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MHD Flow and Heat Transfer a Nanofluid over a Permeable Stretching Sheet

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Abstract*— In this paper, Magneto hydrodynamic steady flow and heat transfer of an incompressible nanofluid over a nonlinearly stretching and permeable sheet is investigated. Slipping condition of fluid flow at the surface was considered and fluid flow was only due to surface motion. Surface temperature assumed to be constant. Partial differential equations (PDE) of momentum and energy equations were transformed to ordinary differential equations (ODE) using similarity method and suitable dimensionless similarity parameters. Obtained ODE's were solved numerically using shooting method for boundary value problems. Dimensionless velocity and temperature distributions and gradients have been illustrated as a function of various boundary conditions parameters and nanofluid volume fraction. Influence of nanoparticles volume fraction on Nusselt number and heat transfer has been discussed and compared to the state that regular base fluid was applied. This is observed that, Nusselt number and heat transfer increase due to nanoparticles in the base fluid. Also, suction at the surface escalates temperature gradient at the surface and heat transfer between nanofluid and stretching surface would rise. On the other hand, blowing decreases heat transfer at the surface. The present results are compared with some reported theoretical results by other investigators and good agreement is found.

***Index Terms*— Stretching sheet, Permeable, Nanofluid, Nanoparticle volume fraction, Fluid slip.**

I. INTRODUCTION

Mass and heat transfer over a stretching sheet is applied in many industrial and engineering branches. The most important applications of flow over a stretching surface are in thermal processing of sheet-like materials that is a necessary operation in production of paper, polymeric sheets extrusion, insulating materials, and fine-fiber mattes [1]. Aerodynamic extrusion of plastic sheet, cooling of an infinite metallic plate in a cooling Bath, drying of papers is thermal application of moving sheets. The rate of stretching and cooling has an important influence on the quality and characteristics of the final product [2]. Flow in the presence of magnetic field (MHD flow) in micro channels makes a significant role in relation between certain problems of the movement of physiological liquid fluid [3]. In recent decades, nanofluids are used instead of conventional fluid to enhance convective heat transfer, because the thermal conductivity of nanofluids is higher than regular base fluids [4]. In fact, nanofluids contain nanometer sized metals or oxide metals particles that dispersed in the base fluid. Behavior of nanoparticles in the base fluid is similar to fluid behavior [5]. Several investigations have been done by researchers regarding mass and heat transfer over a stretching sheet. Regular fluid flow and heat transfer over an impermeable linearly stretching surface in the absence of fluid slip and magnetic field was studied [6]. Viscous dissipation term in energy equation was considered in this study, and surface temperature was assumed to be linear. MHD flow and heat transfer of a regular fluid over a stretching sheet with the presence of chemical reaction solved [7]. Momentum and energy equations for a nanofluid over a linearly impermeable stretching surface in the absence of slip and magnetic field were studied [8]. Nanofluid flow and heat transfer over an unsteady stretching surface has been presented [9]. Flow and mass transfer of nanufluid over a stretching permeable sheet without fluid slip has been investigated [10]. Viscous dissipation term in energy equation was considered in this study. A stretching sheet with rising temperature of sheet that is linearly stretched was also investigated [11]. MHD flow and heat transfer over an exponentially stretching permeable sheet was presented [12]. The infinity temperature (temperature beyond boundary layer) was assumed to be variable in this study. Nanofluid flow and heat transfer over a shrinking or stretching surface with nonzero velocity out of boundary layer was researched [13]. Energy and momentum equations for flow and heat transfer of a regular fluid over a permeable surface with constant velocity in the absence of fluid slip were also solved [14]. Velocity of fluid beyond boundary layer is not zero in this Study. Fluid flow equations over a permeable stretching surface with the presence of magnetic field and fluid slip was solved analytically [15]. In this work, Magneto hydrodynamic steady flow and heat transfer of an incompressible nanofluid over a nonlinearly stretching and permeable sheet is investigated. Slipping condition of fluid flow at the surface is considered and fluid flow is only due to surface motion. Finally, the present results are compared with some reported theoretical results by other investigators and good agreement is found.

II. ANALYSIS

A. Model description

As is illustrated in figure1 a stretching flat sheet is moving along the horizontal direction with nonlinear or linear velocity of $u_w(x)$. The surface is permeable and suction or blowing occurs at the surface in vertical direction. A magnetic field as a body force is exerted in the vertical direction. Velocity of

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fluid, out of boundary layer is zero and surface is isothermal. Fluid slip at the surface is considered.

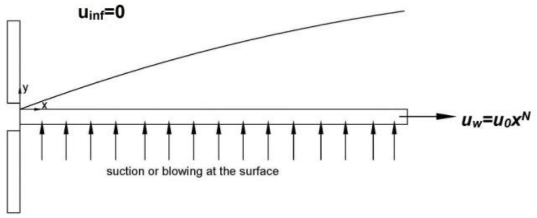

Fig 1. Schematic of permeable stretching surface

The first step for obtaining velocity and temperature distribution is writing Navier-stocks and energy equations and determining boundary conditions. x coordinate is directed at the stretching side and y coordinate is perpendicular to the stretching surface. Also, u and v are velocity components in the x and y directions, respectively. Two equal and opposite direction forces are applied on the sheet to keep the origin fixed. Velocity of surface is as follow:

$$u_w(x)=u_0x^N \tag{1}$$

u_0 is a constant and N called nonlinear parameter. Fluid velocity at the surface is $u_f=u_w+u_s$. Also, u_s is slipping velocity of nanofluid at the surface. Fluid blowing or suction at the surface due to its permeability is considered ($v_w=v_0$). A magnetic field with strength B(x) is applied in the vertical direction as follow:

$$B(x)=B_0x^{(N-1)/2} \tag{2}$$

B_0 is a constant. The basic equations are given as follow:

$$\partial u/\partial x+\partial v/\partial y=0 \tag{3}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{nf}\frac{\partial^2 u}{\partial y^2}-\sigma_{nf}B^2\frac{u}{\rho} \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

σ_{nf} is electrical conductivity of nanufluid. The boundary conditions are as follow:

$$At\quad y=0: u=u_w+u_s,\ \ v=v_w,\ \ T=T_s \quad \text{and}$$
$$At\quad y\to\infty: u=0,\ \ T=T_\infty \tag{6}$$

μ_{nf}, ρ_{nf}, α_{nf} are the properties of nanofluid. Thermo physical properties of nanofluid obtained using the following equations [16]:

$$\frac{\mu_{nf}}{\mu_f}=\frac{1}{(1-\varphi)^{2.5}}$$
$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s \tag{7}$$
$$(\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\varphi(\rho c_p)_s$$
$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)}$$

φ is volume fraction of nanoparticles in the fluid. Subscripts f and s are used for base fluid and nanoparticles respectively. In this case, water as base fluid with nonoparticles of copper oxide is studied. The properties of water with various metals and oxide metals are given in table1 [17].

Table1. Thermophisical properties of water and nanoparticles

	$\rho(kg/m^3)$	$c_p(j/kg.K)$	$k(w/m.K)$
Copper(cu)	8933	385	401
Copper oxide(cuo)	6320	531.8	76.5
Alumina(Al2O3)	3970	765	40
Titanium Oxide	4250	686.2	8.9538
Pure water	997.1	4179	0.6130

B. *Mathematical Method*

It is observed that equations (4) and (5) are PDE and these equations are not homogenous. For solving these equations, similarity method is used. The following suitable similarity parameters are used.

$$f'(\eta)=\frac{u}{u_w}=\frac{u}{u_0x^N}$$
$$\eta=y\sqrt{\frac{u_0(N+1)}{2\nu_f}}x^{(N-1)/2}$$
$$v=-\sqrt{\frac{u_0(N+1)}{2\nu_f}}x^{(N-1)/2}\left[f+\frac{N-1}{N+1}\eta f'\right]$$
$$\theta(\eta)=(T-T_\infty)/(T_w-T_\infty) \tag{8}$$

ν_f is kinetic viscosity of water (base fluid). Substituting equations (7) and (8) in equations (4) and (5) using chain derivative rule, the equations (4) and (5) are transformed to the following ODE equations:

$$\frac{\mu_{nf}}{\mu_f}f'''+\left[(1-\varphi)+\frac{\varphi\rho_s}{\rho_f}\right]\left[ff''-\frac{2n}{n+1}f'^2\right] \tag{9}$$
$$-(1-\varphi)^{2.5}\frac{2M^2}{N+1}f'=0$$

$$\theta''+\frac{k_f}{k_{nf}}\left[1-\varphi+\varphi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]\Pr\theta' f=0 \tag{10}$$

Pr is nanofluid prandtl number and M is defined as follow:

$$M^2=\sigma_{nf}B_0/\rho u_0 \tag{11}$$

Boundary conditions of equations (9) and (10) are as

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follow:

$$\eta = 0 : f'(0) = 1 + Kf''(0), \quad f(0) = f_w, \ \theta(0) = 1$$
$$\eta \to \infty : f'(\infty) = 0, \quad \theta(\infty) = 0 \qquad (12)$$

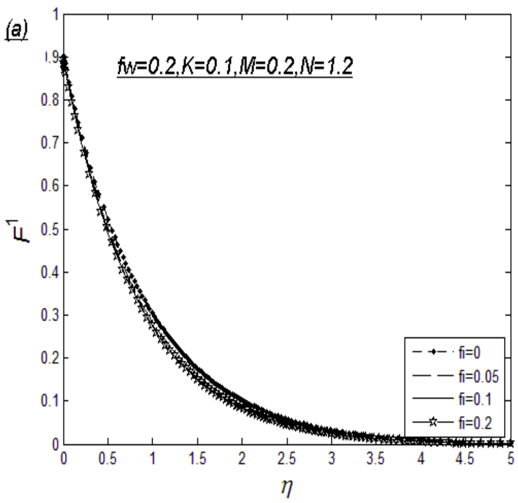
K is slipping coefficient and f_w is nonzero for permeable surface and defended as:

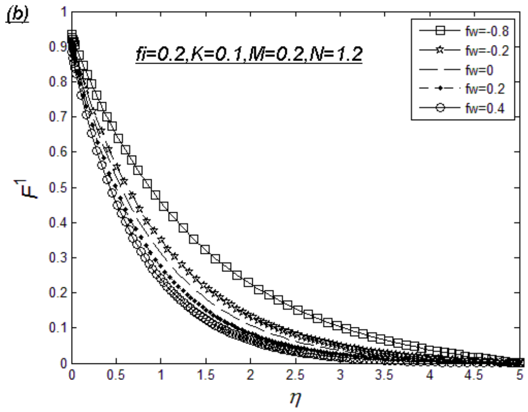
$$K = \frac{L}{x}\sqrt{\frac{u_w(N+1)}{2\nu_\infty}}, \qquad f_w = \frac{-xv_w}{\sqrt{\frac{u_0 xN\nu_\infty(N+1)}{2}}} \qquad (13)$$

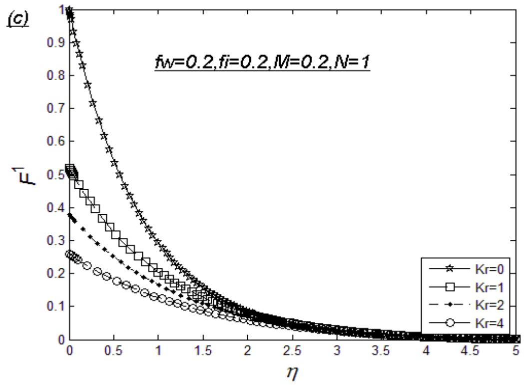
f_w is positive for suction and negative for blowing. Equations (9) and (10) with boundary conditions (12) are solved numerically using shooting method in matlab software. Obtained graphs and results discussed in the next section.

III. RESULTS AND DISCUSSION

Diagrams of dimensionless velocity and its gradient are illustrated in figures 2, 3 and 4 for all boundary conditions parameters. Figures 5, 6, 7 and 8 illustrate dimensionless temperature and its gradient as a function of η for various parameters. As is shown in figure 2(a), influence of nanoparticles on the velocity, especially at the surface is not considerable and can be neglected. Figure 2(b) interprets effect of suction or blowing at the surface on velocity. Velocity variation inside boundary layer is considerable due to permeability. Blowing at the surface, decreases fluid velocity in the boundary layer of flow. However, suction increases velocity of fluid. Influence of parameters variation on the velocity gradient for various volume fractions and permeability is illustrated in figure 3(a-b). As figure 3(a), nanoparticle volume fraction leads to velocity gradient rises. Suction increases velocity gradient at the surface as figure 3(b). However, blowing leads to velocity gradient drops at the surface.







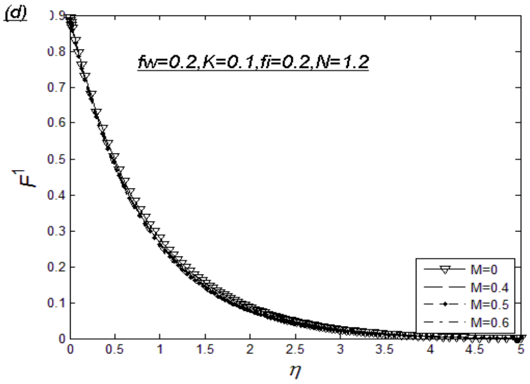
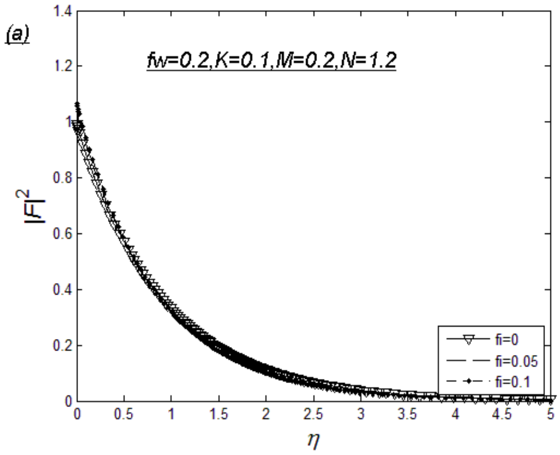


Fig 2. Dimensionless velocity (a-d) for various B.C parameters and φ as a function of η.



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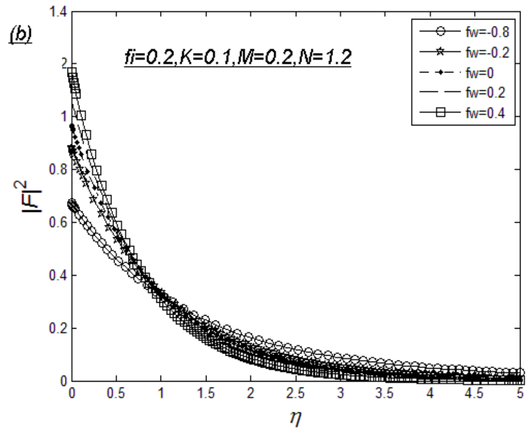


Fig 3. Dimensionless velocity gradient (a-b) for various values of φ and f_w as a function of η

Figure 4(a-b) illustrates Impacts of slip at the surface and magnetic field on the velocity gradient at the surface. Fluid slip at the surface has considerable effect on the velocity gradient. On the other hand, magnetic field has slight effect on the velocity gradient.

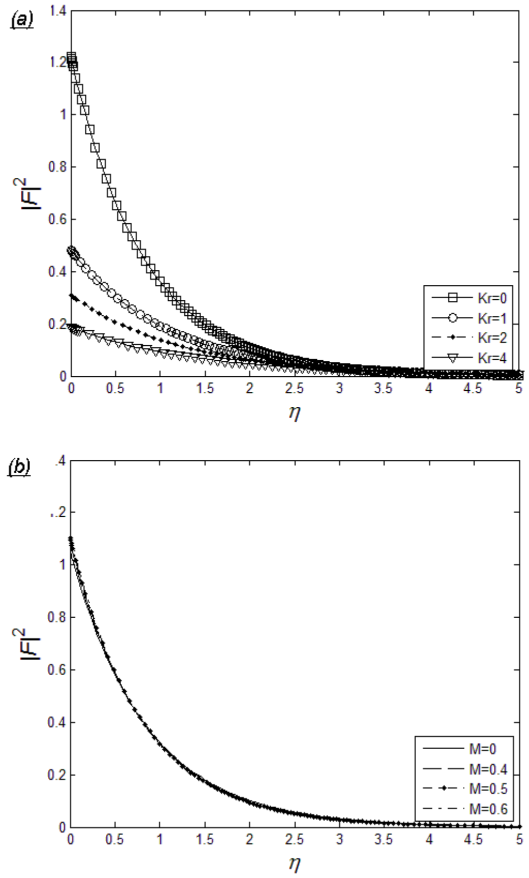


Fig 4. Dimensionless velocity gradient (a-b) for various values of K and M as a function of η

Temperature gradient is shown in figures 5 and 6 as a function of η. Study of Temperature gradient at the surface (η=0) is important, because heat transfer is directly influenced by temperature gradient at the surface. As is illustrated in figure 5(a), presence of nanoparticles in the base fluid, significantly impacts the temperature gradient at the surface (θ′(0)). Rise of nanoparticles volume fraction leads to temperature gradient drops at the surface. Figure 5(b) proves high impact of permeability on the temperature gradient. Suction (positive values of f_w), rises temperature gradient. However, blowing descends temperature gradient at the surface. Influence of fluid slip at the surface is clarified by figure 6(a). Effect of fluid slip on the temperature gradient is considerable. As figure 6(b), Magnetic field has very slight impact on the temperature gradient and can be neglected.

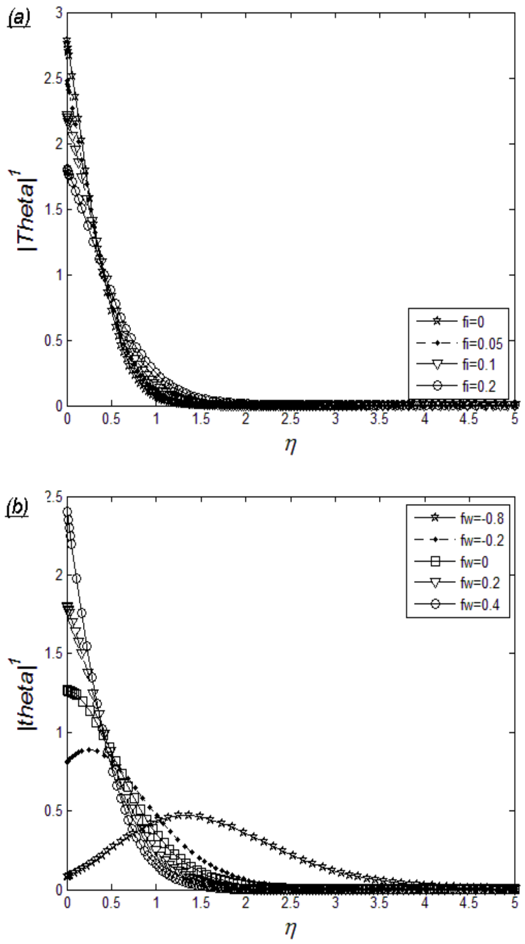
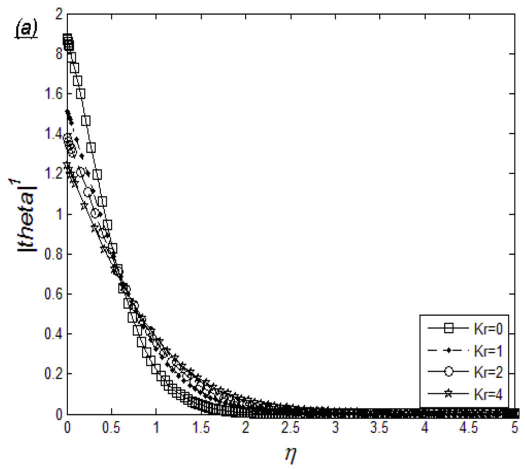


Fig 5. Dimensionless temperature gradient for various values of φ and f_w as a function of η



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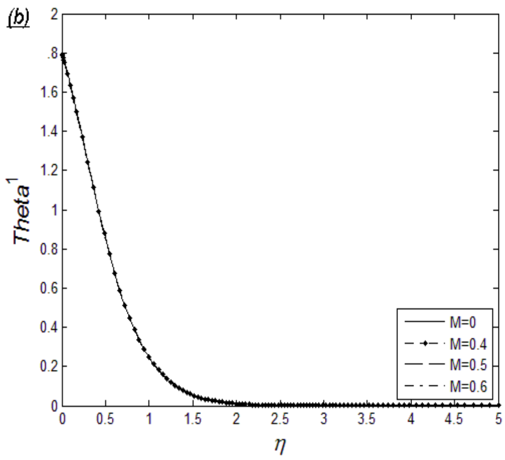

Fig 6. Dimensionless temperature gradient for various values of K and M as a function of η

Figure 7(a) proves that nanofluid temperature profile is thicker than base fluid one. Temperature profile becomes thicker, with rise of nanoparticles volume fraction. As Figure 7(b), blowing makes temperature profile, thicker. However, suction makes it thinner. Impact of fluid slip on the temperature profile is similar to blowing at the surface as figure 8(a). Magnetic field as is illustrated in figure 8(b) does not lead to any considerable change in the temperature profile.

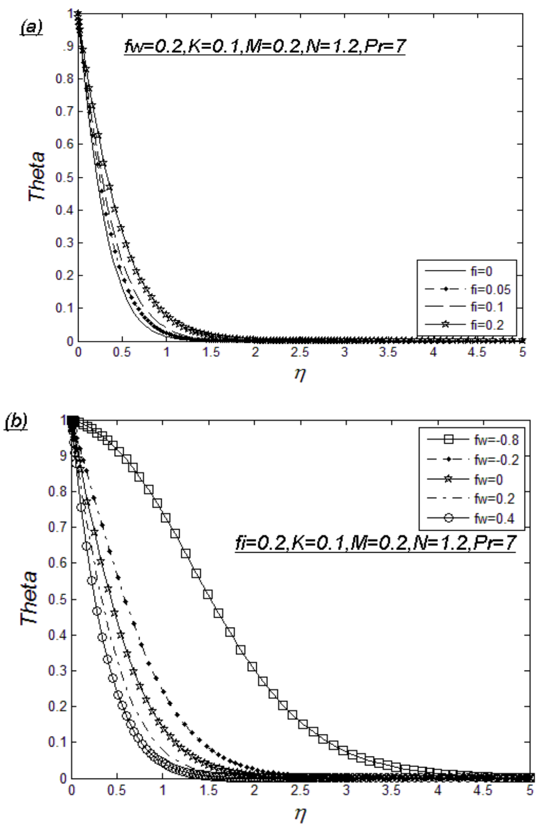

Fig 7. Dimensionless temperature for various values of φ and f_w as a function of η

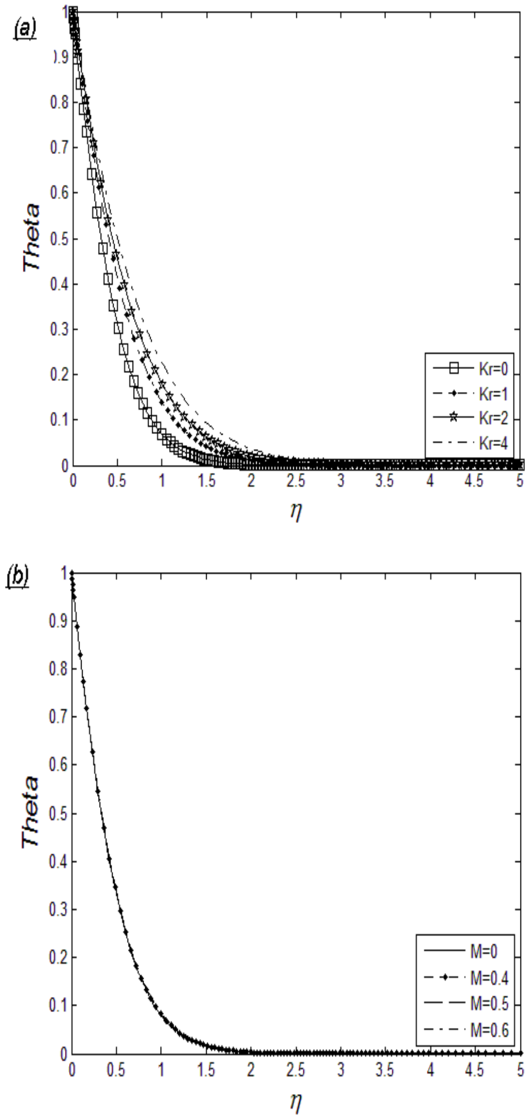

Fig8. Dimensionless temperature for various values of K and M as a function of η

Nusselt number and convection coefficient are obtained from the following equation:

$$Nu_x = hx / k_f, \qquad h_x = \frac{-k_{nf}\,\partial T / \partial x}{(T_w - T_\infty)} \tag{14}$$

Using similarity parameter (η) and chain derivative rule, Nu_x is obtained as follow:

$$Nu_x = -\frac{k_{nf}}{k_f}\theta'(0)\sqrt{\frac{u_0(N+1)}{2\nu_\infty}}\,x^{\frac{N+1}{2}} \tag{15}$$

Nu_x is shown in figure 9 as a function of x. this graph illustrates enhancement of Nusselt with rise in volume fraction of nanoparticles. This graph proves that, rise of thermal conductivity of nanofluid due to nanoparticles, prevails over temperature gradient drop due to nanoparticles in the base fluid. In Fact, the main purpose of using nanofluid is proven by this graph.

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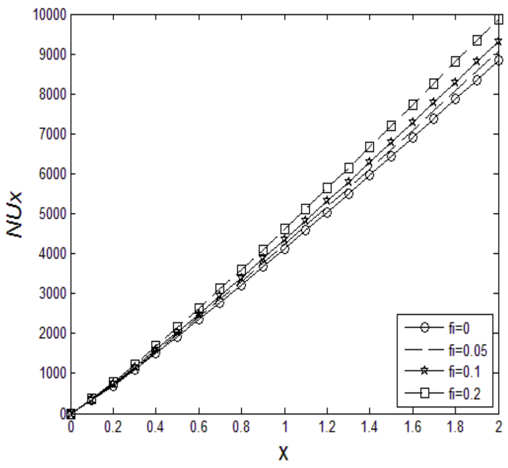

Fig 9. Nusselt number as a function of x for various volume fraction of nanofluid (Pr=7,u_0=2)

Results of this paper for -θ′(0) is compared to some other investigations and is presented in Table 2.

Pr	N	f_w	K	M	φ	Cortell[6]	Present Results
1	0.2	0	0	0	0	0.610	0.613
1	0.5	0	0	0	0	0.595	0.599
5	1.5	0	0	0	0	1.557	1.557
3	1	0	0	0	0	1.164	1.165
7	0.5	0.2	0.5	0.3	0		2.567
6.2	1	0	0.1	0	0.1		1.402
6.2	1	0.5	0	0	0.2		2.365

IV. CONCLUSION

Flow and heat transfer over a permeable MHD stretching surface was presented. Velocity and temperature distribution in the flow and thermal boundary layers studied. Numerical results prove that nanofluid leads to drop of temperature gradient at the surface. However, nanoparticles increased thermal conductivity of fluid. Rise of thermal conductivity prevails over the temperature gradient drop. Consequently, Nusselt number and heat transfer increase due to nanoparticles in the base fluid. Suction at the surface escalates temperature gradient at the surface and heat transfer between nanofluid and stretching surface would rise. On the other hand, blowing decreases heat transfer at the surface. Consequently, convective heat transfer would drop. Fluid slip at the surface significantly descends Nusselt number and convective heat transfer at the surface. Impact of other boundary conditions parameters were illustrated in details.

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