and finite number of edges, joined with a common cutvertex. If Δ be the degree of the cutvertex, then* $\chi'_a(G) = \Delta$.

Proof: According to the previous lemma we know that if a graph contains finite number of cycles of any lengths and they are joined by a common cutvertex with degree Δ, then $\chi'_a(G) = \Delta$. So, if we prove that χ'_a equal to Δ for G, contains n cycles of lengths 5 and q number of edges, joined by a cutvertex v_0 with degree Δ, then we can say that the above statement is true. Here $\Delta = 2n+q$.

Let e_{0i}; $i = 0,1,\ldots,q-1$ be the edges incident on the common cutvertex v_0 of G. Now we colour the edges of n cycles by *avd-colouring* using the rule given in Lemma 5. Then we colour the q edges as

$$\phi(e_{0i}) = (2n-1)+(i+1);\ i = 0,1,\ldots,q-1.$$

Here $\phi(e_{0,q-1}) = 2(n-1)+q = 2n+q-1$. So, $(2n+q)$ colours are required.

Therefore, $\chi'_a(G) = \Delta$.

A. *Avd-edge coloring of sun*

The sun graph is obtained by adding an edge to each of the vertex of any cycle of finite length. If we add an edge to each vertex of C_n, then we get a sun S_{2n} with $2n$ vertices. So C_n is a subgraph of S_{2n}. We prove the following result for sun.

Lemma 7 *For any sun S_{2n},*

$$\chi'_a(S_{2n}) = \begin{cases}\Delta+2, & \text{if } n=5;\\ \Delta+1, & \text{otherwise};\end{cases} \quad \text{where } \Delta = 3.$$

Proof: Let v_i, e_i; $i = 0,1,\ldots,n-1$ be the vertices and edges of the cycle C_n. To construct S_{2n} (shown in Fig. 3), we add an edge $e'_i = (v_i, v'_i)$ to the vertex v_i. The adjacent vertices of v_i of C_n are v_{i-1} and v_{i+1}. And v'_i's are all pendent vertices. To colour the edges of sun by *avd-colouring*, we consider the following four cases.

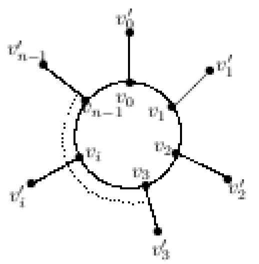

Fig. 3: Sun (S_{2n})

Case 1: If $n \equiv 0 \pmod 3$, i.e., C_{3k}.

Here we colour the edges of C_{3k} as per the rule given in case 1 of Lemma 2. And other edges of S_{2n} as

$$\phi(e'_i) = \begin{cases}1, & \text{if } i \equiv 0 \pmod 3;\\ 2, & \text{if } i \equiv 1 \pmod 3;\\ 0, & \text{if } i \equiv 2 \pmod 3.\end{cases}$$

Case 2: If $n \equiv 1 \pmod 3$, i.e., C_{3k+1}.

The colouring procedure of the edges of C_{3k+1} are same as given in case 2 of Lemma 2. And the colours of the edges e'_i; $i = 0,1,\ldots,3k-1$ are given by

$$\phi(e'_i) = \begin{cases}1, & \text{if } i \equiv 0 \pmod 3;\\ 2, & \text{if } i \equiv 1 \pmod 3;\\ 0, & \text{if } i \equiv 2 \pmod 3.\end{cases}$$

and $\phi(e'_{3k}) = 0$.

Case 3: If $n \equiv 2 \pmod 3$, i.e., C_{3k+2}.

Here we colour the edges of C_{3k+2} as per the rule given in case 3 of Lemma 2. The edges e'_i; $i = 0,1,\ldots,3k-2$ of S_{2n} as

$$\phi(e'_i) = \begin{cases}1, & \text{if } i \equiv 0 \pmod 3;\\ 2, & \text{if } i \equiv 1 \pmod 3;\\ 0, & \text{if } i \equiv 2 \pmod 3.\end{cases}$$

and $\phi(e'_i) = \begin{cases}2, & \text{for } i = 3k-1;\\ 0, & \text{for } i = 3k, 3k+1.\end{cases}$

Case 4: If $n = 5$.

Then we colour the edges e'_i; $i = 0,1,2,3,4$ of S_{2n} as

$$\phi(e'_i) = \begin{cases}1, & \text{if } i = 0;\\ 2, & \text{if } i = 1;\\ 0, & \text{if } i = 2,3,4.\end{cases}$$

Thus, from all the above cases we find that 4 or 5 colours are required to colour the edges of a sun. For sun $\Delta = 3$.

Therefore, $\chi'_a(S_{2n}) = \begin{cases}\Delta+2, & \text{if } n=5;\\ \Delta+1, & \text{otherwise.}\end{cases}$

Let a graph be obtained from the sun S_{2n} by joining an edge to each of the pendent vertex. We obtain the following result for such a graph.

Lemma 8 *Let G be a graph obtained from S_{2n}, by adding an edge to each of the pendent vertex, then*

$$\chi'_a(G) = \begin{cases}5, & \text{if } n=5;\\ 4, & \text{otherwise.}\end{cases}$$

The is another important result of avd-colouring of a subgraph of cactus graph which is stated below.

Lemma 9 *Let G be a graph contains a cycle of any length and finite number of edges. If they are joined by a common cutvertex v_0 with degree Δ, then* $\chi'_a(G) = \Delta$.

Corollary 1 *If the length of the cycle is 5, then,*

$$\chi'_a(G) = \begin{cases}\Delta+2, & \text{if one edge incident on cutvertex};\\ \Delta+1, & \text{if two edges incident on cutvertex}.\end{cases}$$

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

Lemma 10 *If a graph G contains two cycles of finite lengths and they are joined by an edge, then*

$$\chi'_a(G) = \begin{cases} \Delta+2, if\ lenghts\ of\ two\ cycles\ are\ 5; \\ \Delta+1,\ otherwise; \end{cases}$$

here $\Delta = 3$.

Proof: Let C_n and C_m be two cycles joined by an edge (v_0, v'_0) (shown in Fig. 4).

Let v_i, e_i; $i = 0,1,\ldots,n-1$; v'_j, e'_j; $j = 0,1,\ldots,m-1$ be the vertices and edges of C_n and C_m respectively.

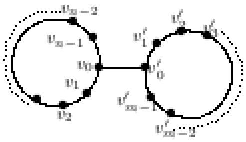

Fig. 4: The graph contains two cycles, joined by an edge

We colour the edges of C_n as per the rule given in Lemma 2. And we colour the edge (v_0, v'_0) by 1, i.e., $\phi(v_0, v'_0) = 1$. Now the *avd-edge colouring* procedure of the cycle C_m is given bellow.

Case 1: For $n = 3k$, $m = 3k+i$, $i = 0,1,2$.

When $m = 3k$, then we colour the first $3k-1$ edges of C_m as

$$\phi(e'_j) = \begin{cases} 0, & if\ i \equiv 0\ (mod\ 3); \\ 1, & if\ i \equiv 1\ (mod\ 3); \\ 2, & if\ i \equiv 2\ (mod\ 3); \end{cases}$$

and the last edge as $\phi(e'_{3k-1}) = 3$.

When $m = 3k+1$ and $3k+2$, the colouring process is same as above.

Case 2: For $n = 3k+1$, $m = 3k+i$, $i = 1,2$.

When $m = 3k+1$, then we colour the first $3k-1$ edges of C_m as per second subcase of case 1. We colour the remaining two edges as

$$\phi(e'_j) = \begin{cases} 3, if\ \ j = 3k; \\ 2, if\ \ j = 3k+1. \end{cases}$$

When $m = 3k+2$, then we colour the first $3k-1$ edges by using the rule given in above subcase. Now we colour the last three edges as

$$\phi(e'_j) = \begin{cases} 3, if\ \ j = 3k; \\ 0, if\ \ j = 3k+1; \\ 2, if\ \ j = 3k+2. \end{cases}$$

Case 3: For $n = 3k+2$, $m = 3k+2$.

Here we colour the edges of C_m according to the rule of second subcase of case 2.

When $n = 5$ and $m = 5$, then we colour the edges as $\phi(e'_0) = 0$, $\phi(e'_1) = 1$, $\phi(e'_2) = 2$, $\phi(e'_3) = 0$ and $\phi(e'_4) = 3$ respectively.

So, from the above cases we see that χ'_a equal to 5 when two cycles are of lengths 5, and 4 for other cases. In this case $\Delta = 3$.

Therefore,

$$\chi'_a(G) = \begin{cases} \Delta+2, \text{ if lenghts of two cycles are } 5; \\ \Delta+1, \text{ otherwise.} \end{cases}$$

Lemma 11 *If a graph G contains two suns and they are joined by only one edge, then*

$$\chi'_a(G) = \begin{cases} \Delta+2, if\ lenghts\ of\ two\ cycles\ are\ 5; \\ \Delta+1,\ otherwise; \end{cases}$$

here $\Delta = 3$ is the degree of G.

Lemma 12 *Let G be a graph contains one cycle of finite length and each vertex of the cycle contains another cycle of finite length. If $\Delta = 4$ be the degree of the graph, then,*

$$\chi'_a(G) = \begin{cases} \Delta+1, if\ length\ of\ main\ cycle\ is\ 5; \\ \Delta,\ otherwise; \end{cases}$$

Lemma 13 *For any path P_n of length n,*

$$\chi'_a(P_n) = \begin{cases} 1, if\ \ n = 2; \\ 2, if\ \ n = 3; \\ 3, if\ \ n > 3. \end{cases}$$

Proof: Let v_0, $v_1, \ldots, v_{n-1}$; e_0, $e_1, \ldots, e_{n-1}$ be the vertices and edges of P_n,

where $e_i = (v_i, v_{i+1})$; $i = 0,1,\ldots,n-2$.

Then we colour the edges of the path as follows:

$$\phi(e_i) = \begin{cases} 0, & \text{if } j \equiv 0 \text{ (mod 3);} \\ 1, & \text{if } j \equiv 1 \text{ (mod 3);} \\ 2, & \text{if } j \equiv 2 \text{ (mod 3).} \end{cases}$$

So, we need 1 and 2 colours when $n = 2,3$ and 3 colours when $n > 3$.

Therefore, $\chi'_a(P_n) = \begin{cases} 1,2, \text{ if } n = 2,3; \\ 3, \text{ if } n > 3. \end{cases}$

V. AVD-EDGE COLOURING OF CATERPILLAR

Definition 1 *A caterpillar C is a tree where all vertices of degree ≥ 3 lie on a path, called the backbone of C. The hairlength of a caterpillar graph C is the maximum distance of a non-backbone vertex to the backbone.*

The result for any caterpillar graph is give below.

Lemma 14 *For any caterpillar graph G,*

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

$$\chi'_a(G)=\begin{cases}\Delta, \text{ if two vertices of maximum} \\ \quad \text{degree are not adjacent;} \\ \Delta+1, \text{ if two vertices of maximum} \\ \quad \text{degree are adjacent.}\end{cases}$$

Proof: Let the length of the path of the caterpillar be n. Then we colour the edges of the path by *avd-colouring* using Lemma 13.

Here only we prove $\chi'_a(G)=\Delta+1$, when two vertices with maximum degree (Δ) are adjacent. The graph of Fig. 5 is a caterpillar. Let u, v be the vertices with maximum degree of G and they are adjacent. That is, $d(u)=d(v)=\Delta$. Again let u_i's and v_i's be the adjacent vertices of u and v respectively, where $i=1,2,\ldots,\Delta-1$.

Without loss of generality let the colour of the edge (u,v) be 0.

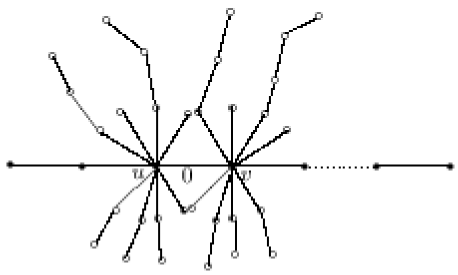

Fig. 5: A caterpillar

Then $\phi(u,v)=0$, $\phi(u,u_i)=i$ ($i=1,2,\ldots,d(u)-1$), $\phi(v,v_i)=i+1$ ($i=1,2,\ldots,d(v)-1$).

So, there are $0,1,2,\ldots,\Delta$, i.e., total $\Delta+1$ colours are required.

In the caterpillar all the vertices degree ≥ 3 must lie on the path. So the vertices which are at a distance $2,3,4,\ldots$, from the path are of degree either 1 or 2. Let u'_i, u''_i, u'''_i, $\ldots$, be the vertices at distance $2,3,4,\ldots$. So we colour the edges (u_i,u'_i), (u'_i,u''_i), (u''_i,u'''_i), $\ldots$, from any of the colour $0,1,2,\ldots,\Delta$, such a way that no two adjacent vertices have the same set of colours. The rule is similar for the edges (v_i,v'_i) (v'_i,v''_i), (v''_i,v'''_i), $\ldots$ etc.

VI. AVD-EDGE COLOURING OF LOBSTER

Another subclass of cactus graph is called lobster graph. The definition of lobster graph is given below.

Definition 2 *A lobster is a tree having a path (of maximum length) from which every vertex has distance at most* k*, where* k *is an integer.*

The maximum distance of the vertex from the path is called the diameter of the lobster graph. There are many types of lobsters given in literature like diameter 2, diameter 4, diameter 5, etc.

From definition of lobster we know that lobster is a one kind of tree. So, by using the result of *avd-edge colouring* for any tree [15], we can conclude the result for lobster which is stated below.

Lemma 15 *For any lobster* G,

$$\chi'_a(G)=\begin{cases}\Delta, \text{ if two vertices of maximum} \\ \quad \text{degree are not adjacent;} \\ \Delta+1, \text{ if two vertices of maximum} \\ \quad \text{degree are adjacent.}\end{cases}$$

Lemma 16 *Let* G_1 *and* G_2 *be two cactus graphs. If* $\Delta_1\leq\chi'_a(G_1)\leq\Delta_1+3$ *and* $\Delta_2\leq\chi'_a(G_2)\leq\Delta_2+3$, *then,* $\Delta\leq\chi'_a(G)\leq\Delta+3$, *where* $G=G_1\bigcup_v G_2$.

Proof: Let G_1 and G_2 be two cactus graphs and Δ_1, Δ_2 be the degrees of them. Now if we merge two cactus graphs with the vertex v then we get a new cactus graph G ($=G_1\cup_v G_2$). Let Δ be the degree of G and we will prove that max $\{\Delta_1,\Delta_2\}\leq\Delta_1+\Delta_2$. For the graph G_1 , $\Delta_1\leq\chi'_a(G_1)\leq\Delta_1+3$ and for the graph G_2 , $\Delta_2\leq\chi'_a(G_2)\leq\Delta_2+3$. Here we have to prove that the lower and upper bounds will preserve for the new graph G.

Let maximum degree Δ_1 of G_1 be attained at u and v be a vertex of G_2 whose degree is Δ_2. Again, let u_i's and v_j's be the adjacent vertices of u and v respectively, where $i=1,2\ldots,\Delta_1$ and $j=1,2\ldots,\Delta_2$. We merge G_1 and G_2 at v and let the adjacent vertices of v be v_i; $i=1,2\ldots,\Delta_1+\Delta_2$. If we merge one vertex of G_1 other than u with v of G_2 then the adjacent vertices of v are v_i; $i=1,2\ldots,\max\{\Delta_1,\Delta_2\}$. So we can say that Δ lies between max $\{\Delta_1,\Delta_2\}$ and $\Delta_1+\Delta_2$. Now from this result we can conclude that the upper and lower bounds of χ'_a are remains same as G.

The *avd-edge colouring* of all subgraphs of cactus graphs and their combinations are discussed in the previous lemmas. From these results we conclude that χ'_a-value of any cactus graph can not be more than $\Delta+3$. Hence we have the following theorem follows.

Theorem 1 *If* Δ *is the degree of a cactus graph* G, *then* $\Delta\leq\chi'_a(G)\leq\Delta+3$.

Proof. The *avd-edge colouring* of all possible subgraphs of cactus graph are discussed and have shown that $\Delta\leq\chi'_a(G)\leq\Delta+3$.. Let G be obtained by v-union of two cactus graphs then G becomes a cactus graph and it is proved that $\chi'_a(G)$ should satisfy the inequality $\Delta\leq\chi'_a(G)\leq\Delta+3$. (Lemma 16).

Hence the theorem.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

VII. THE ALGORITHM AND ITS TIME COMPLEXITY

To start the algorithm of *avd-edge colouring* of a cactus graph, we first construct a graph G' which is equivalent to the given graph G.

A. Construction of an equivalent graph G' of G

Using DFS we obtain all blocks and cutvertices of a cactus graph $G = (V, E)$. Let the blocks be B_0, B_1, B_2,..., B_{N-1} and the cutvertices be C_1, C_2, $C_3, \ldots, C_R$ where N is the total number of blocks and R is the total number of cutvertices.

The blocks of the cactus graph shown in Fig. 6 are

$\{B_0 = (7,8,9,10,11,12,13), B_1 = (11,14,15,16,17,18,19),$
$B_2 = (12,32,33,34), B_3 = (7,38), B_4 = (7,39), B_5 = (7,1),$
$B_6 = (7,35,36,37), B_7 = (9,44,45), B_8 = (18,28,29,30,31),$
$B_9 = (16,20), B10 = (16,23,24,25), B11 = (16,26),$
$B_{12} = (1,40), B_{13} = (1,2,3,4,5,6), B_{14} = (20,21),$
$B_{15} = (20,22), B_{16} = (26,27), B_{17} = (3,41), B_{18} = (4,42,43)\}.$

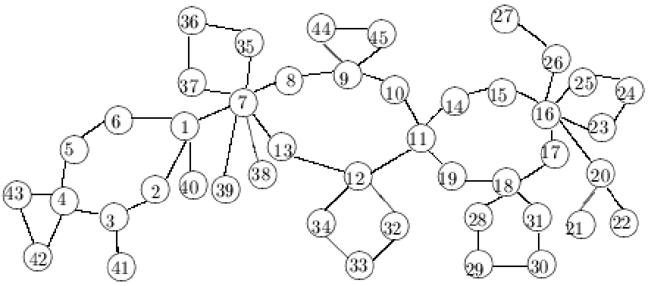

Fig. 6: A cactus graph G

and the cutvertices are {7,1,4,3,9,11,12,18,16,20,26} respectively.

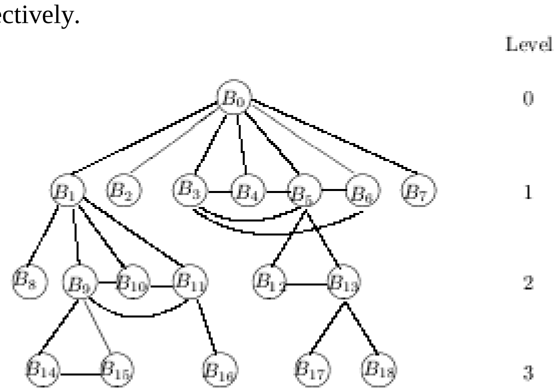

Fig. 7: The equivalent graph G' of G

Now we have in a position to construct an *equivalent graph* G' of G whose vertices are the blocks of G and an edge is defined between two blocks if they are adjacent blocks of G.

i.e., $G' = (V', E')$ where $V' = \{B_0, B_1, \ldots, B_{N-1}\}$ and $E' = \{(B_i, B_j) : i \neq j, i, j = 0,1,\ldots, N-1$, B_i and B_j are adjacent blocks $\}$.

The graph G' for the graph G of Fig. 6 is shown in Fig. 7.

B. Avd-colouring of edges

Now, we take any arbitrary cycle as starting block. The block is so chosen that the degree of the cutvertex is maximum. We denote the starting block as B_0 and let the level of B_0 be 0. Now, the blocks adjacent to B_0 are taken at level 1. The blocks adjacent to the blocks of level 1 are taken as the blocks of level 2 and so on. Now we colour the block B_0 using the rule stated at Lemma 2. Now we colour the blocks of level 1 from left to right. Let the blocks of level 1 be $B_{11}, B_{12}, B_{13}, \ldots$. They are either edges or cycles. We consider the first block B_{11} of level 1 which is adjacent to B_0. If the block is an edge then colour the block by using Lemma 7. If it is a cycle of finite length then we colour it by using Lemma 3. The blocks which are adjacent to B_0 we colour them according to the rule of the block B_{11}. Next we colour the edges of the block B_{12}. Suppose it is not adjacent to B_{11}. Then if B_{12} is an edge, we use Lemma 7 to colour the edge. If B_{11} is a cycle is a cycle of finite length, then Lemma 12 is used. Let B_{12} be also adjacent to B_{11}. If B_{11} is an edge and B_{12} is a cycle then, Lemma 6 is used and if B_{11} and B_{12} both are edges then Lemma 9 is used. Let us consider the block B_{1i} for some i at level 1. If it is not adjacent with any block of level 1, then we colour it by Lemmas 7 and 12. But, if it is adjacent at least one block of level 1 then we follow the rules of lemmas 5, 6, 9 and 16. Now we colour the edges of the blocks of level 2, then level 3 and so on as per the procedure mentioned above. Suppose a block at level l, say B_{lj} is an edge and it is adjacent to a block say $B_{l+1,k}$ at level $l+1$ which is also an edge. Then we colour the block by using the Lemma 8. Let a block, say $B_{k-1,i}$ at level $k-1$ be a cycle of finite length, its adjacent block at level k, say B_{kj}, is an edge, its adjacent block at level $k+1$, say $B_{k+1,p}$ is a cycle of finite length. Then we colour the block $B_{k+1,p}$ at level $k+1$ by using Lemma 10. Suppose a block $B_{q-2,i}$ is a block at level $q-2$, which is a cycle of finite length. Its adjacent block say $B_{q-1,j}$ at level $q-1$ is an edge. The adjacent block of $B_{q-1,j}$, $B_{q,j}$ at level q is a cycle of finite length and if its adjacent block $B_{q+1,j}$ is an edge at level $q+1$, then we colour this block by using the Lemma 11.

Algorithm MINAVDEC

Input: The cactus graph $G = (V, E)$.

Output: Avd-colouring of its edges.

Step 1: Compute the blocks and cutvertices of G and

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

construct an equivalent graph G' of G.

Step 2: Let B_0 starting block, where the degree of the cutvertex of B_0 is maximum.

Step 3: We colour the block B_0 using Lemma 2.

Step 4: Consider the blocks B_{1j}, $j = 1, 2, 3, \ldots,$ of level 1. Colour the blocks from left to right as follows.

(i) Take the first block B_{11} which is adjacent to B_0. If it is an edge then we colour B_{11} by using Lemma 7 and if it is cycle then we colour it by Lemma 3.

(ii) Next we consider the second block B_{12}. If it is not adjacent to B_{11}, then colour it by either Lemma 7 or Lemma 3. If B_{12} adjacent to B_{11} then colour it by using lemmas 5, 6, 9 and 16.

(iii) Consider the block B_{1i}. If it is not adjacent to any block of level 1, then colour it by lemma 7 or 3. But if it is adjacent to at least one block of level 1 then follow the rules of lemmas 5, 6, 9 and 16.

(iv) The blocks which are adjacent to B_0 only then colour them by the process similar to B_{11}.

Step 5: Suppose a block at level l, say B_{lj} is an edge and another block $B_{l+1,k}$ at level $l+1$ adjacent to B_{lj}, which is also an edge. Then we colour them by using Lemma 8.

Step 6: Let a block $B_{k-1,i}$ at level $k-1$ be a cycle length and its adjacent block at level k be B_{kj} is an edge. If its adjacent block $B_{k+1,p}$ is a cycle of finite length at level $k+1$, then colour the blocks by using Lemma 10.

Step 7: Suppose $B_{q-2,i}$ is a block at level $q-2$ which is a cycle of finite length. Its adjacent block $B_{q-1,j}$ at level $q-1$ is an edge. The adjacent block of $B_{q-1,j}$ at level q is $B_{q,m}$, cycle of finite length. Let every vertex $B_{q,m}$ contains an edge, if $B_{q+1,n}$ one of them at level $q+1$, then we colour the blocks by using Lemma 11.

Step 8: Consider the blocks of subsequent levels and repeat steps 4 to step 7 to colour all the vertices of G

end MINAVDEC

C. Time Complexity

The correctness of the algorithm follows from the lemmas proved in the paper.

Theorem 2 *The time complexity of the algorithm MINAVDEC is* $O(n)$.

Proof. The blocks and cutvertices of any graph can be computed in $O(m+n)$ time [14]. For the cactus graph $m = O(n)$. Hence step 1 of algorithm MINAVDEC takes $O(n)$ time.

The time complexity to colour the edges of a block of size m_1 is $O(m_1)$. Step 4 colours the vertices of the blocks which are at level 1 of G'. If the number of vertices of all blocks of this level is m_2, then the time complexity for step 4 is $O(m_2)$. That is the time complexity depends upon the number of vertices of the whole graph. Since the number of vertices of the entire graph is n, the time complexity of the algorithm is $O(n)$.

VIII. CONCLUSION

The bounds of avd-edge colouring of a cactus graph and various subclass viz., cycle, sun, star, caterpillar, lobster are investigated. By marging all the results, we have obsereved that for any cactus graph the value of avd-edge chromatic number lies between Δ and $\Delta+3$. Currently we are engaged to find the bounds for different graph labelling like list coloring, graceful labeling. Harmonious labeling, etc, on cactus graphs.

REFERENCES

[1] M. Aigner, E. Triesch, Z, Tuzu, "Irregular assignments and vertex-distinguishing edge-colorings of graphs", Comninatorics 90 (A. Barlotti et al., eds.), Elsevier Science Pub., New York, pp. 1-9, 1992.

[2] S. Akbari, H. Bidkhori and N. Nosrati, " r -strong edge colorings of graphs", Discrete Math., vol. 306, pp. 3005-3010, 2006.

[3] P. N. Balister, E.Györi, "J.Lehel and R.H.Schelp, Adjacent vertex distinguishing edge-colorings", SIAM J. Discrete Math., vol. 21, No. 1, pp. 237-250, 2007.

[4] P.N.Balister, B.Bollabás and R.H.Schelp, "Vertex-distinguishing colorings of graphs with $\Delta(G)=2$ ", Discrete Math., vol. 252 (1-3), pp. 17-29, 2002.

[5] C. Bazgan, A.Harkat-Benhamdine, Hao Li and M.Woz' niak, "On the vertex-distinguishing proper edge-colorings of graphs", J. Combin. Theory B, vol. 75, pp. 288-301, 1999.

[6] A.C.Burris and R.H.Schelp, "Vertex-distinguishing proper edge colorings", J. Graph Theory, vol. 26, No. 2, pp. 73-82, 1997.

[7] J. Čern' y, M. Hor̆nák and R. Soták, "Observability of a graph", Math Slovaca, vol. 46, pp. 21-31, 1996.

[8] K. Edwards, M. Hornk and M. Woznaik, "On the neighbour-distinguishing index of a graph", Graph Com., vol. 22, pp. 341-350, 2006.

[9] M. Ghandehari and H.Hatami, "Two upper bounds for the strong edge chromatic number", preprint.

[10] H. Hatami, " $\Delta+300$ is a bound on the adjacent vertex distinguishing edge chromatic number", J. Comb. Theory Ser. B, vol. 95, pp. 246-256, 2005.

ISSN: 2277-3754
ISO 9001:2008 Certified
International Journal of Engineering and Innovative Technology (IJEIT)
Volume 3, Issue 4, October 2013

[11] M. Ho ř nák and R. Soták, "Observability of complete multipartite graphs with quipotent parts", Ars Combin, vol. 41, pp. 289-301, 1995.

[12] M. Ho ř nák and R. Soták, "Asysmtotic behaviour of the observability of Q_n", Discrete Math, vol. 176, pp. 139-148, 1997.

[13] B. Liu and G. Liu, "On the adjacent vertex distinguishing edge colourings of graphs, International Journal of Computer Mathematics, vol. 87, No. 4, pp. 726-732, 2010.

[14] E. M. Reingold, J. Nivergent and N. Deo, "Combinatorial Algorithms : Theory and Practice, Prentice Hall, Inc., Englewood Chiffs", New Jersy, 1977.

[15] Z.Zhang, L.Liu and J.Wang, "Adjacent strong edge coloring of graphs", Appl. Math. Lett., vol. 15, pp. 623-626, 2002.

AUTHOR'S PROFILE

Dr. Nasreen Khan is Assistant Professor of Mathematics Department, Global Institute of Management and Technology, West Bengal, India. Before that she was a fulltime research scholar of Applied Mathematics Department, Vidyasagar University. She has completed her Phd in 2013 under the guidance of Professor Madhumangal Pal, the Professor of Applied Mathematics Department, Vidyasagar University. Her B.Sc and M.Sc both were from Vidyasagar University with good marks. Dr. Khan has six articles of which two are in national and four in international journals. Her specialization is ***Computational Graph Theory***. Her interest for future research is in ***Fuzzy Game Theory, Fuzzy Graph Theory, Optimization.***

Dr. Khan is the author of the book "Colouring of Cactus Graphs", published by LAP LAMBERT Academic Publishing, Germany. She is the lifelong member of Operation Research Society (Kolkata Chapter). She is also a reviewer of Canadian Journal of Mathematics.

She has participated national and international seminars/ conferences/ winter school/ refresher courses in Indian as author and listener.

Address for communication:
Dr. Nasreen Khan
Applied Science and Humanities
Global Institute of Management and Technology
NH 34, Palpara More, Krishnagar-741102
West Bengal, India
Email: nasreen.khan10@gmail.com

Prof. Madhumangal Pal is currently a Professor of Applied Mathematics, Vidyasagar University, India. He has received **Gold and Silver medals** from Vidyasagar University for rank first and second in M.Sc. and B.Sc. examinations respectively. Also he received jointly with Prof. G.P.Bhattacherjee, "**Computer Division Medal**" from Institute of Engineers (India) in 1996 for best research work. He also received "**Bharat Jyoti Award**" in 2012. Prof. Pal has successfully guided 13 research scholars for Ph.D. degrees and has published more than 125 articles in international and national journals. His ***specializations include Algorithmic Graph Theory, Fuzzy Correlation & Regression, Fuzzy Game Theory, Fuzzy Matrices, Genetic Algorithms and Parallel Algorithms, Fuzzy Graph Theory.***

Prof. Pal is the Editor-in-Chief of "Journal of Physical Sciences" and "Annals of Pure and Applied Mathematics", and member of the editorial Boards of the journals "International Journal of Fuzzy Systems & Rough Systems", "International Journal of Computer Science, Systems Engineering & Information Technology", "Advanced Modelling and Optimization", "International Journal of Logic and Computation", "ISRN Discrete Mathematics" and "International Journal of Engineering Science, Advanced Computing and Bio-Technology". He is also a reviewer of several international journals.

Prof. Pal is the author of the books "Fortran 77 with Numerical and Statistical Analysis" published by Asian Books, New Delhi, "Numerical Analysis for Scientists and Engineers" and "Classical Mechanics" published by Narosa New Delhi and Alpha Sciences, Oxford, U.K., "Engineering Mathematics –Vol. I & II", "Advanced Algebra", PHI Learning, New Delhi, "C Programs including Numerical and Statistical Methods", Narosa.

He has delivered invited talks and chaired in national and international seminars/ conferences/ winter school/ refresher courses in Indian and Abroad.

Address for communication:
Professor Dr. Madhumangal Pal
Department of Applied Mathematics
Vidyasagar University
Midnapore-721102
West Bengal, India
Email: mmpalvu@gmail.com
URL: http://vidyasagar.ac.in/dept_of_mathematics/MMP.pdf
Mobile: (+91)9932218937/ 9832192207